Design Guideline for Continuous Beams Supporting Steel Joist Roof Structures

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Summary

Cantilever-suspended span construction has historically been an economical way of designing roof structures. Research conducted recently has shown that traditional methods of analysis may be either overly conservative or non-conservative. Also, tests conducted on the bracing of overhanging beams has produced more insight as to the effectiveness an optimum locations of braces.

Based on recent research, this paper presents recommendations for bracing requirements and also a new method of calculating the critical lateral-torsional buckling moment for overhanging beams. This method is more consistent with the LRFD specification and yields more accurate results than traditional methods.

DESIGN GUIDELINES FOR CONTINUOUS BEAMS SUPPORTING STEEL JOIST ROOF STRUCTURES

James Rongoe, P.E.

INTRODUCTION

In the design of roof structures, it is common practice to have girders of alternate bays run continuously over columns which support the girders in the other bays, the span of which is shortened by the length of the cantilevers (Figure 1). This type of framing is commonly referred to as a cantilevered-suspended span construction, which was originally developed by Gerber approximately a century ago (Ref. 3). By properly proportioning the cantilever span to the back span, an even distribution of the load can be achieved to minimize positive moments by balancing them with negative moments. This system, being statically determinate, can be readily evaluated and produces efficient design compared to a system consisting of simple spans resulting in lighter, shallower girders. Other advantages of this system are that the beam to column connections are simpler, it is faster to erect and deflections are minimized.

This paper addresses a typical roof structure design, in which a primary girder which supports open web steel joists, which in turn support a metal roof deck. Other applications of continuous beams such as in stub girder systems or crane girders, although similar, have unique design considerations and therefore the recommendations presented are restricted to roof girders and are applicable only to rolled W sections, and do not apply to plate girders or truss girders.

TRADITIONAL METHODS OF ANALYSIS

In order to balance the positive and negative moments in an efficient manner, the optimum location of the hinge between the suspended span and cantilever portion must be determined. The hinge is achieved by considering the suspended girder as a simple span which is pinned to cantilever by means of a simple connection. A connection which has partial or full fixity is not desired and should be avoided since this can produce additional negative moments in the cantilever which it is not designed for. Typical values for optimum hinge locations range from L_C/L from 0.167 to 0.25. On page 4-205 of the AISC, LRFD Manual, optimum hinge locations for various loads are given (Figure 2).

Since the girders produce negative moments over the column, these locations must be carefully examined and, because the girder runs continuous over the column, the web should be checked for crippling and yielding. In the case where there is no joist or brace at the column, special attention must be given to the column/girder joint to avoid compromising its overall stability (Ref. 1).

Traditional methods of analysis used in the past to evaluate the capacity of a continuous girder (Figure 3) involved the following:

1) The moment capacity of the cantilever portion in which the bottom flange in compression was determined by assigning an effective length factor, K, to the cantilevered compression flange. The K values varied according to how the cantilever was braced and the position of the load.

2) The back span moment capacity was determined assuming the point of inflection as a brace point for the backspan. Use of these criteria can lead to designs which are either overly conservative or non-conservative (Ref. 4), as explained as follows.

The inaccuracy of assigning "K" values to the unbraced compression flange is due, in part, to the fact that these "K" values are based on a single span cantilever model which is fixed at its root. This model differs from that of an overhanging beam in which buckling is characterized by twisting of its backspan rather than warping. A fixed cantilever, for example, displays a greater displacement at the top flange than at its bottom when it buckles, whereas on an overhanging beam the opposite occurs, i.e., the bottom flange displaces more than the top flange (Ref. 1). Tests have shown that the built-in cantilever model can overestimate the buckling resistance of overhanging beams (Ref. 4). Typical "K" values listed in design guides range from 0.6L, to 7.5L, depending upon restraint conditions at the tip and root. These values do not consider effects of the backspan. It has been recommended, however, that for overhanging beams the effective length of the cantilever should be taken as not less than the length of the backspan (Ref. 5).

Another common design assumption is that the bottom flange is unbraced between the end support and the flexural point of inflection. This is also not accurate since (1) the flexural point of inflection is not equivalent to a brace; and (2) when a girder has its top flange braced, for example by steel joists, there is a certain amount of restraint provided to the bottom compression flange through the distortional stiffness of the web. Even though both of these assumptions are not completely accurate, they tend to compensate for each other's inaccuracies. These design methods can lead to predicted strength that bears little relationship to the actual strength of the beam (Ref. 1).

Until recently, little information pertaining to the buckling strength of continuous overhanging beams was available. In 1988, in British Columbia, Canada, a parking roof over a supermarket designed with cantilevered-suspended span construction collapsed. As a result, questions were raised concerning the adequacy of bracing design for this type of construction (Ref. 4). Since then, considerable research and testing of continuous overhanging beams has been conducted, such as that sponsored by the Natural Sciences and Engineering Research Council of Canada and the Canadian Institute of Steel Construction.

This research and full scale testing focused specifically on overhanging beams restrained by steel joists. From this work, the adequacy and efficiency of bracing and position of loads was evaluated and a procedure has been developed to determine the elastic critical buckling moment, M_{CR}, for overhanging beams. By being able to determine that the beam is adequately braced and will not buckle, the beam will be able to develop its full moment resistance. These new guidelines, which enable the designer to determine with a greater degree of accuracy the strength and stability characteristics of an overhanging beam, will in turn result in better and more efficient designs.

BRACING

If a beam is adequately braced, it will be able to reach its full capacity without prematurely buckling. Since an overhanging beam has a portion of its bottom flange in compression, loading patterns and bracing requirements are more complex than that of a simple span beam. Tests have been conducted on W-sections to evaluate the effects of bracing on overhanging beams (Ref. 1, 5) and have resulted in new insights as to the beam's behavior under various load conditions. Full scale tests have also been modeled to assess the effectiveness of joists as braces for both the top and bottom compression flanges.

Two types of braces, lateral which prevents translational movement, and torsional, which prevents rotational movement, were studied in these tests (Figure 4). Also, various brace locations were tested to determine where the braces are most effective. One finding, which is not surprising, is that at the column where the beam is continuous, lateral bracing provided to both the top and bottom flange, where compression is the greatest, maintained stability (Ref. 1, 4, 5, 6). Tests have shown that by allowing the bottom flange to translate at the column, the load at failure can decrease as much as 70% (Ref. 4).

When it is not practical to extend the joist bottom chord to laterally brace the beam at the top of the column, it is recommended that the column continuity be extended to the top of the beam by providing a properly designed moment connection between the beam and column, and providing web stiffeners (or pairs of them) which provide enough stiffness to prevent translation at the beam column connection (Ref. 1, 4). It is important to note that torsional restraint at the column-beam joint, without the benefit of web stiffeners or lateral bracing, results in diminished moment resistance, in some cases over 50% (Ref. 1). When designing for column continuity by using a rigid connection and web stiffeners, it is recommended that the unbraced length of the column be extended from the top of the column to the top of the beam (Ref. 3) (Figure 5).

Another strategic brace location which enhances the beam stability and strength is at the tip of the cantilever. Tests have shown that if the cantilever tip is torsionally restrained, the moment resistance can increase significantly (Ref. 1, 4). For beams which are braced at the column, "the additional conditions that most enhance the beam stability are joists with bottom chord extensions to provide lateral support to the bottom flange of the beam at each cantilever tip" (Ref. 6).

Another important finding confirmed by full scale tests is that standard open web steel joists can be effective to restrain not only the top flange but also through the distortional stiffness of the web can provide restraint to the bottom flange. "When the portion of the bottom flange, which is in compression, attempts to deflect sideways, it is restrained by the lateral bending stiffness of the web which is anchored to the torsionally restrained top flange" (Ref. 6) (Figure 6). There is evidence that the standard welded joist shoe connection to the W-section and the flexural stiffness of the joist can provide not only lateral but also torsional restraint of the beam (Ref. 1, 4). It has been demonstrated both in full scale tests and finite element programs that, for a beam which is laterally braced at the column, the restraint provided by the joists at the cantilever tip and backspan is sufficient to stabilize the beam and is often sufficient to allow the beam to reach its full moment resistance (Ref. 1).

The degree of fixity of the restraint provided by the joists is proportional to such parameters as the flexibility and spacing of the joist, how they are connected to the beam, and the beam geometry. Essa and Kennedy (Ref. 4) developed a spring model defining the effective distributed torsional stiffness, K_e, delivered to the bottom flange as:

$$\frac{1}{K_e} = \frac{1}{K_b} + \frac{1}{K_c} + \frac{1}{K_f} + \frac{1}{K_w}$$

where:

K_b = in-plane bending stiffness of the joist, K_j, divided by the bracing spacing

K_c = connection stiffness divided by the bracing spacing

K_f = torsional stiffness of the braced flange between bracing points

K_w = bending stiffness of the web

K_j = $\frac{EI_X}{L_p}$ where E = modulus of elasticity
I_X = moment of inertia of the top chord of the joist
L_p = horizontal distance from the reaction to the first lower panel point

For K_j, a conservative value of 2.7×10^2 in. lb/rad per joist can be used if no data (for the joist properties) are available (Ref. 6). For a standard welded connection of the joist to the beam using 3/16" minimum welds, Milner (Ref. 12) recommended an infinite value of K_c, based on experimental results, therefore $1/K_c$ would equal 0.

The torsional stiffness of the flange, K_f, is given by Essa and Kennedy (Ref. 4) as:

$$K_f = \frac{7.3\, G\, b_f\, T_f^3}{L_b^2}$$

where G = modulus of rigidity
b_f = flange width
T_f = flange thickness
L_b = bracing spacing

The bending stiffness of the web, K_w, is given by Svensson (1985) as:

$$K_w = \frac{E\, T_w^3}{4(1-u^2)d}$$

where T_w = web thickness
u = Poisson's ratio
d = depth of the section

The position of the load above the shear center also has an effect on the stability of the beam. The higher the application of the load above the shear center of the beam (top flange loading), the greater the destabilizing effect. In tests performed by Albert, Essa and Kennedy (Ref. 1), loads were applied at or above the top flange which represents the more severe loading condition, and it was confirmed that the elevated loads diminished the capacity of the beam. It is therefore beneficial to transfer the load of the suspended span beam to the cantilever tip through a connection near the center of the beam.

The live load pattern is also an important consideration when determining the capacity of a continuous beam. For example, partial or no live load on the backspan produces minimum positive moments, thus resulting in a greater length of the bottom flange in compression. In this situation, the beam could possess a lower buckling strength than when under full load. Likewise, full live load on the backspan with partial or no live load on the adjacent spans produces maximum positive moment for which the beam must be sized accordingly.

SUMMARY

1. To maintain stability, it is important to brace the top and bottom flange of the beam laterally at the column where the beam is continuous.

2. If the beam-column connection where the beam is continuous cannot be braced laterally, it is recommended that column continuity be achieved through a rigid beam-column connection and web stiffeners of sufficient rigidity. Torsional restraint only at the beam-column connection results in diminished beam capacity.

3. Standard open web steel joists welded to the top flange provides restraint of the bottom compression flange, the degree of which varies dependent upon parameters such as the beam geometry.

4. Bracing the top flange of the cantilever tip increases the buckling resistance. Also, bracing the bottom flange at the tip may provide additional increase to the beam capacity.

5. The effect of load patterns must be considered when designing continuous overhanging beams.

LATERAL-TORSIONAL BUCKLING RESISTANCE

A primary concern in designing continuous overhanging beams is that the beam is capable of reaching its potential moment resistance without premature buckling occurring. Prior methods used to calculate buckling resistance were based on the wrong models, i.e. fixed cantilevers, and neglected such factors as torsional restraint of the bottom flange due to the web and height of the load application and were based upon incorrect assumptions regarding unbraced lengths. The results of these methods can therefore be inaccurate, either overly or non-conservative.

A procedure developed by Essa and Kennedy (Ref. 6) has been developed which can directly calculate the elastic buckling resistance, M_{CR}, of the beam. This method involves factors which affect the buckling capacity of the continuous beam which prior methods did not consider, such as restraint provided by joists and the distorsional stiffness of the beam, the length of the backspan and the loading conditions. The advantage of this procedure is that it allows us is to directly determine whether the beam can achieve its moment resistance without prior failure due to buckling. In contrast, traditional methods indirectly calculated the beam's capacity by assumed values for unbraced lengths and effective length factors for the cantilever. This new procedure is based on a finite element model and has been corroborated by full scale tests (Ref. 6). The formula for the critical moment is based on the fact that the buckling value for an overhanging beam is due predominantly to its torsional resistance as given in a formula based on Lindner (Ref. 11) as:

$$M_{CR} = \frac{K}{L}\sqrt{E\,I_Y\,G\,J^*}$$

where:

- K = a buckling coefficient
- L = length of the backspan
- E = modulus of elasticity
- I_Y = moment of inertia about the weak axis
- J^* = modified torsional constant

$$= J + \frac{K_e L^2}{\pi^2 G}$$

where: K_e = effective distributed torsional stiffness as previously defined.

The value of the buckling coefficient, K, varies dependent upon how the beam is braced, its boundary conditions, the ratio of the cantilever to the backspan, the load pattern expressed in terms of the moment ratio, R, and the beam geometry as defined by the torsional parameter:

$$X = \frac{\pi}{L}\sqrt{\frac{E\,C_w}{G\,J}}$$

Design curves developed by Essa and Kennedy (Ref. 6) for the determination of the buckling coefficient, K, are included in the Appendix. These curves apply for most practical loading situations and cantilever to backspan ratios.

The moment ratio parameter, R, used in the curves, is defined as the maximum static backspan moment, $\frac{WL^2}{8}$ to the maximum end moment or

$$R = \frac{|M^-_{max}|}{M^+_{max} + |M^-_{max}|}$$

For beams which are laterally braced at the column, the critical loading condition occurs when the negative moment at the column is maximum and the positive backspan moment is minimum, having the largest portion of the bottom flange in compression. Thus, the loading condition with partial live load on the backspan and maximum live loads in the suspended and cantilevered spans must be investigated.

Included are graphs for the situation where the tip of the cantilever is not braced by a bottom chord joist extension. In this case, the buckling coefficient, K, is independent of the $\frac{Lc}{L}$ ratio and therefore the graphs on figures 6 and 7 are in terms of the variables R, the moment ratio, and X, the torsional constant. When there is no joist at the tip of the cantilever, the buckling coefficient, K, is dependent upon the ratio of $\frac{Lc}{L}$ as well as R and X (figures 8-13). As indicated on these graphs the buckling coefficient, K, decreases as the ratio of the cantilever to backspan, $\frac{Lc}{L}$ increases. Note that in all cases the critical moment is the value of the maximum negative moment over the column support, even though the absolute value of the maximum positive moment may be greater (Ref. 6).

Also included in the curves in figures 14 and 15 are the case where there are no joists on the column line to brace the beam so that the only torsional restraint is through a rigid beam-column connection. In this case, two different loading patterns could be critical and must be checked: (1) with full load on all spans; and (2) with partial or no live load on the suspended and cantilever spans and full load on the backspan. In general, for similar loading conditions, beams laterally braced at the column lines have higher critical moment values than beams which do not have lateral restraints. Also, due to potential relative-lateral movement between the top and bottom flange at the column, sideways buckling of the web must also be considered (Ref. 2). Critical moments for beams without lateral bracing at the column can be less than one half of those which have lateral bracing (Ref. 1, 6). Also, since the stability of the beam in this case is dependent upon the rigid connection between the column and beam, the absence of lateral bracing at the beam-column connection is not recommended.

YURA METHOD FOR DETERMINATION OF M_{CR}

Another method to determine the critical moment of an overhanging continuous beam has been developed by Yura (Ref. 8). In this approach, the beam is analyzed in two segments, the overhanging portion and the backspan. Thus two values for the M_{CR} are calculated and the lower one would be the one at which level buckling would initiate.

For the overhanging portion M_{CR} is calculated using the following formula:

$$M_{CR} = C_b \frac{\pi}{L_b} \sqrt{EI_Y GJ}$$

where

$C_b = 1.0$ and L_b = length of the overhanging segment

This formula is based on AISC formula F1-13 without the warping term which in this case is not significant and can be neglected. For the backspan the M_{CR} is calculated using a coefficient of bending, C_b, which is based upon the moment gradient and applying it to AISC LRFD formula F1-13. For a continuous beam which supports steel joists, two C_b factors are as follows:

1. For a beam with the top flange braced continuously under downward (top flange) load:

$$C_b = 3.0 - \frac{2}{3}\left[\frac{M_1}{M_0}\right] - \frac{8}{3}\frac{M_C}{\left[M_0 + M_1'\right]}$$

Where

M_0 = largest negative moment at the continuous end

M_1 = moment at the other end

M_1' = M_1 unless M_1 is positive, in which case $M_1' = 0$

M_C = maximum positive moment or minimum negative moment in back span.

2. For a beam with the top flange braced continuously and under uplift or suction load:

When both end moments are positive or zero:

$$C_b = 2.0 - \left[\frac{M_0 + 0.6\,M_1}{M_C}\right]$$

When $\mathbf{M_O}$ is negative and $\mathbf{M_1}$ is positive or zero:

$$C_b = \frac{2\,M_1 - 2\,Mc + 0.165\,M_O}{0.5\,M_1 - M_c}$$

Where both end moments are negative:

$$C_b = 2.0 - \left[\frac{M_o + M_1}{M_c}\right]\left[0.165 + \frac{1}{3}\left(\frac{M_1}{Mo}\right)\right]$$

Comparison of values for $\mathbf{M_{CR}}$ calculated by the Kennedy and Yura methods are shown in Figure 7. In the first two examples when the backspan moment controls, the Yura values fall within the range of values calculated by the Kennedy method. In the third example, where the overhanging portion controls the Yura value for $\mathbf{M_{CR}}$ is slightly lower (more conservative) than the corresponding ones calculated by the Kennedy method.

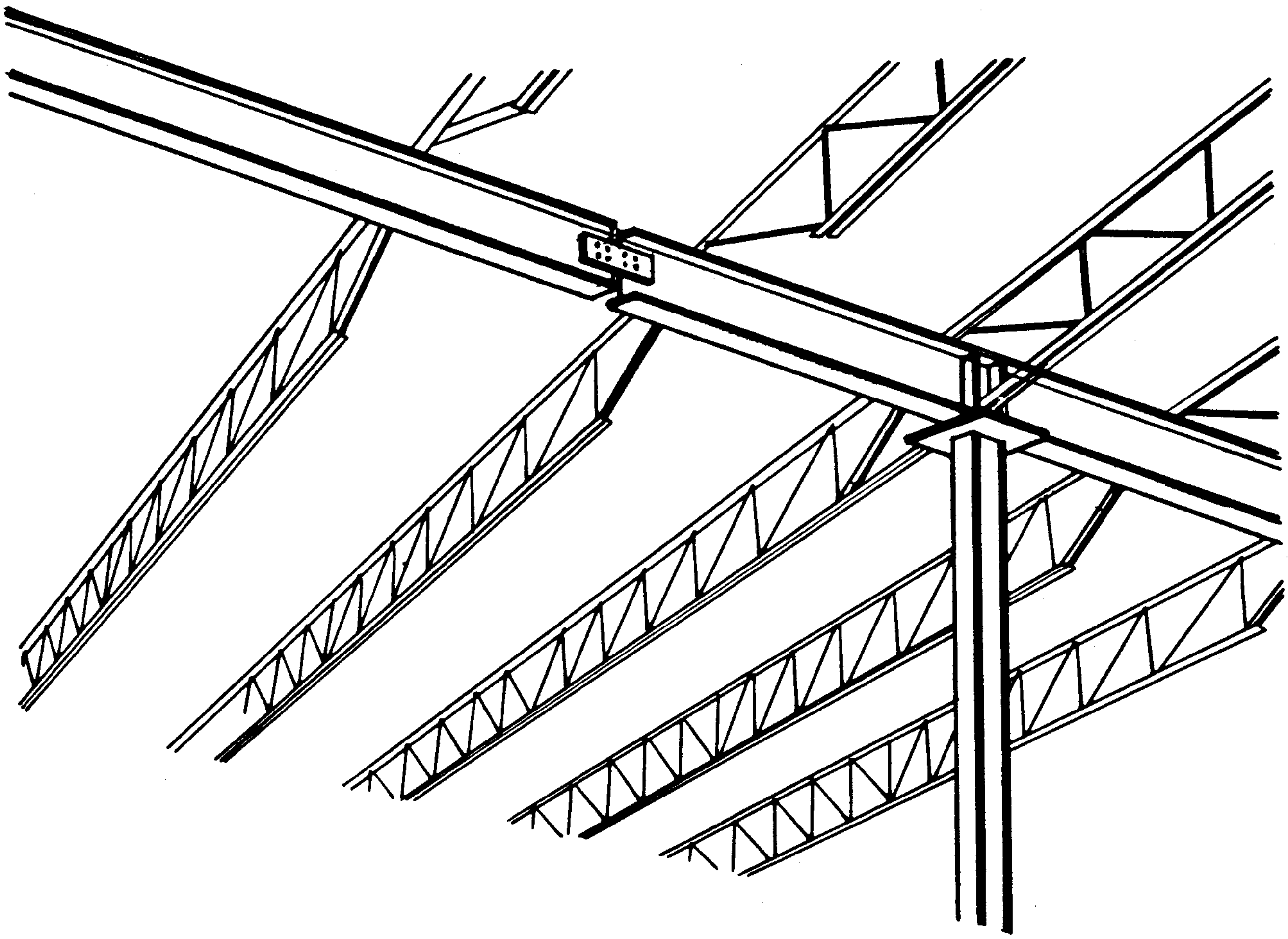
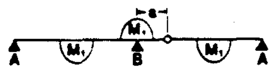
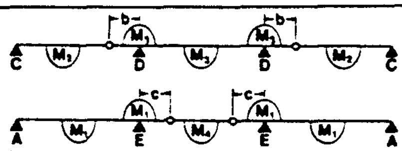
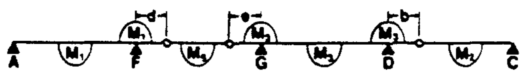
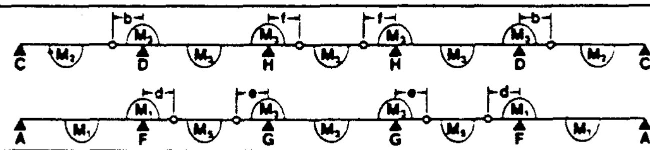
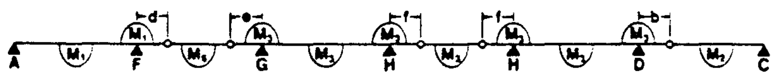
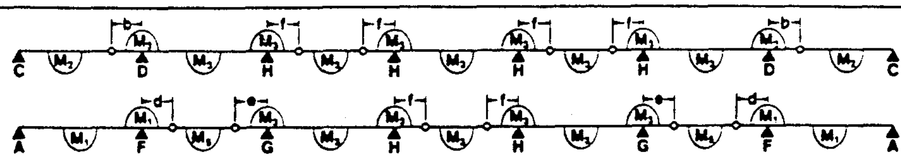


FIGURE 1

BEAM DIAGRAMS AND FORMULAS

Design properties of cantilevered beams

Equal loads, equally spaced

No. Spans	System
2	
3	
4	
5	
≥6 (even)	
≥7 (odd)	

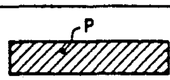
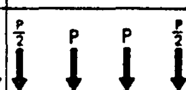
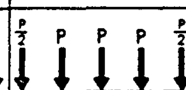
	n	∞	2	3	4	5
	Typical Span Loading					
Moments	M_1	0.086 x PL	0.167 x PL	0.250 x PL	0.333 x PL	0.429 x PL
	M_2	0.096 x PL	0.188 x PL	0.278 x PL	0.375 x PL	0.480 x PL
	M_3	0.063 x PL	0.125 x PL	0.167 x PL	0.250 x PL	0.300 x PL
	M_4	0.039 x PL	0.083 x PL	0.083 x PL	0.167 x PL	0.171 x PL
	M_5	0.051 x PL	0.104 x PL	0.139 x PL	0.208 x PL	0.249 x PL
Reactions	A	0.414 x P	0.833 x P	1.250 x P	1.667 x P	2.071 x P
	B	1.172 x P	2.333 x P	3.500 x P	4.667 x P	5.857 x P
	C	0.438 x P	0.875 x P	1.333 x P	1.750 x P	2.200 x P
	D	1.063 x P	2.125 x P	3.167 x P	4.250 x P	5.300 x P
	E	1.086 x P	2.167 x P	3.250 x P	4.333 x P	5.429 x P
	F	1.109 x P	2.208 x P	3.333 x P	4.417 x P	5.557 x P
	G	0.977 x P	1.958 x P	2.917 x P	3.917 x P	4.871 x P
	H	1.000 x P	2.000 x P	3.000 x P	4.000 x P	5.000 x P
Cantilever Dimensions	a	0.172 x L	0.250 x L	0.200 x L	0.182 x L	0.176 x L
	b	0.125 x L	0.200 x L	0.143 x L	0.143 x L	0.130 x L
	c	0.220 x L	0.333 x L	0.250 x L	0.222 x L	0.229 x L
	d	0.204 x L	0.308 x L	0.231 x L	0.211 x L	0.203 x L
	e	0.157 x L	0.273 x L	0.182 x L	0.176 x L	0.160 x L
	f	0.147 x L	0.250 x L	0.167 x L	0.167 x L	0.150 x L

FIGURE 2

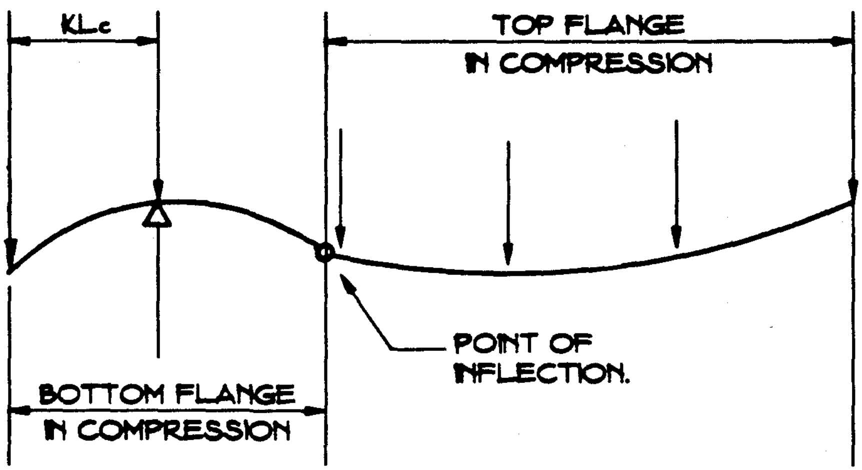
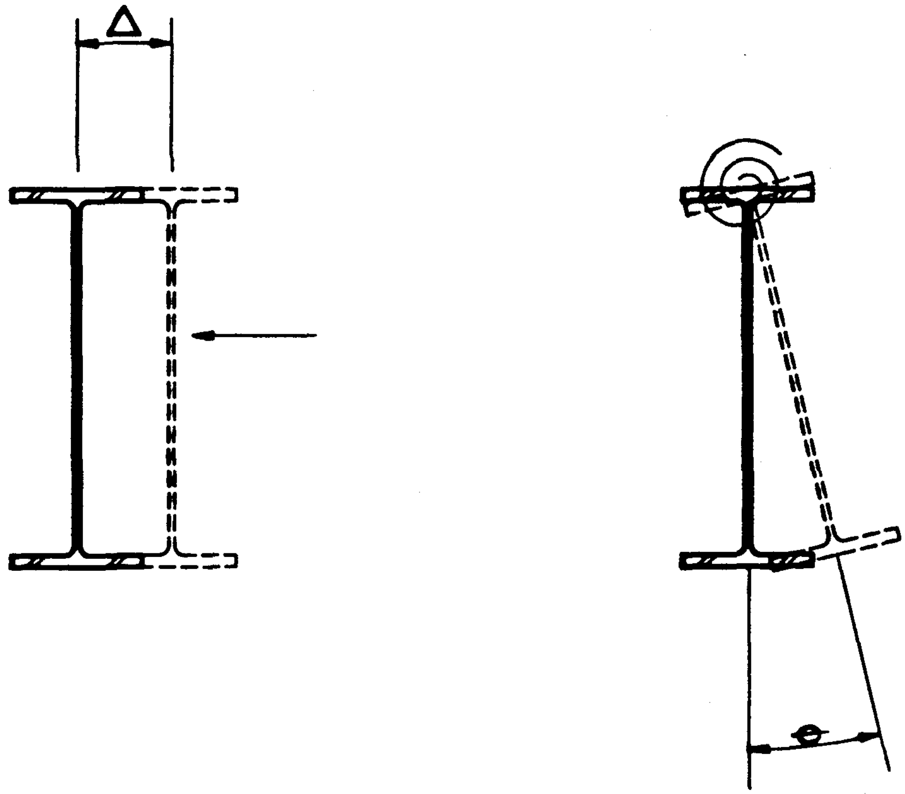

KLc
TOP FLANGE
IN COMPRESSION
POINT OF
INFLECTION.
BOTTOM FLANGE
IN COMPRESSION

FIGURE 3



LATERAL BRACE TO PREVENT TRANSLATION Δ

TORSIONAL BRACE BRACE TO PREVENT ROTATION θ

FIGURE 4

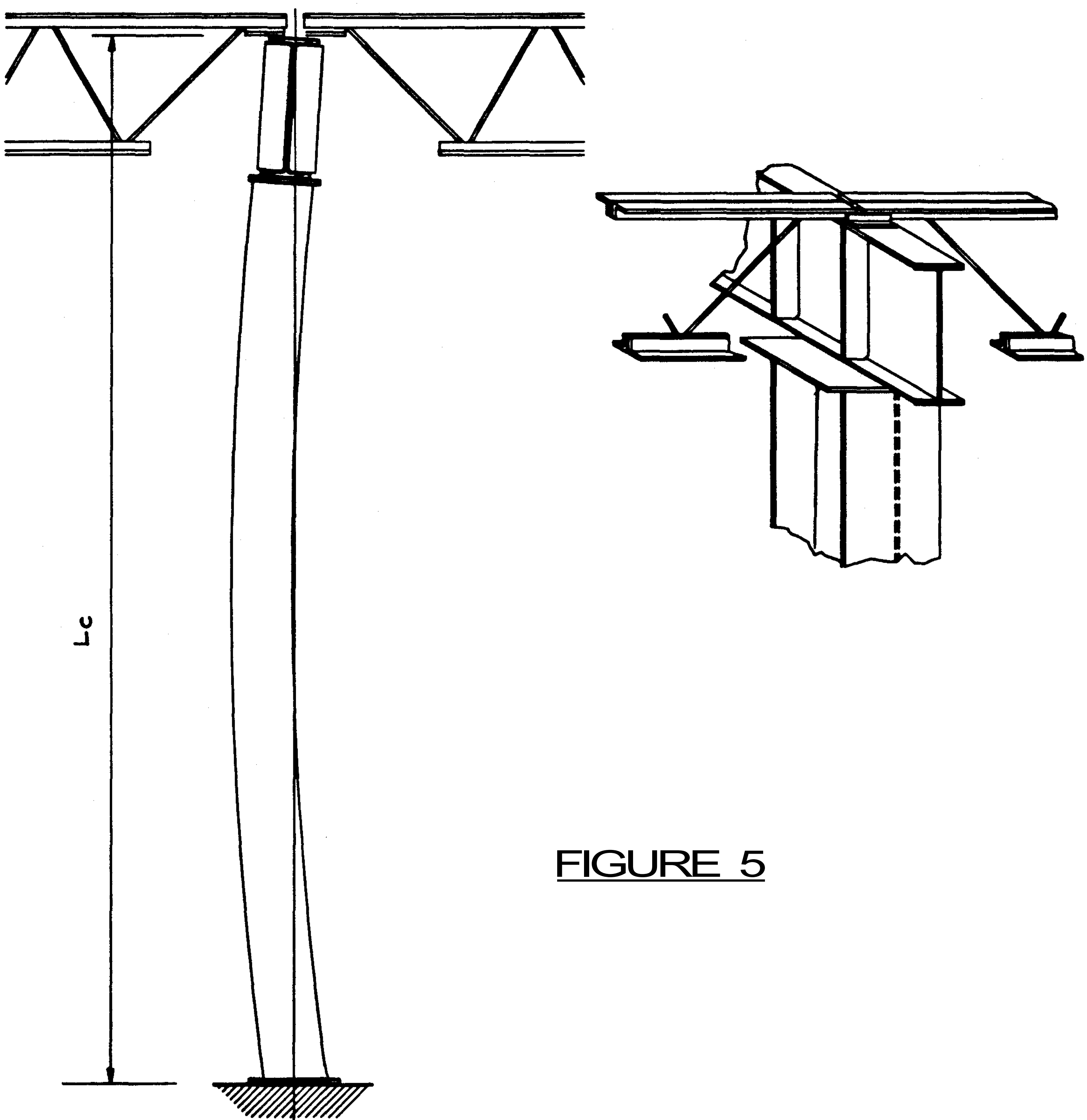


FIGURE 5

REPRINT FROM ROOF FRAMING WITH CANTILEVER GIRDERS & OPEN WEB STEEL JOISTS [REF. 3]

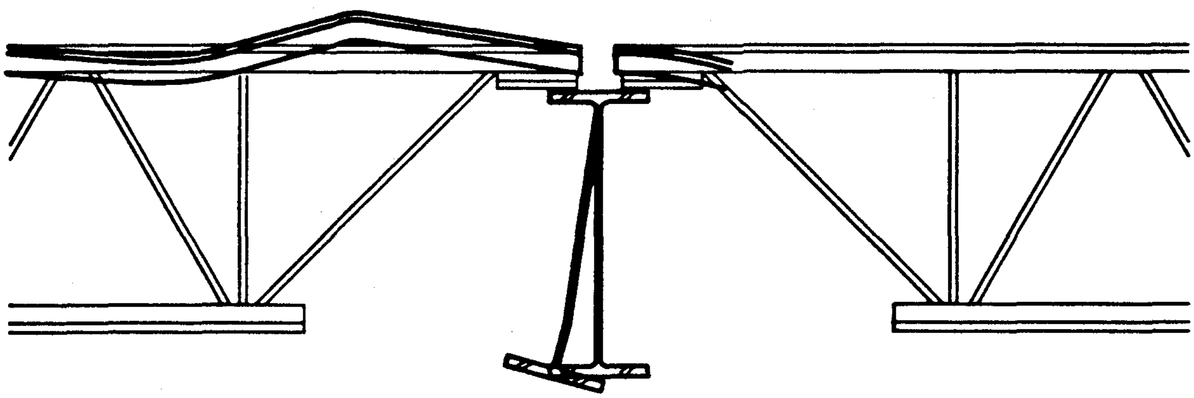


FIGURE 6

REPRINT FROM ROOF FRAMING WITH CANTILEVER GIRDERS & OPEN WEB STEEL JOISTS [REF. 3]

CANTILEVER LENGTH	BACK SPAN	SECTION	$\frac{M^-_{max}}{M^+}$	KENNEDY METHOD		YURA METHOD	
				M_{CR} CANT. TIP BRACED	M_{CR} CANT. TIP UNBRACED	M_{CR} BACKSPAN	M_{CR} CANTILEVER
7 FT	42 FT	W24 x 62	-263/192	410 FT-K	311 FT-K	398 FT-K	428 FT-K
7 FT	35 FT	W21 x 50	-263/105	347 FT-K	239 FT-K	278 FT-K	297 FT-K
6 FT	30FT	W18 x 35	-180/108	258 FT-K	195 FT-K	201 FT-K	181 FT-K

FIGURE 7

DESIGN EXAMPLE - LRFD

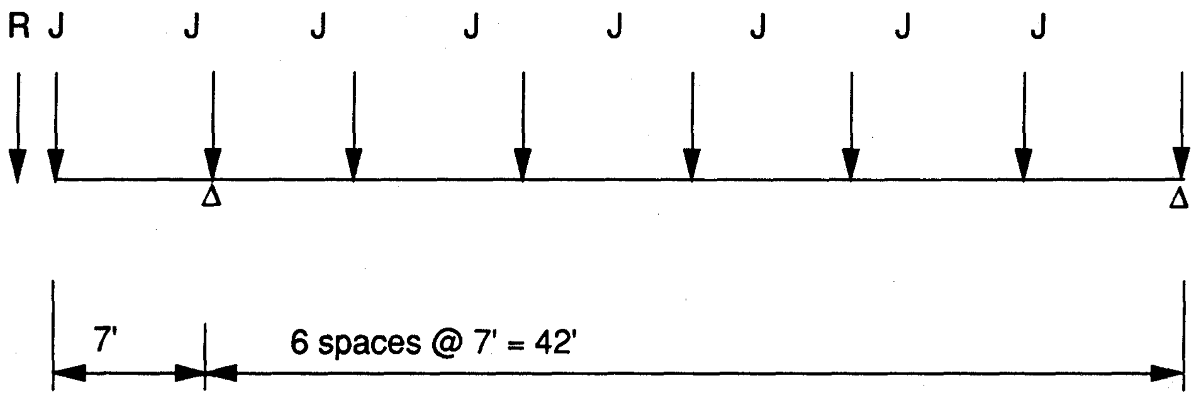
Dead Load = 20 psf   Factored Dead Load = 20 x 1.2 = 24 psf

Live Load = 30 psf   Factored Live Load = 30 x 1.6 = 48 psf

Column grid = 42' x 30'

Joist Spacing = 7' O/C

Beam braced at col. line both top and bottom flanges



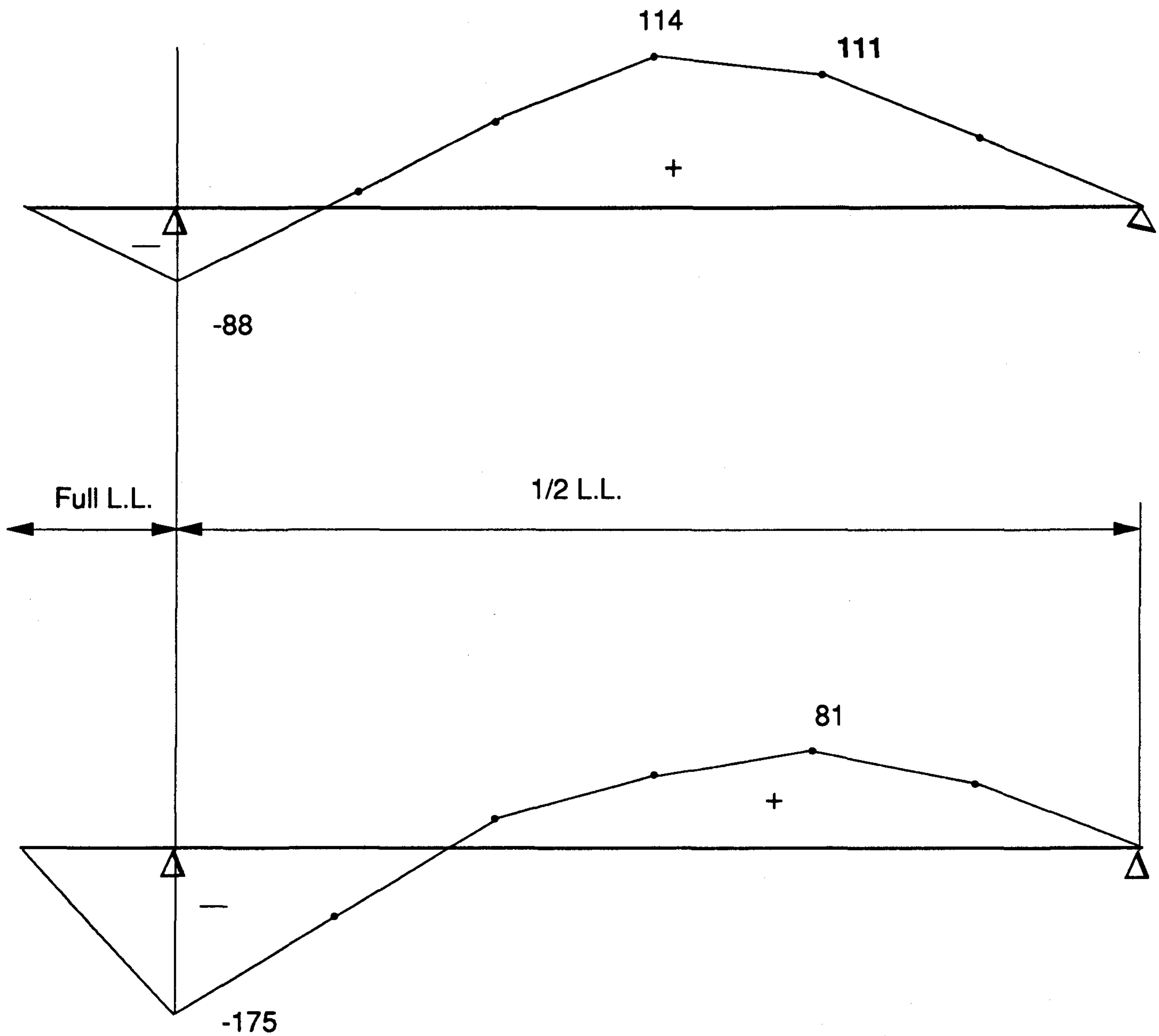
Joist Reactions, J : D.L. = 24 psf x 30' x 7' = 5^K

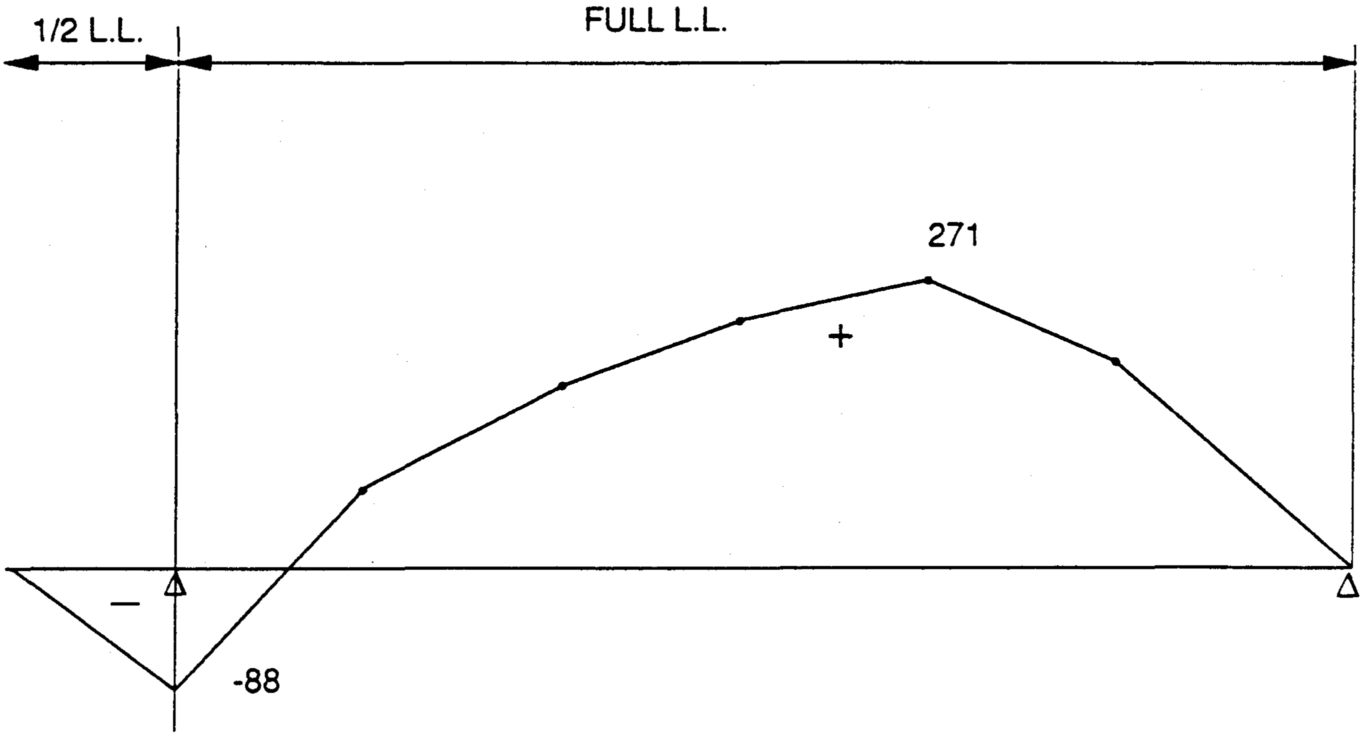
Full L.L. = 30 psf x 30' x 7' = 10^K

Suspended Beam Reaction, R = 1.5 J

FACTORED MOMENT DIAGRAM

Dead Load





SUMMARY

$M_U^- \max = -88 - 175 = -263_{FT-K}$; $M_U^+ = 111 + 81 = 192_{FT-K}$

$M_U^+ \max = 114 + 271 = 385_{FT-K}$

R_U max occurs @ Full Dead & Live Load = 81^K

Try W24 x 62 : Fy = 36 ksi

$M_p = 459_{FT-K}$ $M_R = 284_{FT-K}$

$L_p = 5.8$ FT $L_R = 17.2\ FT$ L = length of back span = 504 in

$T_W = 0.395\ in$ $I_Y = 34.5\ \text{in}^4$ $C_W = 4620\ in^6$

$T_F = 0.505\ in$ $J = 1.71\ in^4$

bF = 7 in $\sqrt{\dfrac{E\,C_W}{GJ}} = 83.6$ in d = 23.57 in

Determine M_{CR}

Calculate K_E, effective torsional stiffness with joists welded to top flange,

$$\frac{1}{K_E} = \frac{1}{K_b} + \frac{1}{K_F} + \frac{1}{K_W}$$

Where

$$K_b = \frac{K_{joist}}{L_b} = \frac{270}{84}\ in - lb./rad = 3.2$$

$$K_F = \frac{7.3\, G\, b_F\, T_F^3}{L_b^2} = \frac{7.3 \quad (11000)\,(7)\,(0.505)^3}{84^2} = 10.26$$

$$K_W = \frac{E\, T_W{}^3}{4\,(1-\mu^2)\, d} = \frac{29000\,(0.395)^3}{4(0.977)\,(23.57)} = 19.39$$

$$\frac{1}{K_E} = \frac{1}{3.2} + \frac{1}{10.26} + \frac{1}{19.39}$$

$$K_E = 2.17$$

Calculate J*, modified torsional constant

$$J^* = J + \frac{K_E L^2}{\pi^2 G} = 1.71 + \frac{2.17\,(504)^2}{\pi^2\,(11000)}$$

$$J^* = 6.79$$

$$For\ M_{CR} : R_M = \frac{|M^-|}{M_+ + |M^-|} = \frac{263}{(192 + 263)} = 0.58 \quad (Use\ 0.6)$$

Calculate X, beam torsional parameter

$$X = \frac{\pi}{L}\sqrt{\frac{EC_W}{GJ}} = \frac{\pi}{504}(83.6) = 0.52$$

From graph, figure 7 with X = 0.52 and R_M = 0.6:

K buckling coefficient = 9.2

$$M_{CR} = \frac{K}{L}\sqrt{E\,I_y\,G\,J^*}$$

$$= \frac{9.2}{504\ x\ (12)}\sqrt{29000\,(34.5)\,(11000)\,(6.79)}$$

$$M_{CR} = 416_{FT-K}$$

$$M_p = 459_{FT-K} \geq M_{cr} \geq M_R = 284_{FTK}$$

Since $\mathbf{M_{CR}}$ is in the inelastic range, determine factored moment resistance as follows:

Find $\mathbf{L_b}$, the equivalent unbraced length using formula [Fl-13].

$$M_{CR} = 416_{FT-K} = \frac{\pi}{L_b}\sqrt{EI_Y\,GJ + \left(\frac{\pi E}{L_b}\right)^2 I_Y\,C_W}$$

$$L_{b_{equiv.}} = 13.7\ FT$$

Solving for $\mathbf{M_N}$ using AISC Formula [F1 -2]:

$$M_N = \left[M_p - \left(M_p - M_R\right)\right] \left[\frac{L_b - L_p}{L_R - L_p}\right]$$

$$= \left[459 - (459 - 284)\right] \left[\frac{13.7 - 5.8}{17.2 - 5.8}\right]$$

$$M_N = 338_{FT-K}$$

$$\phi M_N = 0.9\,(338) = 304_{FT-K} \geq 263_{FT-K} \quad \textbf{O.K.}$$

Determine positive moment resistance for back span.

$Max\ M_U^+ = 385_{FT-K} \quad Use\ L_b = 84\ in\ and\ C_b = 1.0$

$Since\ L_b \geq L_R$ Use Formula F1 - 2

$$M_N = C_b \left[M_p - (M_p - M_R)\right] \left[\frac{L_b - L_p}{L_R - L_p}\right] \leq M_p$$

$$= \left[459 - (459 - 284)\right] \left[\frac{7 - 5.8}{17.2 - 5.8}\right]$$

$$M_N = 441_{FT-K}$$

$$\phi\ M_N = 0.9\ (441) = 397_{FT-K} \geq 385_{FT-K} \quad \text{O.K.}$$

Determine web capacity at continuous end.

Check web crippling

$$\phi\ R_N = 135\ T_W{}^2 \left[1 + 3(\frac{N}{d}) \left(\frac{T_W}{T_F}\right)^{1.5}\right] \sqrt{\frac{F_{YW}\ T_F}{T_W}}$$

Where N = required minimum bearing length $= \dfrac{R_U}{\phi\ F_{YW}\ T_W} - 2.5k$

$\phi = 1.0$

k = 1.375 in

$$N_{MIN} = \frac{81^k}{36\ (0.395)} - 2.5\ (1.375) = 2.2\ in$$

$$\phi R_N = 135\,(0.395)^2\,[1+3]\left(\frac{2.25}{23.57}\right)\left(\frac{0.395}{0.505}\right)^{1.5}\sqrt{\frac{36\,(0.505)}{0.395}}$$

$$= 171^k \geq 81^k \quad O.K.$$

Check web yielding

$$R_N = (5k\ + N)\,F_{YW}\,T_W$$

$$= (5\,(1.375) + 2.25)\,(36)\,(0.395)$$

$$= 130^k \geq 81^k \quad O.K.$$

Live Load Deflection

Check back span at full live load **(max M⁺)**

$$Unfactored\ \ M^{+} = \left[\frac{271}{1.6}\right] = 169_{FT-K}$$

$$Unfactored\ \ M^{-} = \left[-\frac{88}{1.6}\right] = -55_{FT-K}$$

$$D_{LL} \approx \frac{5\ L^2}{48\ EI}\left[M^{+} - 0.1\,M^{-}\right]$$

$$\approx \frac{5\,(42)^2\,(1728)}{48\,(29000)\,(1550)}\left[169 - 0.1\,(55)\right]$$

$$D_{LL} \approx 1.15\ \text{in} = \frac{L}{438} < \frac{L}{240} \quad \text{O.K.}$$

M_{CR} by Yura method.

For beams with end moments and top flange braced.

Calculate M_{CR} for the backspan portion on the beam:

$$C_b = 3.0 - \frac{2}{3}\left(\frac{M_1}{M_o}\right) - \frac{8}{3}\frac{M_c}{(M_o + M_1)}$$

Where M_1 = End moment at continuous end = 263_{FT-K}

M_0 = End moment at other end = 0

M_C = Positive moment at center = 192_{FT-K}

$$C_b = 3.0 - 0 - \frac{8}{3}\left(\frac{192}{-263}\right) = 4.95$$

$$M_{CR} = C_b \frac{\pi}{L_b} \sqrt{EI_Y\, GJ + \left(\frac{\pi E}{L_6}\right)^2 I_Y\, C_W}$$

$$= 4.95\left(\frac{\pi}{504\,(12)}\right)\sqrt{29000\,(34.5)\,(11000)\,(1.71) + \left(\frac{\pi\,(29000)}{504}\right)^2 (34.5)\,(4620)}$$

$M_{CR} = 398_{FT-K}$ (compared to $416_{FT\text{-}K}$ by Kennedy method)

Calculate M_{CR} for the overhang:

$$M_{CR} = C_b \frac{\pi}{L_b}\sqrt{EI_Y\, GJ}$$

$$= \frac{\pi}{84\,(12)}\sqrt{(29000)\,(34.5)\,(11000)\,(1.71)}$$

$$= 428_{FT\text{-}K} > 398_{FT\text{-}K}$$

Therefore, M_{CR} for backspan controls

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Mr. Joseph A. Yura, Professor, University of Texas, Austin, Texas

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APPENDIX

DESIGN CURVES FOR THE BUCKLING COEFFICIENT

(Reprint from Design of Steel Beams in Cantilever-Suspended Span Construction — Essa and Kennedy (Ref. 6))

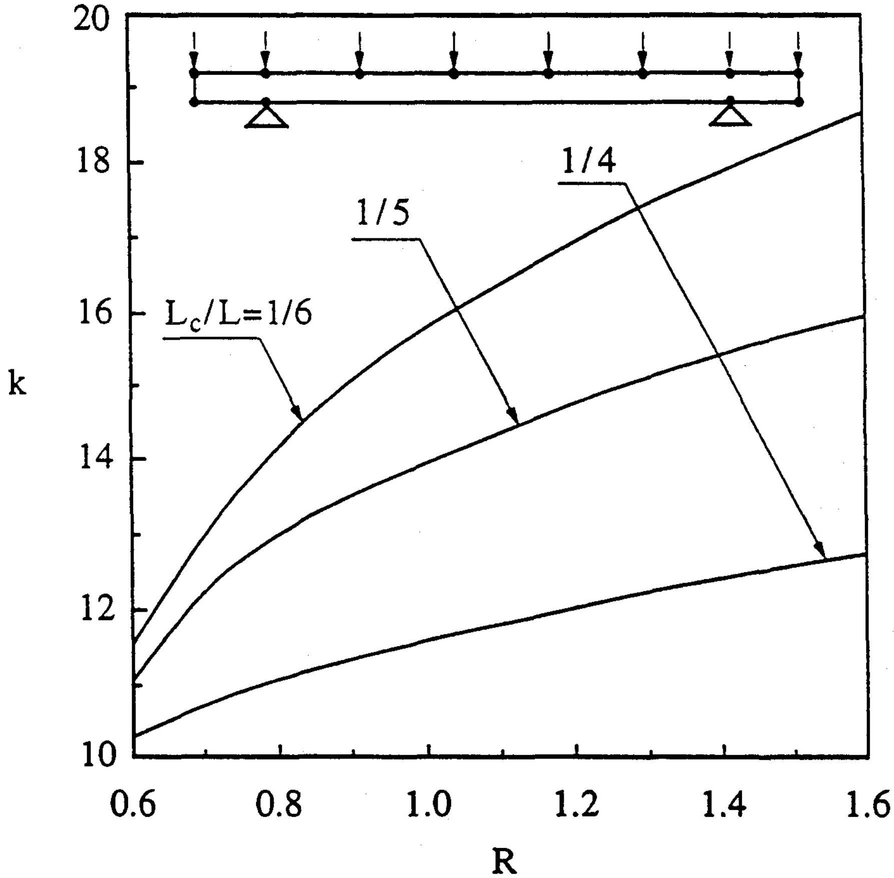

Fig.4 Essa and Kennedy

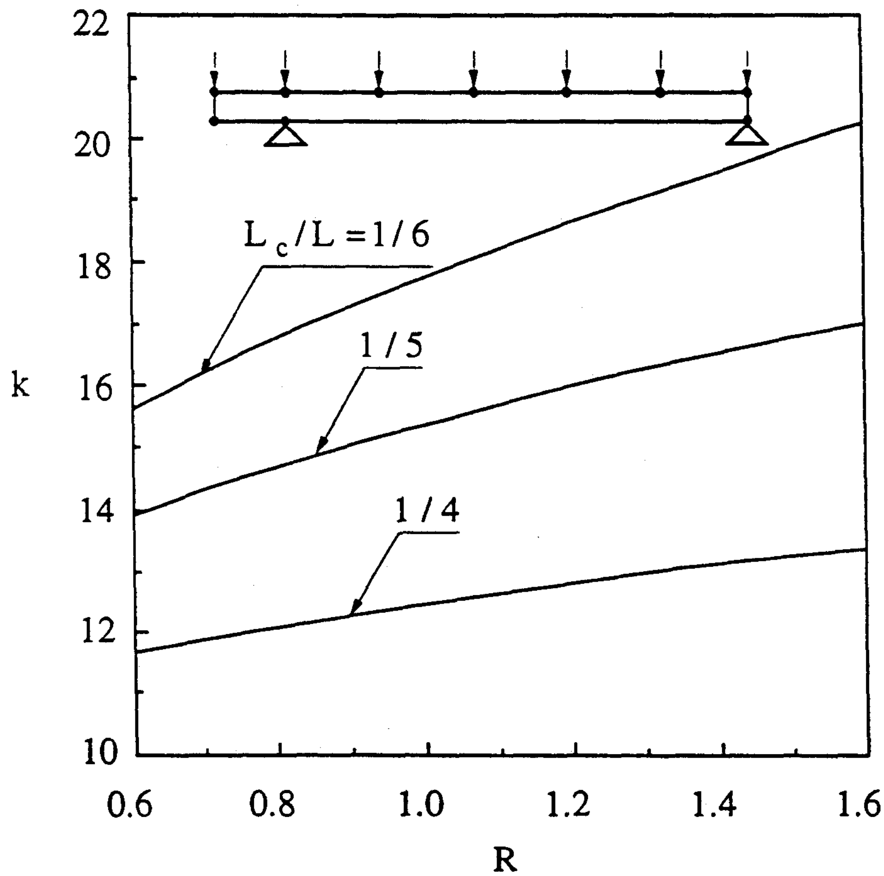

Fig.5 Essa and Kennedy

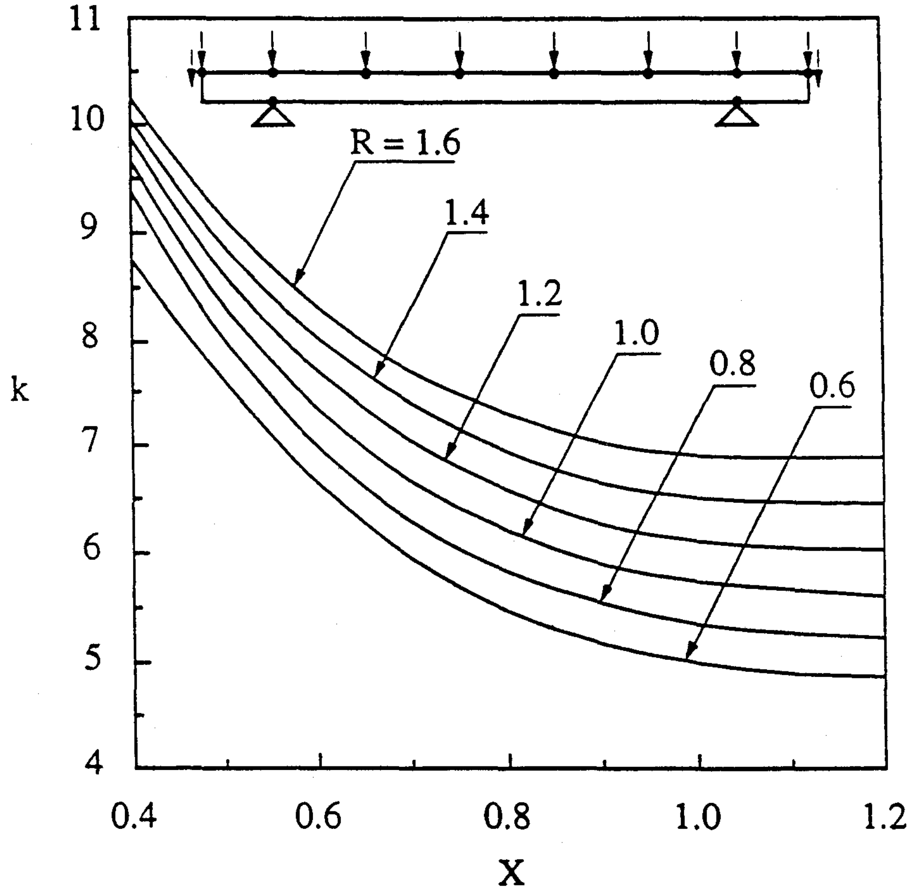

Fig.6 Essa and Kennedy

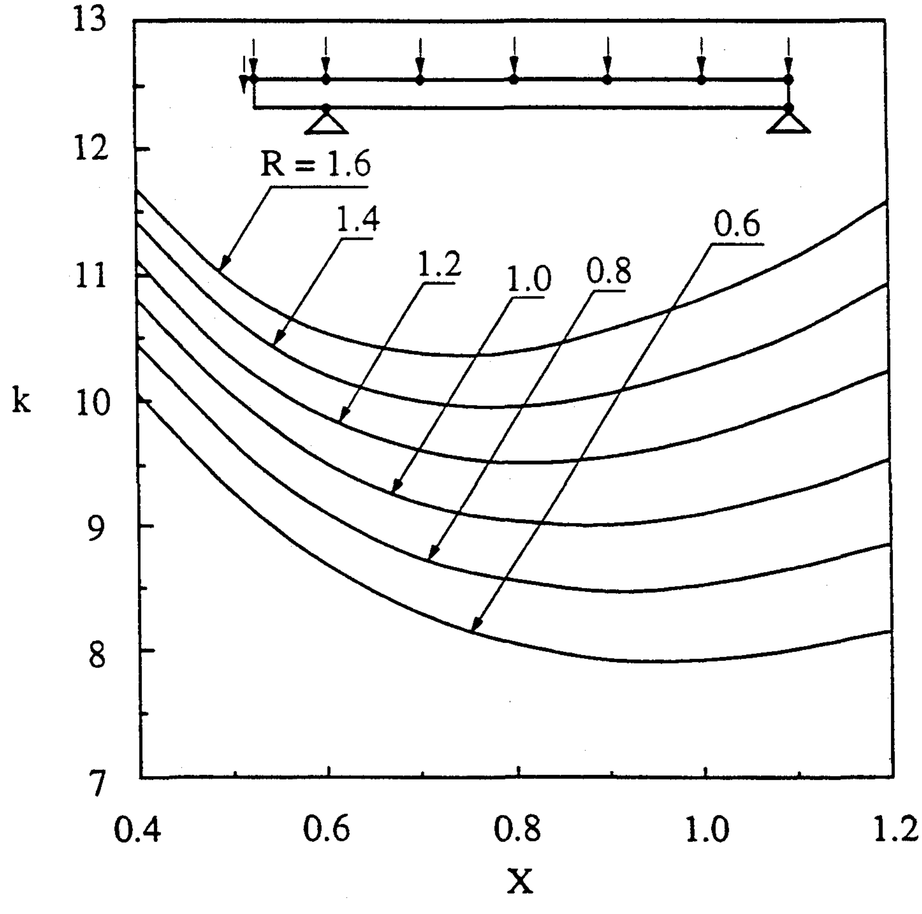

Fig.7 Essa and Kennedy

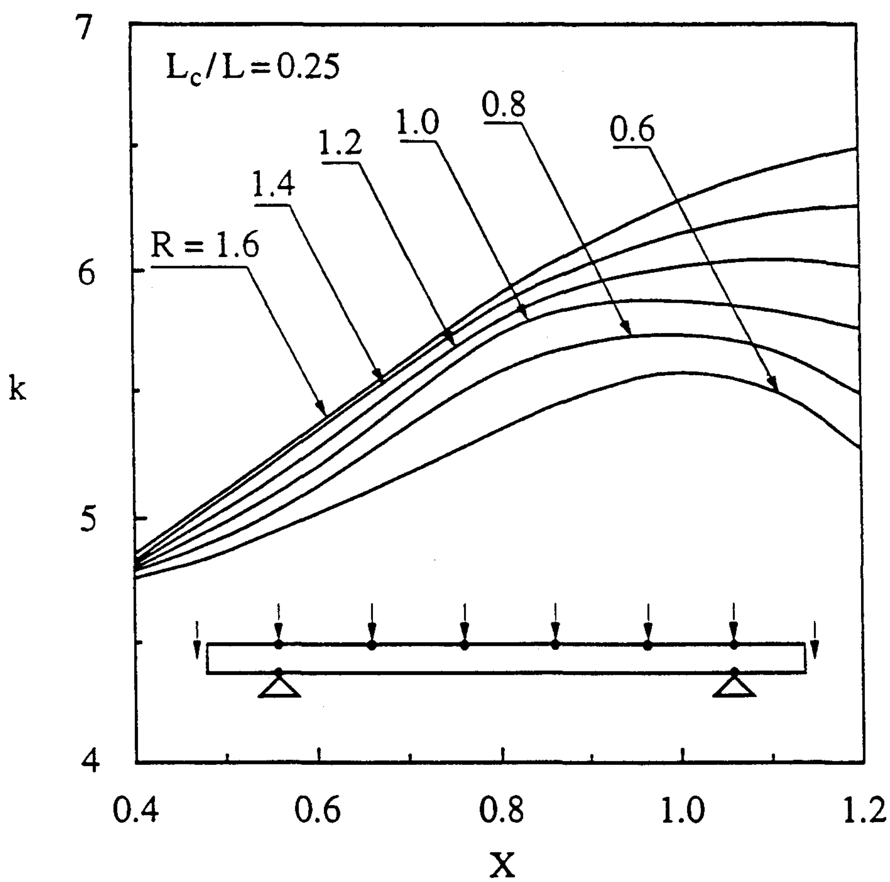

Fig.8 Essa and Kennedy

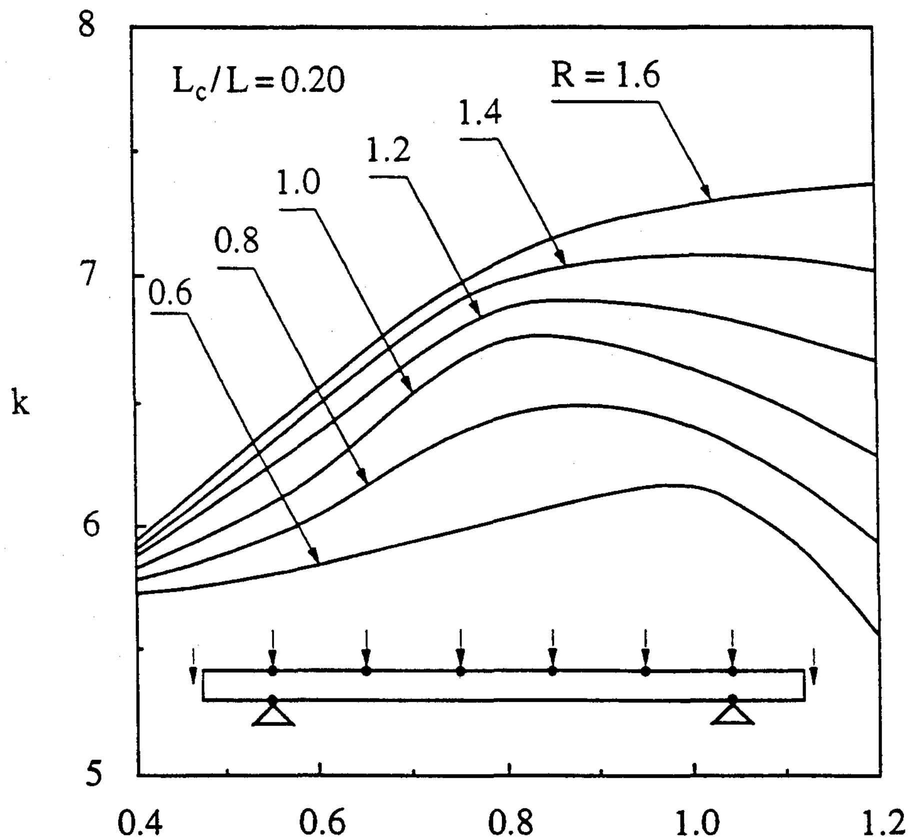

Fig.9 Essa and Kennedy

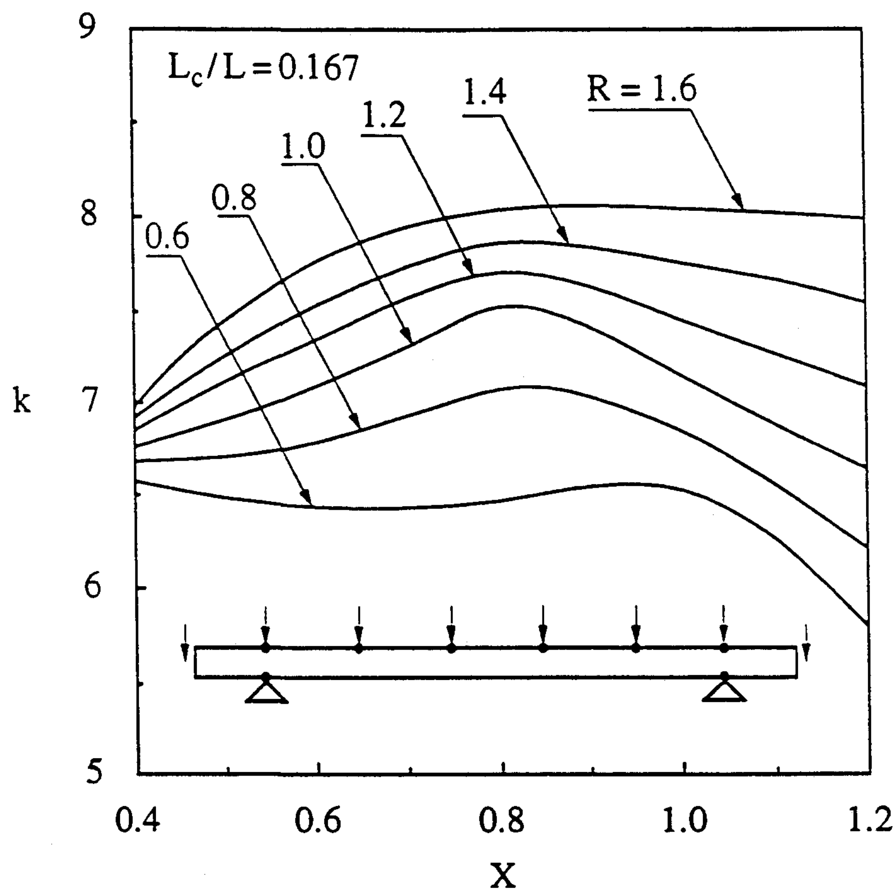

Fig.10 Essa and Kennedy

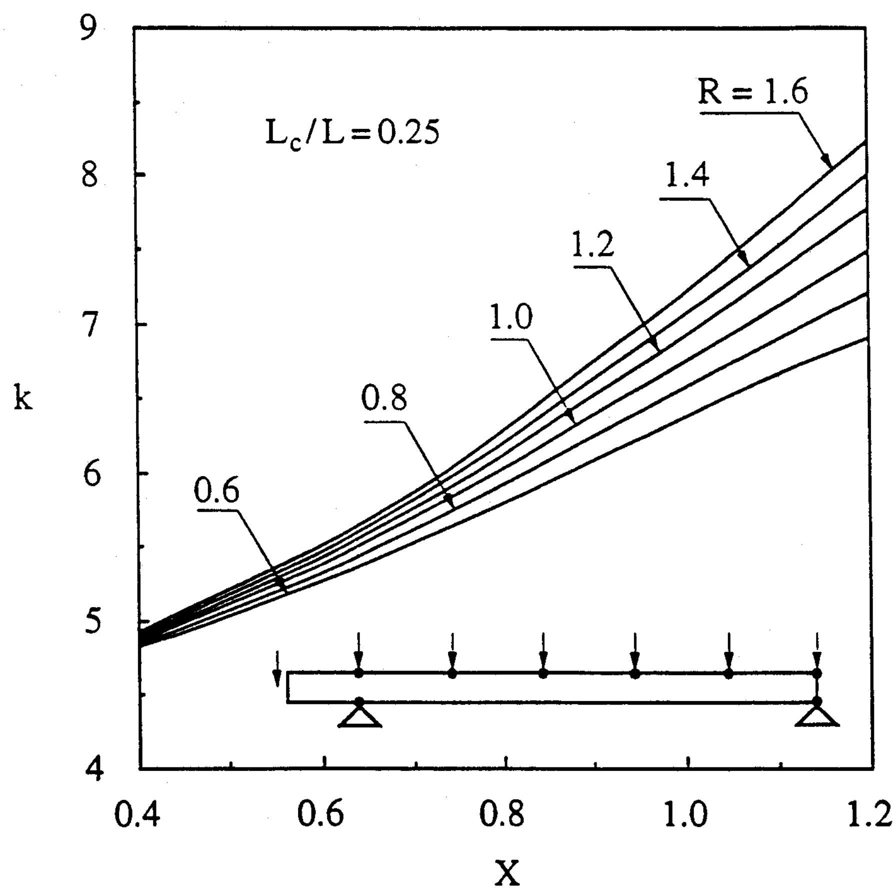

Fig.11 Essa and Kennedy

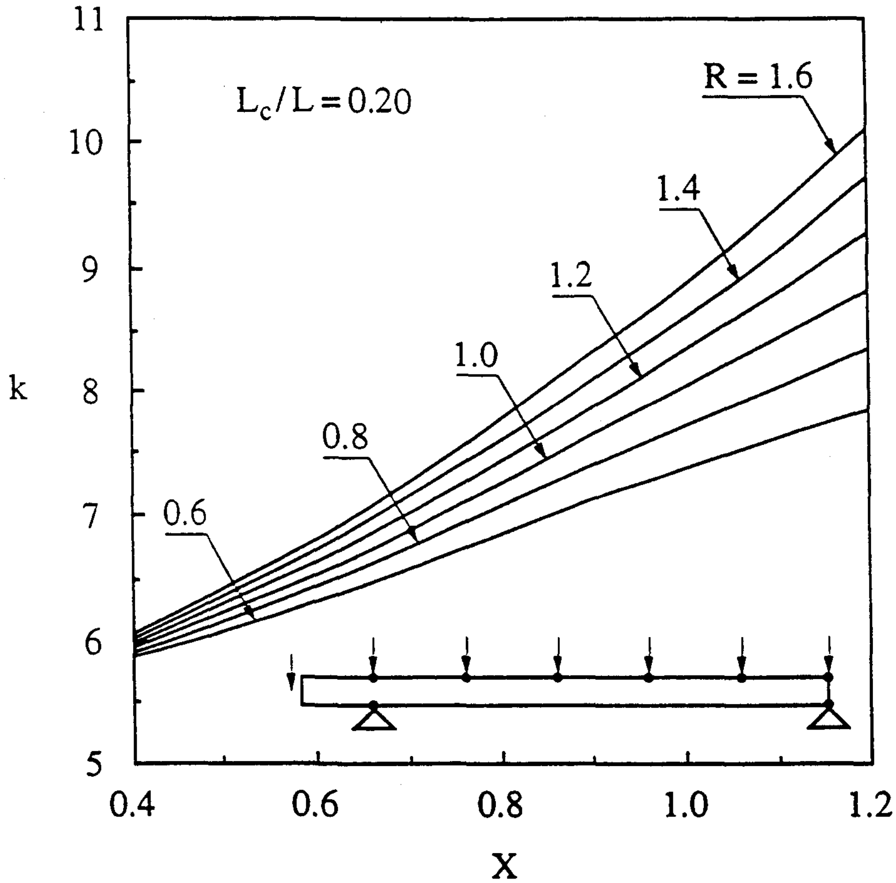

Fig.12 Essa and Kennedy

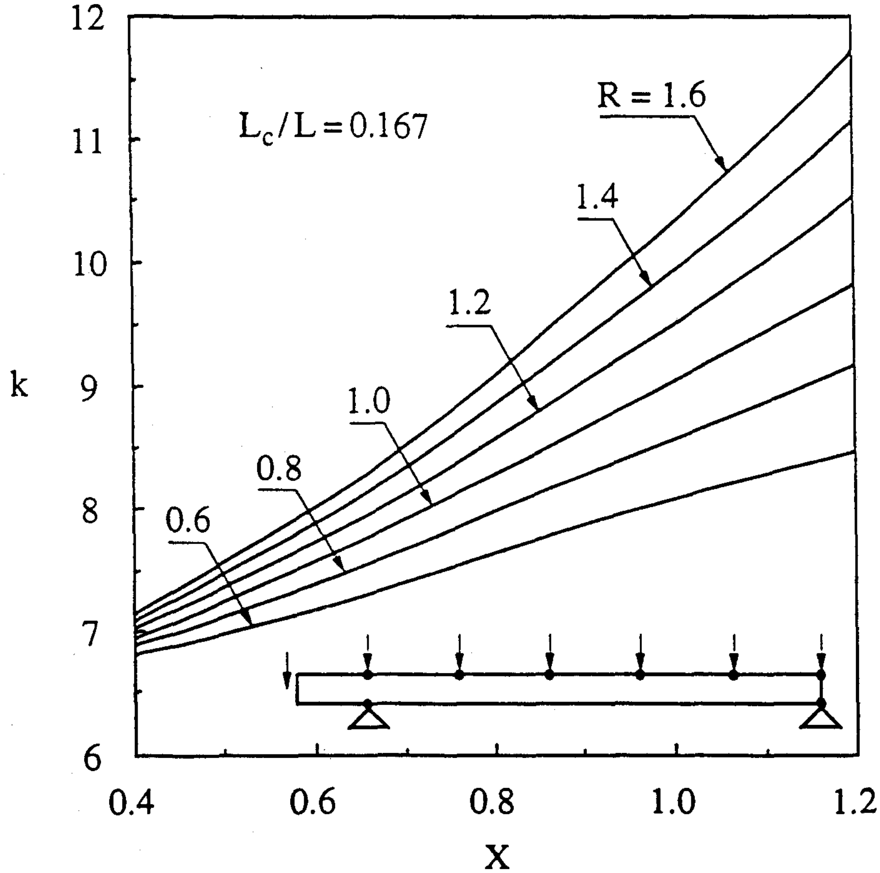

Fig.13 Essa and Kennedy

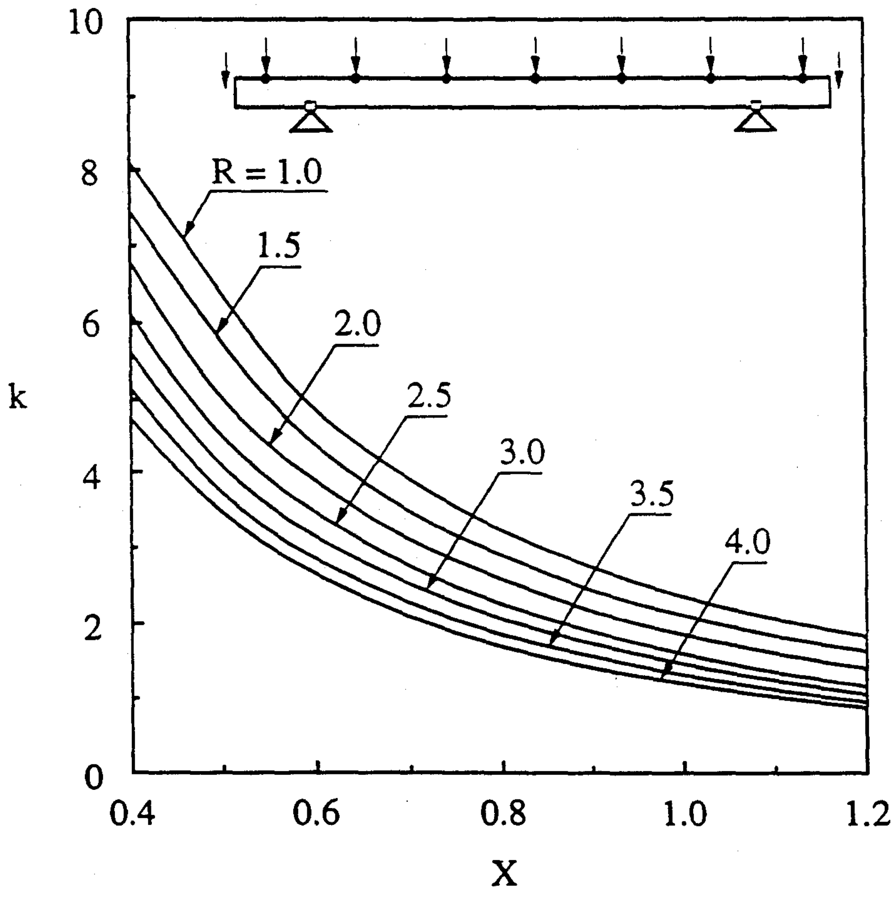

Fig.14 Essa and Kennedy

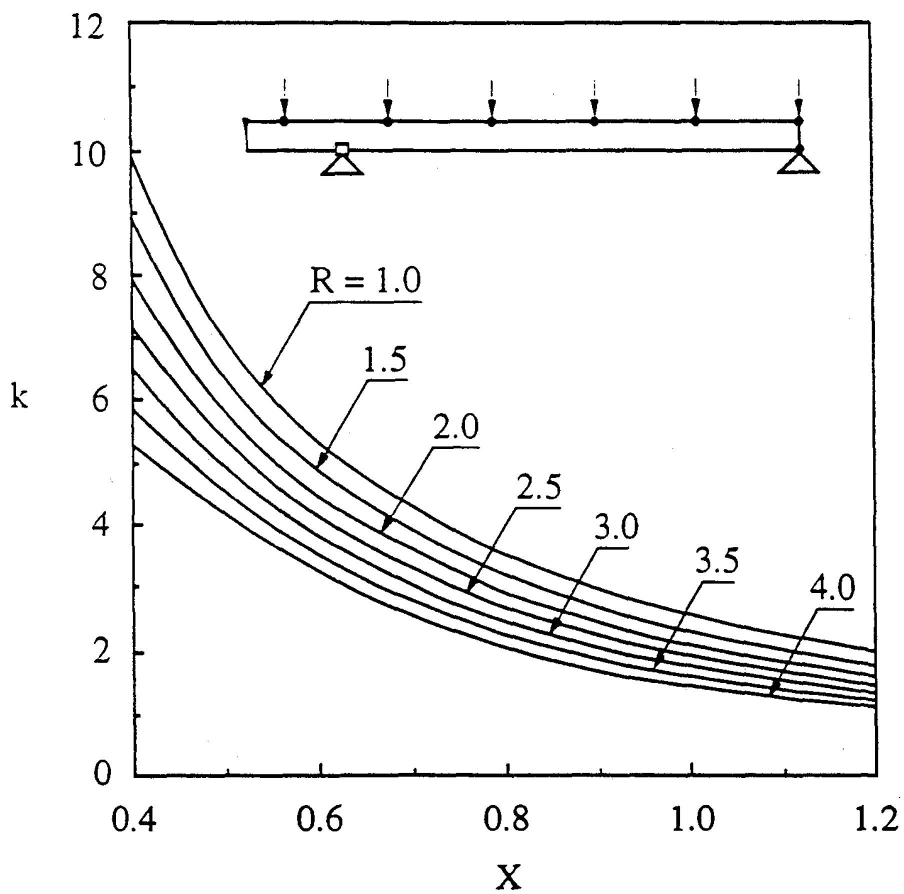

Fig.15 Essa and Kennedy