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Background and Illustrative Examples on Proposed Direct Analysis Method for Stability Design of Moment Frames

Report on behalf of AISC TC 10
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PREFACE

Summarized herein is background information and illustrative examples for new frame stability design provisions proposed by AISC TC 10 for the 2005 AISC Standard. In the latest AISC ballot (July 2003), most of the new provisions appear in a new Appendix 6, entitled "Direct Analysis Method for Moment Frames", which provides an alternative to the frame stability provisions in Section B6 of the Standard. The frame stability provisions of Section B6 are essentially identical to those in the 1999 (3rd edition) of the LRFD Specification for Structural Steel Buildings, except for the addition of a minimum moment requirement. The background and fundamental features of the standard (Section B6) and alternative (Appendix 6) provisions are described herein. Several illustrative example problems are presented to demonstrate and contrast the two stability design approaches.

ACKNOWLEDGMENTS

The developments summarized in this report are the result of contributions by many individuals over the course of deliberations by AISC TC 10 and the AISC-SSRC Ad Hoc Committee on frame stability. The AISC-SSRC Ad Hoc Committee co-chairs (J. Yura and G. Deierlein) and members (W. Baker, J. Hajjar, T. Galambos, R. Heinge, L. Lutz, K. Mueller, S. Nair, C. Rex, R. Tremblay, D. White and R. Zieman) developed the first draft of the provisions. Other noteworthy contributions include those of D. White, A. Maleck, and R. Ziemian for their analysis of numerous benchmark validation studies and L. Lutz for suggestions of illustrative example problems. Portions of this report are excerpts from an earlier paper presented at the 2002 SSRC Annual Meeting, which introduced a preliminary (now superseded) draft of the proposed provisions (Deierlein, Hajjar, Yura, White, Baker, "Proposed new provisions for frame stability using second-order analysis", SSRC 2002 Annual Meeting, Seattle, WA).

INTRODUCTION

The proposed new provisions for frame stability represent the culmination of work by task committees in AISC and SSRC over the past four years, which incorporate concepts of second-order analysis and design whose origins date back over twenty years. Concerted work on this began late in 1999, with the formation of a joint AISC-SSRC Ad-hoc Committee whose charge was to develop improved specification provisions for member and frame stability. The committee's goals were to develop design methods for stability that made more effective use of modern computer analysis methods, while reducing the over-reliance on effective buckling length procedures in the current AISC Specifications. This ad-hoc committee was combined with AISC TC 10 in 2001, and the combined group developed provisions, which are proposed for adoption in the 2005 AISC Standard. The new provisions were first balloted in March 2003 and have since been revised to address comments raised by the AISC Specification Committee. This report reflects the latest version of the proposed provisions for frame stability.

The July 2003 AISC ballot outlines proposed provisions for the 2005 Standard, which will permit two alternative methods to design for stability effects in moment frames. For discussion purposes, the two approaches will be referred to as the *"Effective Length"* and *"Direct Analysis"* methods. Both approaches require evaluation of second-order effects and member force interaction equations. The methods differ in their specific requirements for calculating second-order effects and the axial strength term, P_n, in the member interaction equation. Requirements for the Effective Length method are contained in the proposed Section B6 of the 2005 Standard. This method is essentially the same method as the approach used in Chapter C of the 1999 AISC-LRFD Specification. Requirements for the Direct Analysis method are specified in a newly proposed Appendix 6 to the 2005 Standard.

This report begins with a brief review of key behavioral effects and second-order analysis considerations, which are relevant to stability design. Next, the two proposed approaches to frame stability are summarized and contrasted through a design example of simple cantilever column. This is followed by highlights of validation studies to evaluate the accuracy of the two proposed methods. The report concludes with three design examples to illustrate practical application of the methods.

BEHAVIORAL EFFECTS

There are potentially many parameters and behavioral effects that influence stability of steel-framed structures. The extent to which these factors are modeled in analysis will affect the criteria that one applies in design of the frame, its members and connections. Without repeating more complete presentations given elsewhere (Birnstiel and Iffland, 1980; McGuire, 1992; White and Chen, 1993; ASCE, 1997; Deierlein & White 1998), it is helpful to review three basic aspects of behavior: geometric nonlinearities, inelastic spread-of-plasticity, and member limit states. These ultimately govern frame deformations under applied loads and the resulting internal load effects.

Geometric Nonlinearities and Imperfections: Modern stability design provisions are based on the premise that the member forces are calculated by second-order elastic analyses, where equilibrium is satisfied on the deformed structure. When stability effects are significant, consideration must be given to initial geometric imperfections in the structure due to fabrication and erection tolerances. For the purpose of calibrating the stability requirements described later, initial geometric imperfections are conservatively assumed as equal to the maximum fabrication and erection tolerances permitted by the AISC *Code of Standard Practice* (2000). For columns and frames, this implies a member out-of-straightness equal to $L/1000$, where L is the member length between brace or framing points, and a frame out-of-plumb equal to $H/500$, where H is the story height. The out-of-plumb is also limited by the absolute bounds as specified in the *Code of Standard Practice*.

Inelastic Spread of Plasticity: The proposed analysis/design approaches are calibrated against inelastic distributed-plasticity analyses that account for spread of plasticity through the member cross-section and along the member length. Thermal residual stresses in W-shape members are assumed to have maximum values of $0.3Fy$ and are distributed according to the so-called Lehigh pattern - linearly varying across the flanges and uniform tension in the web (Deierlein & White 1998).

Member Limit States: Member strength may be controlled by one or more of the following limit states: cross section yielding, local buckling, flexural buckling, and torsional-flexural buckling. For structural analyses envisioned for routine frame design, it is assumed that the analysis does not model local flange/web buckling or torsional-flexural buckling. Therefore, these limits must be considered in separate member design checks. For inelastic analyses, the member yield limit is incorporated directly in the analysis; and for elastic analyses, this limit can be checked by an interaction equation that approximates the *P-M* yield surface. Whether or not the analysis captures in-plane flexural buckling depends on the extent to which the maximum moments are affected by distributed plasticity and member straightness. Concerns as to whether the analysis captures this effect suggest the need to apply a member check for in-plane flexural buckling, even when an accurate second-order analysis is used. As will be addressed later, a key consideration for the in-plane flexural buckling check relates to the assumed buckling length used in calculating the design compression strength, ϕP_n.

SECOND-ORDER ELASTIC ANALYSIS

The AISC stability design provisions are developed for use with second-order elastic analysis. In practice, there are alternative approaches one can employ for conducting second-order analyses, some of which are more rigorous than others. For the purpose of this discussion, second-order elastic analyses will be categorized as "rigorous" or "approximate". The difference between these two depends on the extent to which P-δ effects are modeled and whether the problem is "linearized" to expedite the solution.

Rigorous second-order analyses are those that accurately model all significant second-order effects. Rigorous analyses include solution of the governing differential equation, either through stability functions or computer frame analysis programs that model these effects (McGuire 1992; Deierlein & White 1998). Many (but not all) modern commercial computer programs are capable of rigorous analyses, though users should verify this. Methods that modify first-order analysis results through second-order amplifiers (e.g., B_1 and B_2 factors) are in some cases accurate enough to constitute a rigorous analysis, but this depends on the magnitude of second-order effects and other characteristics of the problem.

Approximate second-order analyses are any methods that do not meet the requirements of rigorous analyses. A common type of approximate analyses are those which only capture P-Δ due to member end translations (e.g., interstory drift) but fail to capture P-δ effects due to curvature of the member relative to its chord. Where P-δ effects are significant, errors arise in approximate methods that do not accurately account for the effect of P-δ moments on amplification of both local member moments and the calculated global (Δ) displacements. These errors can arise both with second-order computer analysis programs and with the B_1 and B_2 amplifiers. White and Maleck (2002) propose the following criteria to rule out cases where P-δ effects can be safely ignored:

$$P_u < 0.15\, P_{eL} = 0.15(\pi^2 EI/L^2) \qquad (1)$$

where P_u is the required column strength and P_{eL} is the elastic buckling load in the plane of bending. The alternative to this equation is to verify the accuracy of the second-order analysis by comparisons to known solutions for conditions similar to those in the structure. Examples of the errors one may encounter are discussed by LeMessurier (1977) and Deierlein & White (1998).

BEAM-COLUMN INTERACTION EQUATIONS (SECTION H1 OF THE 2005 STANDARD)

Both the Effective Length (Section B6) and Direct Analysis (Appendix 6) stability procedures utilize the beam-column interaction equations of Chapter H, albeit with differences in how the required strengths (P_u and M_u) and the nominal compressive strength (P_n) are calculated. For reference in the later discussion, the interaction equations for members under combined axial compression and bending are briefly reviewed. For bi-symmetric beam-columns under combined axial compression and uniaxial bending, the 2005 Standard introduces a new interaction equation for checking out-of-plane (lateral-torsional) instability, which is separate from the check for in-plane (flexural buckling) instability. These separate equations are introduced since they provide more accurate predictions of in-plane and out-of-plane limit states, which tests and theory show are independent phenomena. The separate equations reduce the conservatism in the current (1999 AISC-LRFD) provisions, which combine the two limit state checks into one equation, by combining the most severe combinations of in-plane or out-of-plane limits for $P_u/\phi P_n$ and $M_u/\phi M_n$.

Shown here for illustration are the interaction equations in LRFD format for bi-symmetric beam-columns subjected to axial compression and uniaxial bending. For members subjected to compression and minor axis bending, only the in-plane check applies; whereas for columns under compression and major axis bending, both checks apply.

The limit state of in-plane flexural buckling is checked using the following equations, which have the same format as those in the 1999 AISC-LRFD Specification:

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_u}{\phi_b M_n}\right) \le 1.0 \qquad \text{for } \frac{P_u}{\phi_c P_n} \ge 0.2 \qquad (2a)$$

$$\frac{P_u}{2\phi_c P_n} + \frac{M_u}{\phi_b M_n} \le 1.0 \qquad \text{for } \frac{P_u}{\phi_c P_n} < 0.2 \qquad (2b)$$

where P_u and M_u are the required strengths, calculated from second-order analysis under the design loads; and P_n and M_n are the nominal compression and bending strengths, calculated in the plane of the frame. For the Effective Length method, P_n is determined using the effective buckling length KL in the plane of bending, whereas in the proposed Direct Analysis method, P_n is calculated using $K=1$ ($KL=L$) in the plane of bending. For compact member sections, M_n for the in-plane check is equal to M_p.

The out-of-plane lateral-torsional limit state is checked by the following equation:

$$\frac{P_u}{\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}}\right)^2 \le 1.0 \qquad (3)$$

Here the required strengths P_u and M_u are the same as for Eq. 2a and 2b, and P_n and M_n are calculated using the unbraced length in the out-of-plane direction. These out-of-plane nominal strengths would typically be evaluated on the same basis for the Effective Length and Direct Analysis methods.

EFFECTIVE LENGTH METHOD (SECTION B6 OF 2005 STANDARD)

The Effective Length (or critical load) approach for assessing member axial compressive strength has been used in various forms in the AISC Specification since 1961. The provisions proposed for Section B6 of the 2005 Standard are essentially the same as those from the 3rd edition (1999) of the AISC-LRFD Specification, with the exception of a new minimum moment requirement. The approach is based on calculating effective column buckling lengths, KL, which have their basis in elastic (or inelastic) stability theory. The effective buckling length KL, or alternatively the equivalent elastic column buckling load, $P_e = \pi^2 EI/(KL)^2$, is used to calculate an axial compressive strength, P_n, through an empirical column curve that accounts for member geometric imperfections, yielding, and residual stresses. This column strength is then combined with the design moment strength, ϕM_n, and second-order member forces, P_u and M_u, in the beam-column interaction equations.

Differences between the Effective Length and Direct Analysis approaches lie mainly in the in-plane check. Figure 1a shows a plot of the in-plane interaction equation for the Effective Length approach, where the anchor point on the vertical axis, P_{nKL}, is determined using an effective buckling length factor. Also shown in this plot is the same interaction equation with the first term is based on the squash load, P_y. The load-deformation response of a typical member, obtained from second-order spread-of-plasticity analysis and labeled "actual response," indicates the maximum axial force, P_u, that the member can sustain prior to the onset of instability. The load-deflection response of a second-order elastic analysis, as would be done in design practice, is also shown. The "actual response" curve reveals larger moments than the second-order elastic curve due to the combined effects of partial yielding and geometric imperfections, which are not included in the second-order elastic analysis. The intersection of the second-order elastic curve with the P_{nKL} interaction curve represents the design strength. The plots in Fig. 2a show how the effective length procedure has been calibrated to give a resultant axial strength, P_u, consistent with the actual response. For slender columns, accurate assessment of the effective length (and P_{nKL}) is critical to achieving an accurate solution.

While the effective length approach is calibrated to accurately predict the resultant member strength, one consequence of the procedure is that it under-estimates the actual internal moments under the factored loads (see Fig. 1a). This is inconsequential for the beam-column (since the P_{nKL} reduces the effective strength in the correct proportion), but the reduced moment can affect design of the beams and connections, which provide rotational restraint to the column. This is of greatest concern when the calculated moments are small and axial loads are large, where P-Δ moments induced by column out-of-plumb can be significant. As a safeguard for these cases, the Effective Length procedure in Section B6 includes a new minimum required moment strength for beams and connections, which restrain the column ends. This requirement is specified through the following equation (Eq. B6-3 in the July 2003 Ballot),

$$\Sigma M_u > 0.01 \Sigma P_u L \qquad (4)$$

where ΣM_u is the minimum required strength, P_u is the required strength (axial compression force) in the columns being restrained, and L is the column length.

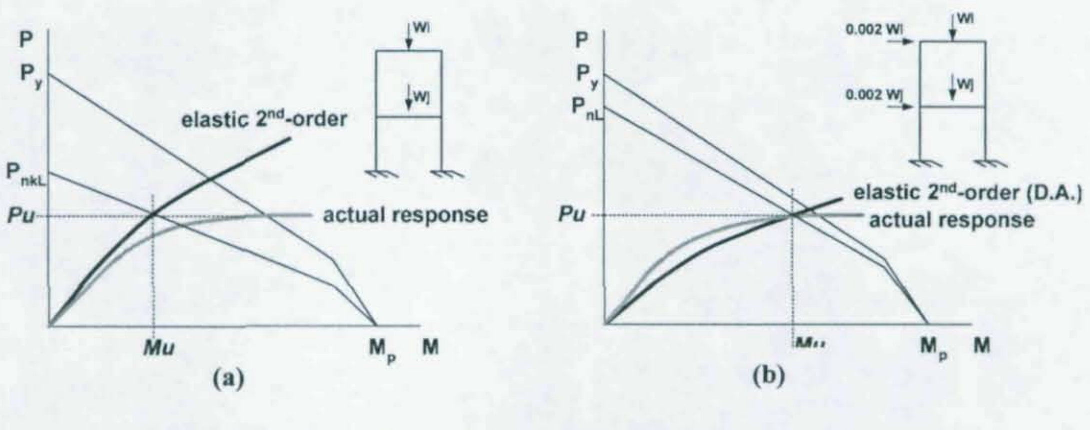


Fig. 1 – Comparison of beam-column interaction checks for (a) the effective length approach and (b) direct analysis approach

DIRECT ANALYSIS METHOD (APPENDIX 6 TO 2005 STANDARD)

The Direct Analysis approach has been developed with the goal to more accurately model frame stability effects in analysis, and thereby, eliminate the need for calculating effective buckling length factors for columns. As summarized below, the new provisions in Appendix 6 of the 2005 Standard involve reducing the nominal elastic stiffness and applying a notional load to the frame. Some aspects of the proposed provisions are similar to so-called "notional load" methods found in steel standards in other countries, e.g., Canadian and Australian Standards and the Eurocode, however, many aspects of the proposed provisions are unique to the AISC Standard and address known shortcomings of conventional notional load approaches in other standards.

Like the Effective Length procedure, the Direct Analysis method begins with a basic requirement to calculate internal member forces using a second-order elastic analysis. As will be shown later in the examples, the Direct Analysis method places a greater reliance on the second-order analysis (primarily in the accurate calculation of second-order moments, M_u), and for this reason, the method stipulates requirements to ensure accuracy of the second order analysis. Analysis rigor is most important where second-order amplifications are large, one measure of which is given by the ratio of member axial compression forces to their elastic buckling strengths (see Eq. 1). Two additional requirements for Direct Analysis are as follows:

- A notional load of $N_i = 0.002\ Y_i$ is to be applied in combination with other factored loads, where N_i is the notional lateral load applied at floor i and Y_i is the gravity load (from strength load combinations) acting at floor i. The notional load is applied to represent the destabilizing effect of a geometric imperfections and other effects (yielding, non-ideal boundary and loading conditions, etc.). The notional load magnitude of 0.002 corresponds to a frame out-of-plumb equal to H/500 (where H is the story height).

- The nominal elastic flexural stiffness assumed in the second-order elastic analysis is equal to 0.8τEI, where τ is calculated as follows:

  For members where $P_u \leq 0.5P_y$: $\tau = 1$
  For members where $P_u > 0.5P_y$: $\tau = 4[P_u/P_y(1-P_u/P_y)]$

  Alternatively, where $P_u > 0.5P_y$ for any members in the frame, $\tau = 1$ provided that an additional notional load of $N_i = 0.001\ Y_i$ is applied to the frame.

There are two reasons for imposing the reduced stiffness for analysis. For frames with slender members, where the limit state is governed by elastic stability, the 0.8 factor on stiffness results in a system design strength equal to 0.8 times the elastic stability limit. This is roughly equivalent to the margin of safety implied by design of slender columns by the effective length procedure where the design strength $\phi P_n = 0.9(0.877)P_e = 0.79P_e$ where P_e is the elastic critical load, 0.9 is the specified resistance factor, and 0.877 is a reduction factor in the column curve equation. For frames with intermediate or stocky columns, the 0.8τ factor reduces the stiffness to account for inelastic softening prior to the members reaching their design strength. The τ is similar to the inelastic stiffness reduction factor implied in the column curve to account for loss of stiffness under high compression loads ($P_u > 0.5P_y$), and the 0.8 factor accounts for additional softening under combined axial compression and bending. It is a fortuitous coincidence that the reductions coefficients for the slender and stocky columns are close enough, such that the single reduction factor of 0.8τ works over the full range of slenderness.

The reduced stiffness and notional load requirements only pertain to analysis of the strength limit state, and they do not apply to analysis of other serviceability conditions for excessive deflections, vibration, etc. For ease of application in design practice, the reduction on EI can be applied by modifying E in the analysis; however, in doing so, one should consider whether the possible side-effects of reducing EA. Moreover, for computer programs that do semi-automated design, one should be sure that the reduced E is only applied for the second-order analysis. The elastic modulus should not be reduced in design equations, which involve E to evaluate the design strength (e.g., M_n for laterally unbraced beams).

As shown in Fig. 1b, the net effect of modifying the analysis in the manner just described is to amplify the second-order moments to be closer to the actual internal moments in the member. It is for this reason that the beam-column interaction for in-plane flexural buckling is checked using an axial strength P_n calculated from the column curve using the actual unbraced member length L, i.e., with $K = 1$. In fact, arguments have been made to use $P_n = P_y$ in the interaction equation, but this would require recalibration of the analysis adjustments, including additional adjustments to account for member out-of-straightness (sweep). After considering alternative strategies, TC 10 decided to use the proposed method (with P_n based on L) as a pragmatic and conservative approach for practical design.

CANTILEVER EXAMPLE

To illustrate an application of the two stability design methods, consider the design of the cantilever beam-column shown in Fig. 2. The cantilever is subjected to the vertical and proportional horizontal load shown, such that the design is controlled by the combined P_u and M_u at the base of the column. Maximum strengths are calculated for three different column lengths, with slenderness ratios of L/r = 20, 40 and 60 (equivalent to KL/r = 40, 80, and 120). Bending is about the major axis, and the column has full out-of-plane (lateral) restraint. The design checks are based on the in-plane interaction check (Eq. 2a-b). Note that the checks were made using a resistance factor of $\phi_c = 0.85$ in compression (consistent with the 1999 AISC-LRFD Specification), so the results would be slightly different if made with the revised value of $\phi_c = 0.9$ as proposed for Chapter E of the 2005 Standard.

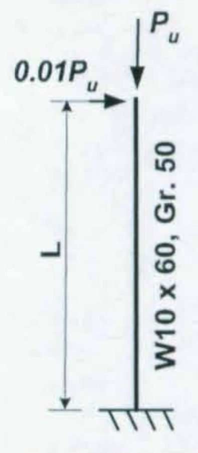


Fig. 2 – Cantilever Example

Shown Fig. 3a-c are plots of the axial load versus moment at the column base for the three column lengths, determined according to the Direct Analysis (DA) and Effective Length (EL) methods. Notice that the internal moments M_u increase much faster with P_u for the Direct Analysis method, due to the reduced stiffness (0.8τEI) and added notional loads. Most of the stiffness adjustment is due to the 0.8 factor, since τ only affects the column with $L/r = 20$ (Fig. 4a,) where the maximum load $P_u > 0.5P_y$ = 440 kips. Overlaid on these force-point traces are the beam-column strength interaction diagrams, where the P_n anchor point for the Effective Length method $P_{n,KL}$ is based on $KL=2L$ and for the Direct Analysis method $P_{n,L}$ is based on L.

The calculated strengths, as determined by the two methods, are summarized in Table 1 in terms of the maximum vertical load P_u (shown in bold). Net strengths for the two methods are within 10%, even though the interim results are quite different. For example, as shown in Figs. 3a-c and summarized in Table 1, the maximum internal moments at the strength limit point are much larger for the Direct Analysis method; whereas the $P_u/\phi P_n$, ratios, which indicated the relative significance of the axial load and moment terms in the governing interaction equation, are consistently larger for the Effective Length method. Moreover, the $P_u/\phi P_n$ ratios for the Effective Length procedure do not change much with increasingly slenderness, because this procedure relies to a much greater degree on capturing stability effects in the P_n term. Conversely, in the Direct Analysis procedure the $P_u/\phi P_n$ contribution decreases and the moment term dominates the solution for cases with increasing slenderness.

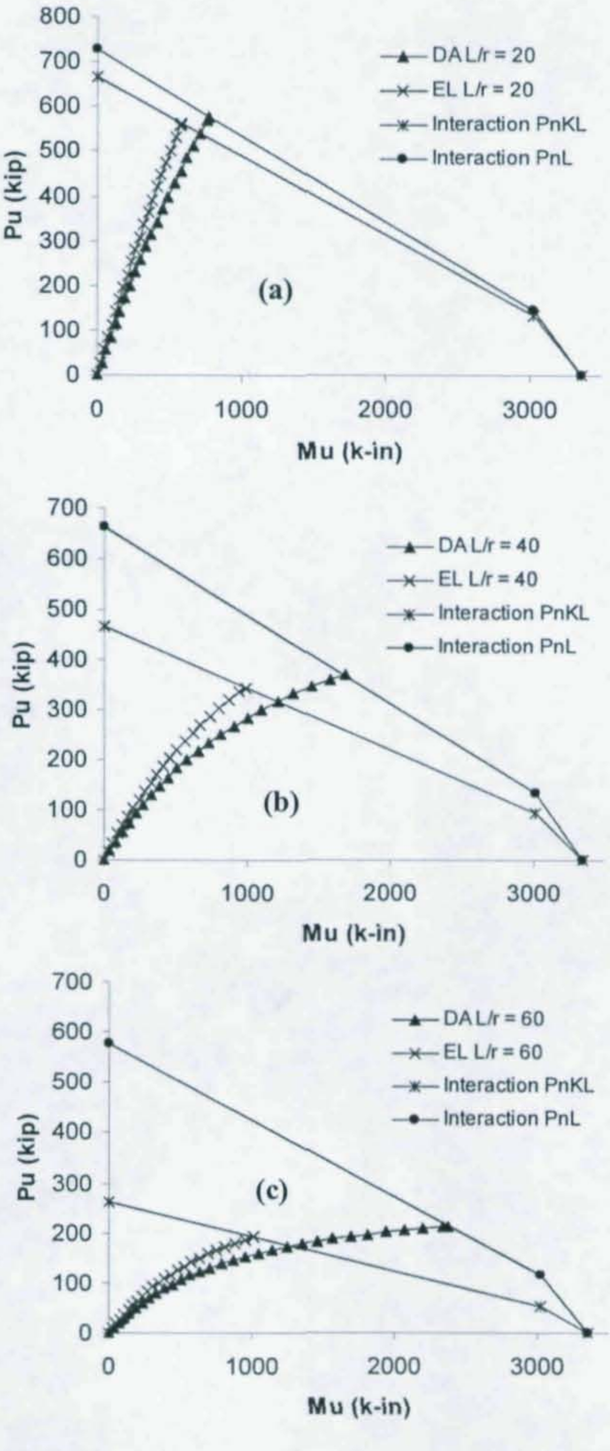


Fig. 3 – Comparison of P-M interaction curves for cantilever column example
(a) short L/r = 20, (b) medium L/r = 40, (c) long L/r = 60

Table 1 – Results for Cantilever Column Example

	Effective Length Method				Direct Analysis Method			
L/r	P_u*(kips)*	M_u*(k-in)*	M_2/M_1	$P_u/\phi P_n$	P_u*(kips)*	M_u*(k-in)*	M_2/M_1	$P_u/\phi P_n$
20	**562**	580	1.17	0.85	**578**	777	1.27	0.80
40	**345**	988	1.63	0.74	**371**	1680	2.16	0.56
60	**193**	1007	1.98	0.74	**213**	2376	3.53	0.37

This comparison highlights the pros and cons of each method. Compared to Direct Analysis procedure, the Effective Length method has the advantage of being less sensitive to the accuracy of the second-order analysis. On the other hand, the method requires calculation of effective column buckling lengths (KL), which can be difficult for complicated structures. Direct Analysis eliminates the need to calculate effective buckling lengths and provides more accurate measures of the true second-order moments. This latter point is important for the design of members and connections, which restrain the beam-column. For example, in the cantilever column example, the base moments from the Direct Analysis procedure take into account initial out-of-plumb and inelastic second-order effects, which are not captured in the Effective Length procedure. Referring to the second-order moments reported in Table 1, the difference in moments between the two methods can be quite large. Subject to the assumed geometric imperfections (out-of-plumb) and residual stresses, validation studies have shown that the moments calculated by the Direct Analysis procedure are generally conservative and closer to the true values. Observations of the type described here about the underestimation of design moments the Effective Length method, led to the new minimum moment requirement (Eq. 4) for the Effective Length method. One should recognize, however, that this minimum does not address cases such as shown in this cantilever example, where the calculated moments in the Effective Length procedure are above the minimum of $0.01PL$, but still less than the actual values, which are calculated more accurately by the Direct Analysis procedure.

VALIDATION STUDIES

Over the course of developing the proposed stability provisions, hundreds of validation analyses have been investigated by members of the SSRC-AISC Ad Hoc Committee. Some of the early investigations (e.g., Maleck 2001, Maleck and White 2003) helped guide development of the provisions, and two recent papers by Maleck and White (2002) and Martinez-Garcia (2002) provide selected case studies to validate the final version of the proposed design methods. These two studies investigated twenty-five frame configurations under multiple load cases, representing several hundred analyses with about 150 comparison points between the two design approaches and refined nonlinear analyses. These studies focus on the limit state of combined axial load and bending in the beam-columns and do not specifically address design checks in restraining beams and connections.

Examples of the frame configurations considered in the benchmark studies by Maleck and White (2002) are shown in Fig. 4. These two-column portal frames and individual column structures provide rigorous test cases of non-redundant systems of varying slenderness, levels of axial compression, and leaning column effects. Other multi-story and multi-bay frames investigated by Martinez-Garcia (2002) embody attributes of realistic structures that pose particular challenges in evaluating stability, three of which are presented in the next session of *Illustrative Examples*. The problems investigated for the benchmark studies are ones where second-order effects are large and where errors between the stability design methods and more exact methods are accentuated. In this sense, these benchmark studies represent extreme cases, which tend to exaggerate the differences one would typically encounter in design practice.

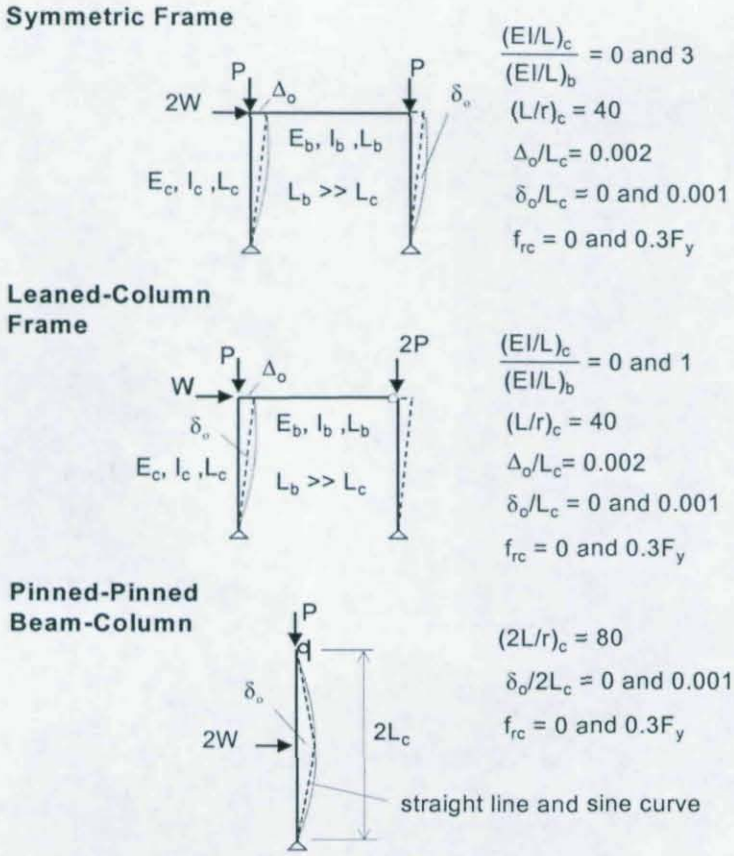


Fig. 4 – Test structures used for validation study (Maleck 2001)

Detailed analysis solutions based on second-order spread-of-plasticity analyses are used as benchmarks against which the proposed design methods were validated. These benchmark solutions incorporate the effects of gradual yielding, initial geometric imperfections, and residual stresses, as outlined previously in the section of this report on *Behavioral Effects*. Thus, they represent the state-of-art in simulating inelastic stability of beam-columns and frames. Material properties (E and Fy) in the spread-of-plasticity analyses were reduced using a resistance factor of 0.9, such that the maximum strength calculated in these analyses corresponds to the structural "design strength" – as opposed to a "nominal strength".

Overall, the two studies (Maleck and White 2002 and Martinez-Garcia 2002) confirm that both the Effective Length and Direct Analysis methods are sufficiently accurate (relative to current methods) for design and that the errors (relative to the spread-of-plasticity solutions) are comparable for the two methods. Maleck and White report that on average the two approaches give strengths within 2% to 7% of those obtained by refined analyses. The maximum discrepancies they observed for the Direct Analysis approach, relative to the refined analyses, range from –6% (unconservative) to +13% (conservative) for members subjected to strong-axis bending and –13% to +15% for members subjected to weak-axis bending. For the Effective Length approach, the maximum discrepancies range from –8% to +18% for strong-axis bending and –17% to +17% for weak-axis bending. These errors are based on design checks made with rigorous second-order elastic analyses. Maleck and White caution that the Direct Analysis design checks based on approximate P-Δ analyses can be up to –23% (unconservative) for members subjected to weak axis bending. This is an example of why the Direct Analysis provisions specify the need for a rigorous analysis when second-order effects are large. Maleck and White further note that for frames where $Pu < 0.15\ (\pi^2 EI/L^2)$, the maximum unconservative errors associated with approximate P-Δ

analyses for the Direct Analysis approach are limited to the maximum errors present in existing Effective Length procedures.

ILLUSTRATIVE EXAMPLES

Three example problems are presented next to illustrate practical application of the proposed stability design methods. The first two examples are frames with heavy gravity loads and large second-order amplification factors. The third example is a stiffer six-story frame, which is more representative of multi-story building frames. Each example includes a comparison of results from the Effective Length and Direct Analysis approaches and more rigorous spread-of-plasticity solutions. The spread-of-plasticity solutions have been independently reported by Maleck (2001) and Martinez-Garcia (2002), and the design solutions have been prepared by multiple members of AISC TC 10 and the AISC-SSRC Ad Hoc Committee. All design checks are based on the LRFD approach and load combinations. Resistance factors used in the design checks are $\phi_b=0.9$ for bending and $\phi_c=0.85$ for compression, the latter of which is slightly smaller than the proposed change to $\phi_c=0.9$ in Chapter E of the 2005 Standard.

Low-Rise Industrial Example

The first example, see Figure 5, is a framing bent from a large floor plan single story industrial building, such as an automobile plant. With heavy material handling equipment hung from the roof and a small wind exposure, such structures are dominated by gravity loads with large second-order effects (Springfield, 1991). Loading shown in Figure 5 represents an eleven bay configuration with ten leaning columns (only two of which are shown) and two lateral-load resisting columns. The concentrated load P has a tributary roof area of 35 ft x 35 ft, and the wind load $W = 5.12$ kips.

The member sizes satisfy a drift limit of $H/400$ for the service load wind of $0.7W$, and the design strength of the frame exceeds the minimum requirement of the LRFD strength load combinations. Based on refined spread-of-plasticity analyses, the frame has a design strength ratio 17% larger than the required strength for gravity ($\phi\lambda_{1.2D+1.6L}=1.17$, where $\phi=0.9$) and 20% larger than required under gravity plus wind ($\phi\lambda_{1.2D+0.5L+1.6W}=1.20$). Using the equation B5-5 (from the July 2002 ballot of Chapter B for the 2005 Standard), the second-order amplification factor under design gravity loads is $B_2 = 2.41$. Under the design gravity plus wind loading combination, $B_2 = 1.74$.

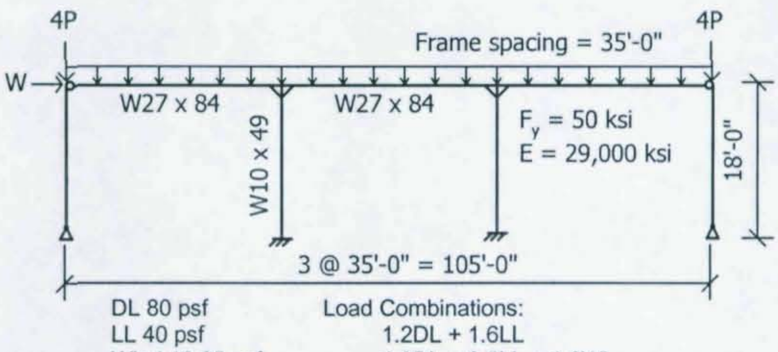


Figure 5 – Single-Story Industrial Building

Axial column forces and maximum moments under the factored load combinations are summarized in Table 2. The Effective Length results are from a second-order analysis of a model based on the ideal geometry of the frame under the factored load combinations (no geometric imperfections or notional loads are introduced). The Direct Analysis results incorporate initial geometric imperfections through the notional load of 0.2% times the factored gravity loads (1.2D + 1.6L for the first combination and 1.2D + 0.5L for the second combination); and stiffness degradation is incorporated by reducing the flexural stiffness of all framing members by to 0.8EI. Since the axial load ratio $P/P_y<0.5$, no additional τ-factor stiffness adjustments are required. The "spread of plasticity" results are from a second-order inelastic analysis, which models gradual yielding through the member cross sections and along their length due to the combined effects of thermal residual stresses and the applied loads.

Table 2: Member Effects Under Factored Load Combinations for Low-Rise Industrial Example

Load Case	Member Check	Analysis/Design Method		
		Spread of Plasticity	Effective Length	Direct Analysis
1.2D+1.6L	P_{col} (kip)	215	216	218
	M_{col} (k-in)	930	407	1220
	M_{bm} (k-in)	8660	8410	8690
1.2D+0.5L+1.6W	P_{col} (kip)	154	158	160
	M_{col} (k-in)	1310	1040	1550
	M_{bm} (k-in)	6490	6360	6630

Referring to Table 2, the Effective Length and Direct Analysis methods both predict the maximum beam moments and axial column forces within about 4% of those from the spread-of-plasticity analysis. On the other hand, there are significant differences in the column moments, particularly for the gravity load case (1.2D + 1.6L). The Direct Analysis method predicts the column moments on average about 25% higher than the spread-of-plasticity solution, and the Effective Length method predicts column moments on average about 40% smaller than the spread-of-plasticity solution. These differences are also reflected in the calculated displacements. This small moments calculated according to the Effective Length method illustrate the need for the newly proposed minimum connection moment requirement (Eq. 4, $\Sigma M_u > 0.01\Sigma P_u L$). Without this minimum requirement, the connection would be under-designed for the second-order moment induced by the combined effects of gravity load and column out-of-plumb.

Using the member forces from Table 2, the columns are checked using the interaction formula for in-plane or out-of-plane (torsional flexural) failure, and the resulting interaction ratios are summarized in Table 3. For the Effective Length method, the in-plane checks are based on a column strength of $\phi P_{nx,KL}$ = 236 kips, obtained with an effective length factor of K = 2.3 using Eq. C-C2-6 of AISC (1999). In-plane checks for the modified stiffness and notional load methods are based on $\phi P_{ns,L}$ = 511 kips, and out-of-plane checks

Table 3: Interaction Values for Low-Rise Industrial Example

Load Case	Member Check	Analysis/Design Method	
		Effective Length	Direct Analysis
1.2D+1.6L	In-plane	1.05	0.83
	Out-of-plane	0.62	0.81
1.2D+0.5L+1.3W	In-plane	1.01	0.82
	Out-of-plane	0.58	0.77

are all based on $\phi P_{nu,L}$ = 361 kips. The column design moment is ϕM_p = 2718 k-in. In-plane interaction is checked using Eqs. 2a & 2b, and the out-of-plane check is made using Eq. 3.

Referring to Table 3, both the Effective Length and Direct Analysis checks are governed by the in-plane strength (shown shaded). The Effective Length method is more conservative, as evidenced by a larger interaction value as compared to the direct analysis method. The in-plane checks can be compared to inelastic limit load ratios of $\phi\lambda_{1.2D+1.6L}$=1.17 and $\phi\lambda_{1.2D+0.5L+1.6W}$ =1.20, obtained from the spread-of-plasticity analyses. The inverse of these limits (0.85 and 0.84 for gravity and gravity+wind, respectively) help to gauge the conservatism in the methods, where larger interaction checks would be conservative and smaller checks unconservative. Compared to these values (0.85 and 0.84) the in-plane checks for the Effective Length method about 15% conservative, whereas the Direct Analysis results appear slightly unconservative (e.g., 0.83 < 0.85 and 0.82 < 0.84). However, since the member forces vary nonlinearly with load (due to second order effects), this simple comparison is approximate and a more accurate comparison would be obtained by scaling up the loads in the Direct Analysis to the point that the in-plane interaction check is equal to 1.0. Scaling the loads in this way results in a limit load of $\phi\lambda_{1.2D+1.6L}$=1.10 for the Direct Analysis which is about 6% smaller (conservative) as compared to the in-plane limit from the spread-of-plasticity solution. Thus, this case demonstrates that the Direct Analysis is conservative (safe) and provides the potential for a more efficient design as compared to the Effective Length method.

Example Connection Design for Effective Length Method:
For the gravity load case, the minimum beam-column connection strength provision of the Effective Length procedure (Eq. 4) provides for a minimum required connection strength of $\Sigma M_u > 2330$ k-in. This is calculated using the vertical load in one lateral load resisting column and one of the leaning columns (a total of $\Sigma P_u = 1078$ kips). Comparing this to the more exact required column moment from the spread-of-plasticity solution (M_u = 930 k-in) indicates that the minimum required by the Effective Length method is quite conservative in this case. The required strength using the Direct Analysis method would be 1220 k-in. To help gauge the impact of these provisions, a connection designed for the Effective Length method moment of 2330 k-in is shown in Fig. 6. This connection would require eight – 1 inch diameter bolts, which is not excessive for the connected members.

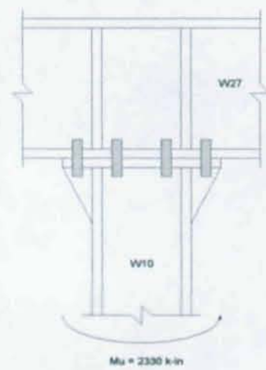


Figure 6: Example Connection Design (8-1" dia. bolts)

Grain Storage Bin

The second example is the support rack for a grain storage bin with the dimensions and loading shown in Figure 7. This is a case where calculation of the column effective lengths is not obvious, and where the Direct Analysis method offers a clear benefit. Columns are assumed to be braced out-of-plane and the cross-beams and bracing are pin-connected to the columns. For the diagonal bracing, one-inch diameter round bars are assumed. Using an elastic critical load analysis, the second-order amplification factors are B_2 = 2.75 and B_2 = 2.20 for the gravity and wind load combinations, respectively. The spread-of-plasticity analyses predict inelastic limit load ratios of $\phi\lambda_{1.4G}$= 1.13 and $\phi\lambda_{1.2G+1.6W}$ = 1.07.

As in the previous example, results for the Effective Length method are calculated for the ideal geometry and stiffness; whereas the Direct Analysis method is based on a reduced stiffness (0.8EI) and with notional loads applied in combination with the design loads. Like the previous example, no additional τ adjustment of stiffness properties is required for Direct Analysis since the axial load ratio $P/P_y < 0.5$.

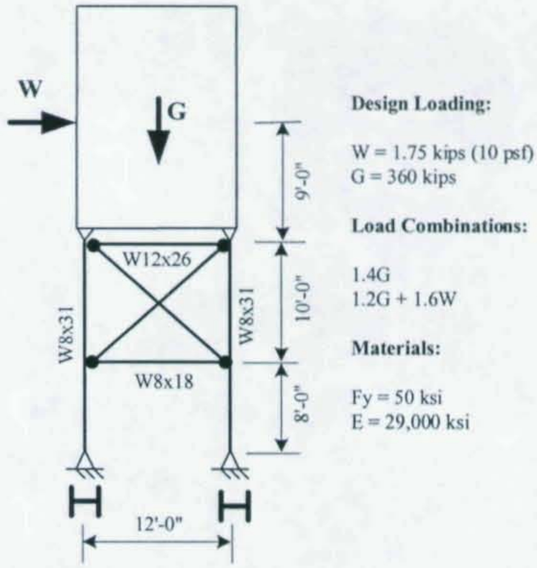


Figure 7 – Grain Bin Support Frame

Maximum column forces and moments (required strengths) are summarized in Table 4 and the interaction checks are summarized in Table 5. As in the previous examples, there is not much difference in axial loads between the methods, but there are large variations in the calculated moments. This is particularly prevalent for the gravity load case, where the calculated column moments for the Effective Length method are essentially zero, and the moments for the Direct Analysis method are about 36% larger than those in the spread-of-plasticity analysis. Under the lateral load case, the differences are less, with moments for the Effective Length method about 24% less (unconservative) than the spread-of-plasticity results and those for Direct Analysis about 38% larger (conservative).

Table 4: Member Effects for Grain Storage Bin Example

Load Case	Member Check	Analysis/Design Method		
		Spread of Plasticity	**Effective Length**	**Direct Analysis**
1.4G	$P_{c,top}$ (kip)	233	247	249
	$P_{c,bot}$ (k-in)	255	252	257
	M_c (k-in)	161	2	220
1.2G+1.6W	$P_{c,top}$ (kip)	203	217	220
	$P_{c,bot}$ (k-in)	224	225	230
	M_c (k-in)	380	288	526

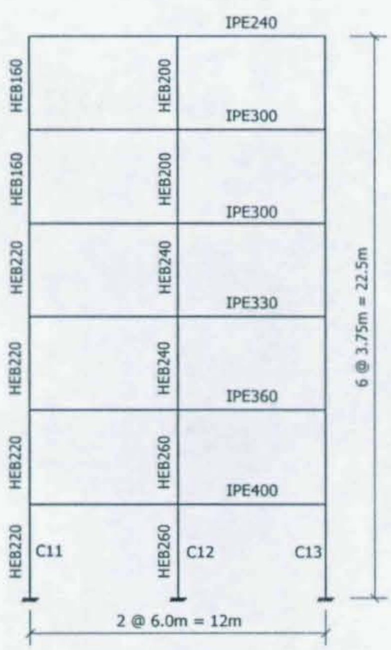
Table 5: Interaction Values for Grain Storage Bin Example

Load Case	Member Check	Analysis/Design Method	
		Effective Length	**Direct Analysis**
1.4GL	Top Column	1.07	0.84
	Bot. Column	1.04	0.85
1.2GL+1.6W	Top Column	1.12	0.96
	Bot. Column	1.11	0.99

The interaction checks, shown in Table 5, are based on the following in-plane column strengths: critical load method $\phi P_{nKL,top} = 232$ kips ($K = 2.4$), $\phi P_{nKL,,bot} = 243$ kips ($K = 2.9$); and the direct analysis method, $\phi P_{nL,top} = 355$ kips, $\phi P_{nL,,bot} = 366$ kips. The K factors for the Effective Length procedure are based on an elastic critical load analysis of the structure under gravity loads. The results in Table 5 show that strength interaction checks based on the Effective Length method are roughly 20% more conservative than the Direct Analysis method. Using the spread-of-plasticity analysis as a benchmark of the actual behavior, interaction values larger than 0.89 (for *1.4G*) and 0.93 (for *1.2G+1.6W*) are conservative. The Effective Length interaction values (1.07 and 1.12) exceed these and are about 20% conservative. The Direct Analysis method is slightly conservative for the gravity plus wind case (0.99 > 0.89) and appears slightly unconservative for the gravity load case (0.85 < 0.93). However, as mentioned in the industrial frame example these linear comparisons are only approximations. When the gravity loads are scaled in the direct analysis to provide an interaction value of 1.0 for the bottom column, the limit load ratio was $\phi\lambda_{1.4G} = 0.99$, which exceeds the value of $\phi\lambda_{1.4G} = 1.07$ from the spread-of-plasticity solution. This indicates that the Direct Analysis is, in fact, 9% lower (conservative) as compared to the spread of plasticity solution.

Multi-story Frame Example

The final example is the multi-story frame shown in Figure 8. One load case is investigated ($1.0G + 1.0W$, where the specified loads are already factored), and member forces and interaction checks are presented for the three columns in the first story. Unlike the previous examples, this frame is fairly stiff with $B_2 = 1.10$ for the first story. The second-order spread-of-plasticity analysis predicts an inelastic design strength ratio of $\phi\lambda_{1.4G} = 1.06$ for this frame, which combined with the low B_2 indicates that it is dominated more by yielding than second-order effects. The center columns of the first two stories have high axial forces and are subject to the τ–factor adjustment in the Direct Analysis method. As discussed earlier, when $P/P_y > 0.5$ for any column, the τ–factor adjustments can be used or an additional notional load of $0.001Y_i$ can be used. In this example, both approaches are presented and compared.



Section	A (cm^2)	I_x (cm^4)	Z_x (cm^3)
HEB 160	54.3	2,492	354
HEB 200	78.1	5,696	643
HEB 220	91.0	8,091	827
HEB 240	106	11,260	1,053
HEB 260	118	14,920	1,283
IPE 240	39.1	3,892	367
IPE 300	53.8	8,356	628
IPE 330	62.6	11,770	804
IPE 360	72.7	16,270	1,019
IPE 400	84.5	23,130	1,307

Figure 8 – Multistory frame

The first floor column forces, summarized in Table 6, reveal that differences between the three methods are much smaller than in the previous examples. This follows from the fact that the second-order amplification is smaller in this example, which is more typical of most multi-story frames than the prior examples.

Table 6: Member Effects for Multistory Frame Example

Location and Effect (1.0G+1.0W)	**Analysis/Design Method**			
	Spread of Plasticity	**Effective Length**	**Direct Analysis with Notional Load**	**Direct Analysis with τ Reduction**
P_{c11} (kN)	683	672	659	662
P_{c12} (kN)	1720	1770	1770	1770
P_{c13} (kN)	921	884	897	894
M_{11} (kN-m)	67	48	58	59
M_{12} (kN-m)	115	118	143	135
M_{13} (kN-m)	99	87	96	96

Results of the beam-column interaction checks (Table 7) show that the Effective Length method is slightly less conservative than the Direct Analysis method, which is in contrast to the previous two examples where the opposite was true. Based on the AISC (1999) alignment charts, the effective buckling length factors for the first story columns are $K = 1.35$, assuming the AISC suggested value of $G = 1.0$ for the foundation support. Accordingly, the in-plane interaction checks for the Effective Length procedure are based on design compression strengths of $\phi P_{nKL\ 11} = \phi P_{nKL\ 13} = 1580$ kN and $\phi P_{nKL\ 12} = 2140$ kN. The in-plane Direct Analysis checks (with $K = 1$) are based design strengths of $\phi P_{nL\ 11} = \phi P_{nL\ 13} = 1680$ kN and $\phi P_{nL\ 12} = 2230$ kN. All out-of-plane checks are based on $K = 1$, with $\phi P_{nL\ 11} = \phi P_{nL\ 13} = 1460$ kN and $\phi P_{nL\ 12} = 2010$ kN; and moment strengths of $\phi M_{p11} = \phi M_{p13} = 175$ kN-m and $\phi M_{p12} = 275$ kN-m are used throughout. As summarized in Table 7, the resulting interaction checks were all close, with the Direct Analysis solutions about 2% to 3% conservative, relative to the Effective Length method. With reciprocal of the inelastic limit load factor equal to 0.94, the average interaction values ranging from 0.94 to 0.99 indicate that all of the stability design methods are conservative in this case.

Table 7: Interaction Values for Multistory Frame Example

Location (1.0GL+1.0W)	**Analysis/Design Method**		
	Effective Length	**Direct Analysis with Notional Load**	**Direct Analysis with τ Reduction**
C11 – in-plane	0.67	0.69	0.69
C11 – out-of-plane	0.54	0.56	0.57
C12 – in-plane	1.21	1.25	1.23
C12 – out-of-plane	1.06	1.15	1.12
C13 – in-plane	1.00	1.02	1.02
C13 – out-of-plane	0.85	0.92	0.91
Average in-plane	0.96	0.99	0.98

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