PROPOSED NEW PROVISIONS FOR FRAME STABILITY USING SECOND-ORDER ANALYSIS

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INTRODUCTION

Assessment of frame stability remains one of the more challenging aspects in the design of steel buildings. The issue is complicated by the interdependence of member and frame response, which requires that system stability effects be incorporated within member-based specification design equations. In addition, permitting geometrically and materially nonlinear structural response at the nominal limit state of the structure is fundamental to achieving economical design. As such, most specifications worldwide couple some form of nonlinear analysis with design provisions to account for significant behavioral effects (Galambos et al. 1998). In the U.S., the AISC LRFD Specification (AISC 1999) requires, as a minimum, that second-order elastic analysis (or first-order analysis coupled with moment amplification) be used to compute required element strengths.

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Two basic approaches commonly used for assessing member and frame stability within the context of using second-order elastic analysis are *critical load* and *direct analysis* approaches (ASCE 1997; White and Clarke 1997; Galambos 1998). *Critical load* approaches, such as used in AISC (1999), involve calculation of the member elastic or inelastic critical loads (or, alternately, effective lengths) as input to a column curve to determine the nominal column compressive strength. This strength is then combined in the beam-column interaction check. *Direct analysis* approaches, used in various forms in several other countries, establish beam-column strength by applying member or frame imperfections, equivalent notional lateral loads, or modified member stiffnesses in the analysis. Two *direct analysis* procedures, so-called *notional load* and *modified stiffness* methods, are investigated herein.

In 2000, an AISC-SSRC Task Committee was formed to develop improved specification provisions for member and frame stability. This paper presents initial findings of this effort. The paper begins with a summary of behavior effects and second-order analysis techniques that must considered when assessing member stability. *Critical load* (i.e., effective length), *notional load*, and *modified stiffness* procedures are then presented within the context of the AISC Specification. This is followed by a summary of benchmark problems used to verify the accuracy of the proposed procedures. Three examples are then presented to illustrate application of the methods to practical problems.

BEHAVIORAL EFFECTS

There are potentially many parameters and behavioral effects that influence stability of steel-framed structures, and the extent to which these factors are modeled in analysis will affect the criteria that one applies in design of the frame, its members and connections. Without repeating more complete presentations given elsewhere (Birnstiel and Iffland, 1980; McGuire, 1992; White and Chen, 1993; ASCE, 1997; Galambos, 1998), it is important to review three basic aspects of behavior: geometric nonlinearities, inelastic spread-of-plasticity, and member limit states. These ultimately govern frame deformations under applied loads and the resulting internal load effects.

Geometric Nonlinearities and Imperfections: Modern stability design provisions are based on the premise that the member forces are calculated by second-order elastic analyses, where equilibrium is satisfied on the deformed structure. Where stability concerns are significant, consideration must be given to initial geometric imperfections in the structure due to fabrication and erection tolerances. For the purpose of calibrating the stability requirements described later, initial geometric imperfections are conservatively assumed as equal to the maximum fabrication and erection tolerances permitted by AISC (2000). For columns and frames, this implies a member out-of-straightness equal to $L/1000$, where L is the member length (between brace or framing points) and a frame out-of-plumb equal to $H/500$, where H is the story height. The out-of-plumb is also limited by the absolute bounds as specified in AISC (2000).

Inelastic Spread of Plasticity: The proposed analysis/design approaches are calibrated against inelastic distributed-plasticity analyses that account for spread of plasticity through the member cross-section and along the member length. Thermal residual stresses in W-shape members are assumed to have maximum values of $0.3Fy$ and are distributed according to the so-called Lehigh pattern - linearly varying across the flanges and uniform tension in the web (Galambos, 1998).

Member Limit States: Member strength may be controlled by one or more of the following limit states: cross section yielding, local buckling, flexural buckling, and torsional-flexural buckling. For the types of frame analyses envisioned for design, it is assumed that the analysis does not model local flange/web buckling or torsional-flexural buckling. Therefore, these limits must be considered in separate member design checks. For inelastic analyses, the member yield limit is incorporated directly in the analysis; and for elastic analyses, this limit can be checked by an interaction equation that approximates the P-M yield surface. Whether or not the analysis captures in-plane flexural buckling depends on the extent to which the maximum moments are affected by distributed plasticity and member straightness. Uncertainty regarding whether the analysis captures this effect suggests the need to apply a member check for in-plane flexural buckling.

SECOND-ORDER ELASTIC ANALYSIS

The stability design provisions discussed in this paper are intended for use with second-order elastic analysis. This implies that that analysis provides equilibrium on the deformed structural configuration, with the material stiffness held constant during the analysis. As described later, this may include cases where the elastic stiffness is adjusted to account for (in an approximate way) inelastic effects. In practice, there are alternative approaches one can employ for conducting second-order analyses, some of which are more rigorous than others. For the purpose of this discussion, second-order elastic analyses will be categorized as "rigorous" or "approximate". The difference between these two depends on the extent to which P-δ effects are modeled and whether the problem is "linearized" to expedite the solution.

Rigorous second-order analyses are those which accurately model all significant second-order effects. Rigorous analyses include solution of the governing differential equation, either through stability functions or computer frame analysis programs that model these effects (McGuire 1992; Galambos 1998). Many (but not all) modern commercial computer programs are capable of rigorous analyses, though users should verify this. In some cases, modification of first-order analysis results through second-order amplifiers [e.g., B_1 and B_2 factors as per AISC (1999)] may constitute a rigorous analysis, but this depends on the magnitude of second-order effects and other aspects of the problem.

Approximate second-order analyses are any methods that do not meet the requirements of rigorous analyses. A common type of approximate analyses are those which only capture P-Δ (due to member end translations, e.g., interstory drift) but fail to capture P-δ effects (due to curvature of the member relative to its chord). This can arise both in computer analysis programs and when applying B_1 and B_2 amplifiers.

The cantilever column shown in Fig. 1 represents a simple test case to determine the accuracy of a second-order analysis. A rigorous analysis will capture the curved second-order moment diagram, which has two effects on response. First, where the member is very flexible, relative

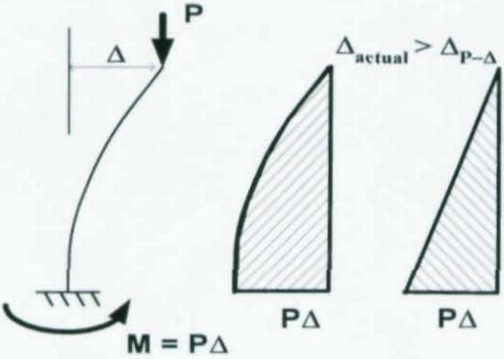

Fig. 1 – Second-order effects in cantilever column

to its boundary conditions, the curved geometry can lead to a condition where the maximum moment occurs along the member length, as opposed to at the member ends. Second, the increase in moments and corresponding curvatures will increase the member end deflection, Δ. The latter point can be explained by the fact that the critical buckling load inferred by the linear moment diagram (which is representative of a story stiffness approach for calculating the *B2* factor) is 22% larger than the actual buckling load. Examples of the differences one may encounter are noted in the illustrative examples presented later, and further discussed by LeMessurier (1977) and Galambos (1998).

CRITICAL LOAD APPROACH

The critical load, or effective length, approach for assessing member axial compressive strength has been used in various forms in the AISC Specification since 1961. The approach is based on the critical elastic (or inelastic) buckling load, $P_e = \pi^2 EI/(KL)^2$, accounting for the restraint offered to the member by the surrounding frame. The critical load is then related to the axial compressive strength, P_n, through an empirical column curve that accounts for member geometric imperfections, yielding, and residual stresses. This column strength is then combined with the moment capacity and second-order forces in an interaction equation.

Given P_n, the beam-column strength is computed in AISC (1999) through the following interaction equation:

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \quad \text{for } \frac{P_u}{\phi_c P_n} \geq 0.2 \qquad (1a)$$

$$\frac{P_u}{2\phi_c P_n} + \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \leq 1.0 \qquad \text{for } \frac{P_u}{\phi_c P_n} < 0.2 \qquad (1b)$$

where P_u and M_u are the axial load and moment determined from second-order analyses, and $\phi_c P_n$ and $\phi_b M_n$ are the design axial compressive and flexural strengths, respectively. Figure 2a shows a plot of this interaction equation, with the anchor point on the vertical axis being represented by P_{nKL} to clarify that an effective length factor is used to calculate P_n. Also shown is the same interaction equation where the first term is based on the squash load, P_y. The load-deformation response of a typical member, obtained from second-order spread-of-plasticity analysis (SSRC 1993) and labeled "actual response," indicates the maximum axial force, P_u, that the member can attain prior to instability. The results of a second-order elastic analysis, as would be done in design practice, are then shown. The moment is amplified in this analysis such that the load-deformation curve intersects the member interaction diagram where the axial strength is

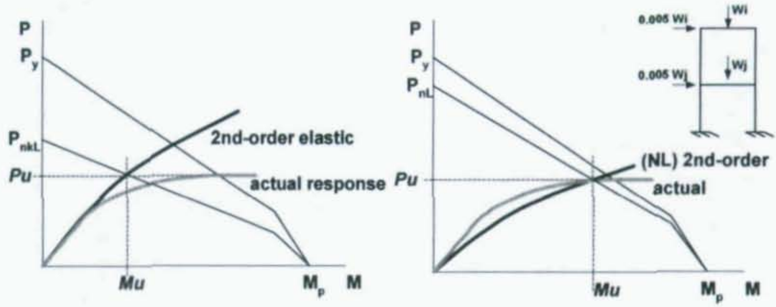

Fig. 2 - Interaction strength for a) critical load approach and b) notional load and modified stiffness approaches

limited to P_u. Accurate assessment of P_{nKL}, and thus of the member critical load or effective length, is key to achieving an accurate solution.

Many approaches have been proposed for computing the member critical load (ASCE, 1997). These may be classified as subassemblage [most typically through the use of the nomographs in AISC (1999)], story-based, or system critical load calculations. Story-based critical loads have the advantage of accounting for the destabilizing effects of weak or leaning columns in a story, relative to strong columns in the story One such story-based approach, based on the work of LeMessurier (1977), is expressed as:

$$P_e = 0.85 \frac{P_u}{\sum P_u} \frac{\sum HL}{\Delta_{oh}} \leq \frac{\pi^2 EI}{L^2} \quad (2)$$

where Δ_{oh} is the first-order sidesway deflection due to the lateral shear H, P_u is required column strength, and the summations are taken across a given story in a building. The 0.85 coefficient accounts for the approximations in column moments, such as shown in Fig. 1.

NOTIONAL LOAD AND MODIFIED STIFFNESS METHODS

Calculation of member critical loads (effective lengths) is non-trivial, particularly where the assumptions of the AISC nomographs often break down. The use of direct second-order analysis provides an attractive alternative for many structural framing systems. In these approaches, geometric imperfections (primarily initial out-of-plumbness) and inelastic effects (including residual stresses) are accounted for through modifications to the second-order elastic analyses. In checking the member interaction equations (Eq. 1a and b), the nominal axial compressive strength, P_n, is then based on the actual member length, termed P_{nL}. Use of the column strength P_{nL}, rather than the squash load P_y, is a practical measure to ensure that braced modes of flexural buckling are captured by the interaction equation.

For both the notional load and modified stiffness approaches, geometric imperfections equal to the maximum out-of-plumbness *(H/500)* are accounted for through application of an equivalent notional lateral load equal to 0.002 times the summation across each floor of the gravity load applied on that floor. Alternatively, this imperfection could be directly incorporated in the analysis model. Where equivalent loads are applied, they must be included with each load combination and applied in the direction of lateral load, or for load combinations with no lateral load, in the direction of sway of the frame under gravity loads.

The notional load approach accounts for inelastic effects through application of an additional notional load of 0.003 times the summation of gravity loads. This value was determined by calibration to more exact solutions in a series of benchmark problems (ASCE, 1997). Combining this factors with the one for geometric imperfections, the notional load method consists of applying notional loads at each floor equal to 0.005 times the factored gravity load applied at that story.

The modified stiffness approach accounts for inelastic effects through a reduced flexural rigidity, EI^*, calculated for each column in the lateral resistance system as follows:

$$EI^* = \tau EI \quad \text{for } M_n < 1.2\, M_y \tag{3a}$$

$$EI^* = 0.8\tau EI \quad \text{for } M_n > 1.2\, M_y \tag{3b}$$

where the stiffness reduction factor $\tau = 1.0$ for $P_u/P_y < 0.5P_y$ and $\tau = 4[P/P_y(1-P/P_y)]$ otherwise. The distinction between yield and nominal moment, M_y and M_n, accounts for the influence of shape factor and residual stresses on progressive yielding and inelastic softening.

Figure 2b shows schematically how the direct second-order analysis approaches capture interaction strength. The interaction diagram based on using P_{nL} $(K=1)$ is closer to the cross section member strength, and the notional loads and modified stiffnesses both cause additional amplification of moments and deflections. The notional loads and stiffness modifications are calibrated with the interaction equation, such that the resulting axial strength P_u is close to the true strength. One result of this is that these methods are more sensitive to the accuracy of the second-order analysis than the critical load method, whereas the critical load method is more sensitive to accurate determination of the critical buckling load (or effective buckling lengths).

These analysis/design approaches are most beneficial if coupled with separate interaction equations for assessing in-plane and out-of-plane strength for beam-columns that are loaded about the strong axis. Considerable research has been conducted on appropriate interaction equations (e.g., see White and Chen, 1993), several of which are under consideration presently for inclusion in the AISC Specification.

VALIDATION STUDIES

Shown in Fig. 3 are a set of test structures that were analyzed to evaluate the accuracy of the three analysis/design approaches. The three structures represent the range of conditions in practice, including symmetric and unsymmetric framing, leaning columns, and individual member behavior. For each structure, multiple conditions were analyzed to investigate the effects of strong versus weak axis bending, geometric imperfections, residual stresses, boundary conditions, and slenderness. Over fifty cases were considered with each analyzed for up to nine different ratios of axial loads to bending moments. Each case was evaluated using the three analysis/design methods described previously and compared to results from detailed second-order spread of plasticity analyses. In total, over 1800 separate analyses were run.

Referring to Fig. 4, the typical analysis results consisted of *P-M* interaction plots of the limiting strengths from the proposed analysis/design approach compared to the "actual" strength determined from a spread-of-plasticity analysis. These spread-of-plasticity analyses captured second-order distributed plasticity effects, including the initial geometric imperfections and residual stresses, as outlined previously.

Differences (errors) between the analysis methods were measured in terms of the radial distance from the interaction plots (Fig. 4). A summary of the overall error statistics is presented in Table 1. Included are results for the three analysis/design methods described previously. Data are presented for cases where the second-order elastic analyses were either "rigorous" (2^{nd}-order) or "approximate" (P-Δ).

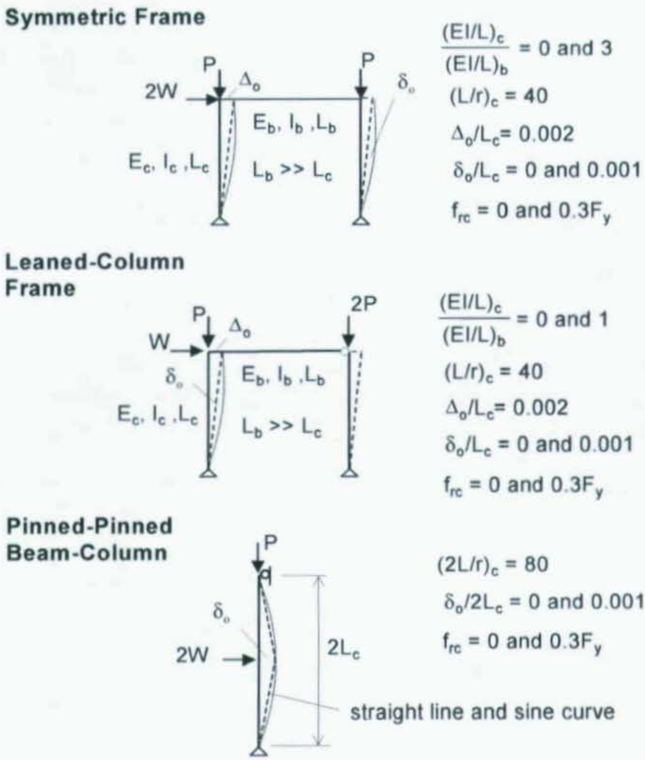

Fig. 3 – Test structures used for validation study

The analyses and error statistics clearly indicated that none of the three methods are exact, each with its own shortcomings and limitations. The average errors range between 8% unconservative and 9% conservative, and the extreme errors were up to 17% unconservative for rigorous 2^{nd}-order analyses and 25% unconservative for approximate (P-Δ) analyses. Differences between the rigorous and approximate analyses within each method demonstrate that the modified stiffness and notional load methods are more sensitive to the second-order analysis accuracy. This follows from the basic approach in the methods (Fig. 2)

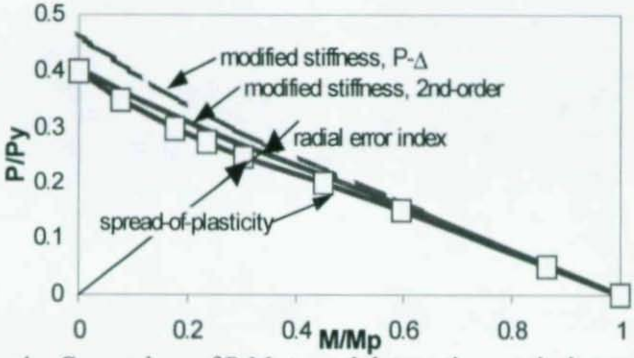

Fig. 4 – Comparison of P-M strength interaction results between spread-of-plasticity solutions and proposed design approaches

and implies that one needs to be more careful with the second-order analysis with the notional load and modified stiffness methods. Finally, while some of the extreme errors are large, one should remember that (1) statistics for the critical load method are nothing new and simply reflect current design provisions, (2) the imperfections and other assumptions applied in the benchmark analyses are conservative, and (3) many of the problematic cases are ones with very large second-order amplification factors that are not common in design practice.

Table 1– Error Statistics Summary from Validation Studies

Bending Axis	Error	Critical Load		Modified Stiff.		Notional Load	
		2nd-order	P-Δ	2nd-order	P-Δ	2nd-order	P-Δ
Weak	Avg.	7 (9)	8 (8)	5 (5)	7 (4)	4 (6)	6 (6)
	Extreme	17 (20)	17 (20)	10 (20)	16 (20)	13 (20)	19 (20)
Strong	Avg.	1 (9)	1 (8)	5 (3)	8 (3)	2 (7)	3 (6)
	Extreme	8 (20)	10 (20)	13 (15)	25 (15)	8 (16)	13 (15)

Note – Error values shown are percent differences between the proposed methods and results of detailed spread-of-plasticity solutions. Unconservative errors are shown first, followed by conservative errors in parentheses ( ).

ILLUSTRATIVE EXAMPLES

Three examples are presented to illustrate application of the alternative stability checks. The strategy in each example is to perform a second-order analyses and design checks for critical members using the three approaches described earlier. For comparison purposes, results from detailed second-order spread of plasticity analyses (by Maleck and White 2001), are also presented to represent the actual response. Results summarized herein are based on a more comprehensive study by Maleck and White (2001).

Low-Rise Industrial: The first example, see Fig. 5, is a framing bent from a large floor plan single story industrial building, such as an automobile plant. With heavy material handling equipment and piping hung from the roof and relatively small wind exposure, these structures are dominated by gravity loads with large second-order effects (Springfield, 1991). Loading shown in Fig. 5 represents an eleven bay configuration with ten leaning columns (only two of which are shown) and two lateral-load resisting columns. The concentrated load P has a tributary area of 35 ft x 35 ft, and the wind load $W = 6.3$ kips.

The member sizes satisfy a conventional AISC (1999) LRFD strength design and meet an $H/400$ drift limit for the service load combination.

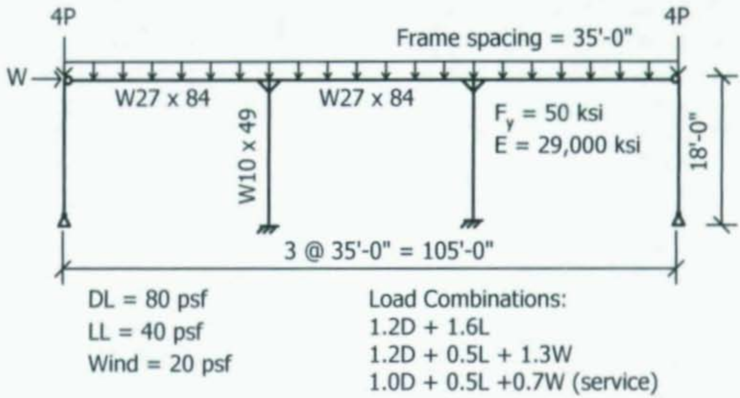

Fig. 5 – Example 1: Single-story industrial building

Using the story stiffness equation (C1-4) of the AISC (1999) under the *1.2D+1.6L* load combination, $B_2 = 2.31$. This is about 4% less than the more exact value of $B_2 = 2.41$ obtained from a critical buckling analysis and the AISC story buckling equation (C1-5). For the gravity plus wind loading case $B_2 = 1.74$ from a critical load analysis.

Axial column forces and maximum moments under the factored load combinations are summarized in Table 2a. The critical load results are from a second-order analysis of the "ideal" structure (no geometric imperfections) under the factored loads (without any notional loads). The modified stiffness analysis incorporates initial geometric imperfections through an equivalent notional load of 0.2% times the factored gravity loads (1.2D + 1.6L for the first combination and 1.2D + 0.5L for the second combination). Since the columns are bent in major axis bending and the axial load $P/P_y < 0.5$, no stiffness adjustments are required. In the notional load analyses, notional loads equal to 0.5% of the factored gravity loads are applied.

Table 2a – Example 1: Member Load Effects

Load Case	Member Check	Analysis/Design Method			
		Spread of Plasticity	Critical Load	Modified Stiffness	Notional Load
1.2D+1.6L	P_{col} (kip)	215	216	215	215
	M_{col} (k-in)	930	430	960	1770
	M_{bm} (k-in)	8660	8490	8670	8940
1.2D+0.5L+1.3W	P_{col} (kip)	154	155	154	155
	M_{col} (k-in)	1310	1060	1340	1760
	M_{bm} (k-in)	6490	6410	6500	6650

Referring to Table 2a, the critical load, modified stiffness, and notional load analyses all predict the maximum beam moments and axial column forces within about 3% of those from the spread-of-plasticity analysis. On the other hand, there are significant differences in the column moments. The modified stiffness predicts the column moments within about 3% of the spread-of-plasticity solution, but the column moments are 50% smaller for the critical load analysis and 90% larger for the notional load method. These differences are further reflected in the calculated displacements. Shown in Fig. 6 is a comparison of the

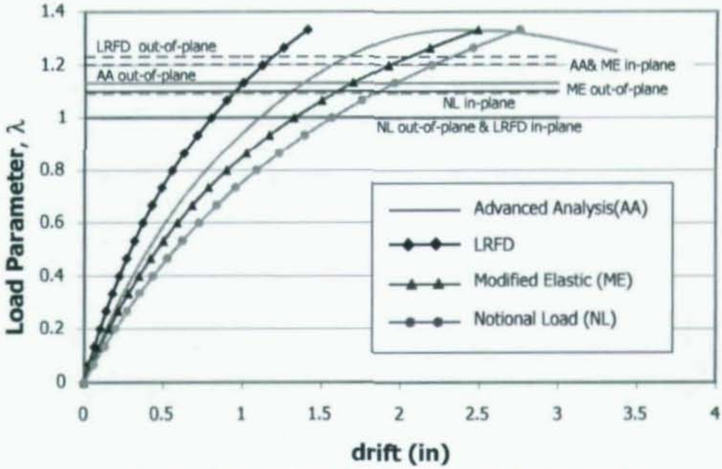

Fig. 6 – Example 1: load versus deflection under strength wind load combination

load versus drift response for the wind load combination. Here the critical load (LRFD) analysis under predicts the lateral deflections, compared to the spread-of-plasticity (Advanced Analysis) solution; and the modified stiffness and notional load methods overestimate the drift, with the modified stiffness coming closest to actual displacements.

Using the member forces from Table 2a, the columns are checked using the interaction formula for in-plane or out-of-plane (torsional flexural) failure. The resulting interaction ratios are summarized in Table 2b. For the critical load method, the in-plane checks are based on a column strength of $\phi P_{nx,KL}$ = 236 kips, obtained with an effective length factor of K = 2.3 using Eq. C-C2-6 of AISC (1999). In-plane checks for the modified stiffness and notional load methods are based on $\phi P_{ns,L}$ = 511 kips, and out-of-plane checks are all based on $\phi P_{nu,L}$ = 361 kips. The column moment capacity is ϕM_p = 2718 k-in.

Referring to Table 2b, due to the large difference in P_n used in the first term of the interaction equation, the critical load method is governed by the in-plane strength whereas the other two methods are governed by the out of plane check. The notional load method is most conservative (giving the largest values), followed by the critical load and modified stiffness methods. The in-plane checks can be compared to inelastic limit load ratios of $\phi\lambda_{1.2D+1.6L}$=1.17 and $\phi\lambda_{1.2D+0.5L+1.3W}$ =1.20 obtained from the spread-of-plasticity analyses. The inverse of these limits (0.85 and 0.84 for gravity and gravity+wind, respectively) provide a gage as to the conservatism in the methods. Compared to these values (0.85 and 0.84) the in-plane checks for both the critical and notional load methods are conservative, whereas the modified stiffness method is unconservative (e.g., 0.74 < 0.85). Since the member forces vary nonlinearly with load (due to second order effects), the degree of unconservatism of the modified stiffness method cannot be determined directly from this comparison. However, by scaling up the load until the in-plane check is equal to 1.0, further analysis would show that the modified stiffness method implies a limit load of $\phi\lambda_{1.2D+1.6L}$=1.29, which is about 10% larger (unconservative) than the in-plane limit from the spread-of-plasticity solution. This is not ideal, but is within the bounds identified in the verification problems described earlier.

Table 2b – Example 1: Column Interaction Checks

Load Case	Check	Analysis/Design Method		
		Critical Load	Modified Stiffness	Notional Load
1.2D+1.6L	in-plane	1.06	0.74	1.00
	out-of-pl.	0.74	0.91	1.17
1.2D+0.5L+1.3W	in-plane	1.00	0.74	0.88
	out-of-pl.	0.78	0.86	1.00

Grain Storage Bin: The second example is the support rack for a grain storage bin with the dimensions and loading shown in Fig. 7. In this case, it is assumed that the columns are braced out-of-plane and that the cross beams and bracing are pin-connected to the columns. Using an elastic critical load analysis, the amplification factors are B_2 = 2.75 and B_2 = 2.20 for the gravity and wind load combinations, respectively. The spread-of-plasticity analyses predict inelastic limit load ratios of $\lambda_{1.4G}$=1.21 and $\lambda_{1.2G+1.3W}$=1.17.

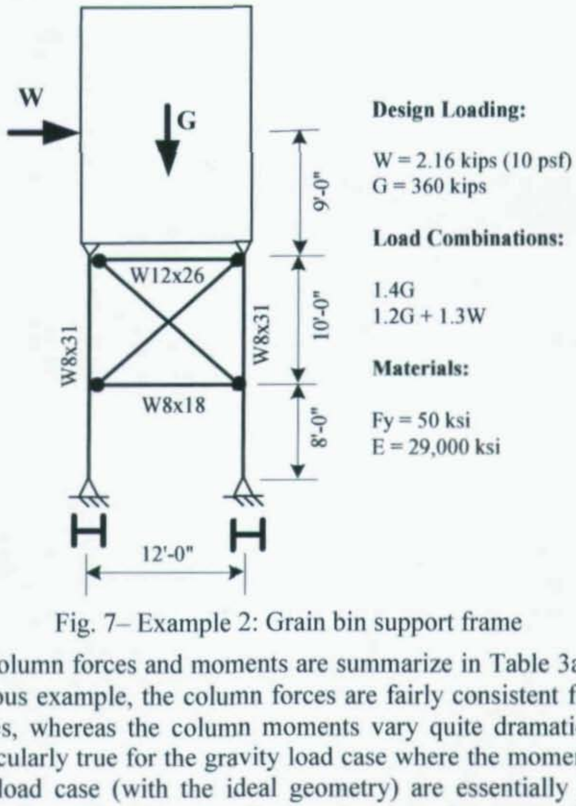

Fig. 7– Example 2: Grain bin support frame

Maximum column forces and moments are summarize in Table 3a. As in the previous example, the column forces are fairly consistent for all four analyses, whereas the column moments vary quite dramatically. This is particularly true for the gravity load case where the moments in the critical load case (with the ideal geometry) are essentially zero. Like the previous example, no adjustment of member properties is required in the modified stiffness approach since the columns are in strong axis bending and the axial load ratio $P/P_y < 0.5$.

The interaction checks (Table 3b) are based on the following in-plane column strengths: critical load method $\phi P_{nKL,top}$ = 232 kips (K = 2.4), $\phi P_{nKL,bot}$ = 243 kips (K = 2.9); and modified stiffness and notional load methods, $\phi P_{nL,top}$ = 355 kips, $\phi P_{nL,bot}$ = 366 kips. Effective length factors for the former are based on an elastic critical load analysis under gravity loads. The results in Table 3b show that critical load method is most conservative, followed by the notional load and modified stiffness method. Based on the spread-of-plasticity analysis results, interaction values larger than 0.92 (for *1.4G*) and 0.95 (for *1.2G+1.3W*) are conservative. Here again, the critical load and notional load methods are conservative, and the modified stiffness method is unconservative. By scaling up the loads, one could show that the modified stiffness method predicts a limit load that is about 7% unconservative, relative to the spread-of-plasticity limit point, under gravity load.

Table 3a – Example 2: Member Load Effects

Load Case	Member Check	Analysis/Design Method			
		Spread of Plasticity	**Critical Load**	**Modified Stiffness**	**Notional Load**
1.4GL	$P_{c,top}$ (kips)	233	237	234	252
	$P_{c,bot}$ (kips)	255	252	256	261
	M_c (k-in)	161	2	130	322
1.2GL+1.3W	P_{ctop} (kips)	203	204	205	221
	$P_{c,bot}$ (kips)	224	225	227	231
	M_c (k-in)	380	289	377	509

Table 3b – Example 2: Column Interaction Checks

Load Case	Member Check	Analysis/Design Method		
		Critical Load	**Modified Stiffness**	**Notional Load**
1.4GL	Top Col.	1.02	0.74	0.92
	Bot. Col.	1.04	0.78	0.92
1.2GL+1.3W	Top Col.	1.07	0.82	0.95
	Bot. Col.	1.11	0.87	0.96

Multi-story Frame: The final example is the multi-story frame shown in Fig. 8, where one load case is investigated (*1.0G + 1.0W*, assuming that the specified loads are already factored), and member forces and interaction checks are presented for the three columns in the first story. Unlike the previous examples, this frame is fairly stiff with a B_2 = 1.10 for the first story. The second-order spread-of-plasticity analysis predicts an inelastic limit load of 1.03 for this frame, which combined with the low B_2 indicates that it is dominated more by yielding than second-order effects. Due to the high axial forces, several columns are subject to the τ–factor adjustment (Eq. 3a) in the modified stiffness

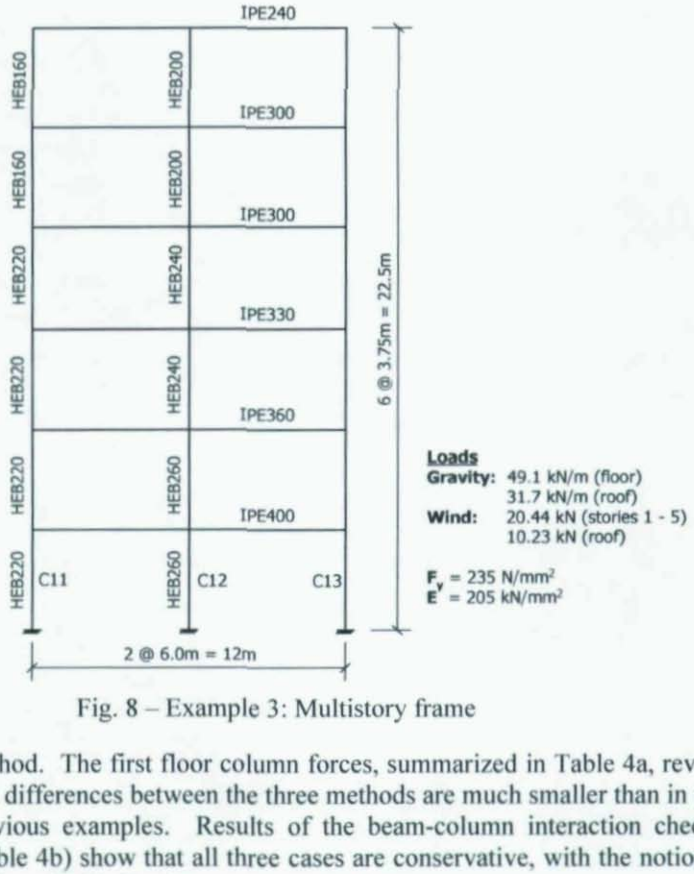

Fig. 8 – Example 3: Multistory frame

method. The first floor column forces, summarized in Table 4a, reveal that differences between the three methods are much smaller than in the previous examples. Results of the beam-column interaction checks (Table 4b) show that all three cases are conservative, with the notional *load method being the most conservative.*

CONCLUDING REMARKS

Two new stability assessment methods, modified stiffness and notional load, are proposed as a practical alternative to the critical load approach, which has in some form been a part of the AISC Specifications since 1961 and revised to include second-order analysis in the first edition of the LRFD Specification in 1986. A key practical

Table 4a – Example 3: Column Load Effects

Location & Effect (1.0GL+1.0W)	Analysis/Design Method			
	Sprd. of Plasticity	Critical Load	Modified Stiffness	Notional Load
P_{c11} (kN)	683	672	666	665
P_{c12} (kN)	1720	1770	1770	1770
P_{c13} (kN)	921	884	891	891
M_{11} (kN-mm10^{-4})	6.7	5.1	5.7	5.3
M_{12} (kN-mm10^{-4})	11.5	12.8	12.9	13.3
M_{13} (kN-mm10^{-4})	9.9	8.7	9.4	9.0

Table 4b – Example 3: Column Interaction Checks

Load Case	Location	Analysis/Design Method		
		Critical Load	Modified Stiffness	Notional Load
1.0GL+1.0W	C11	0.70	0.72	0.70
	C12	1.27	1.27	1.81
	C13	1.04	1.07	1.04
	Wt. Avg.	1.10	1.10	1.38

benefit of the proposed methods is that they eliminate reliance on, and the need to calculate, critical loads and effective length factors. Beyond this, the two methods provide a more accurate and transparent assessment of the actual behavior. They more accurately model second order moments that affect not just column design, but also adjacent members and connections.

Between the two new methods, the notional load is probably the most straightforward to apply, but with the tradeoff that it tends to me more conservative due to calibration of the load factor to account indirectly for inelastic effects. The modified stiffness method can require more work to adjust member stiffness coefficients, but these adjustments more closely reflect the underlying mechanics. The modified stiffness approach also bears some commonalities with the approached of ACI-318 (1999) for slender concrete columns. Perhaps, though, the most important attribute of both approaches is that by placing the emphasis more on more realistic system analysis, they provide a consistent framework that will facilitate further developments, such as the practical use of second-order inelastic analysis methods, in the future.

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