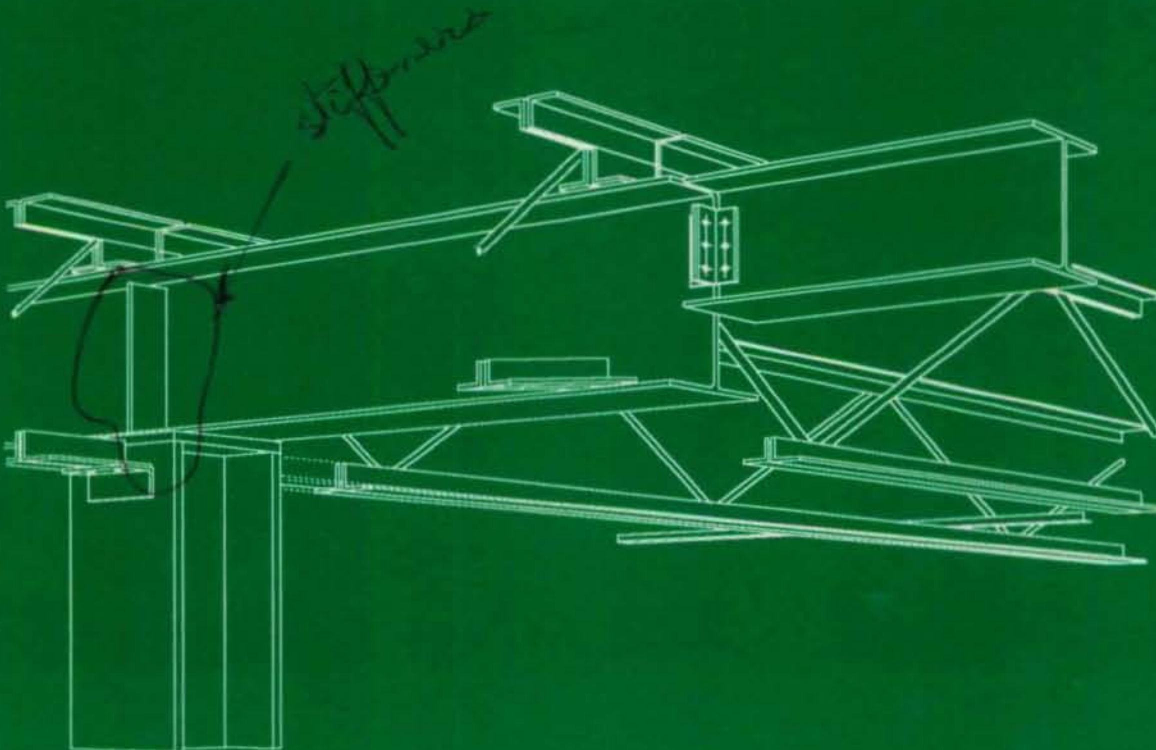
Cantilever

1353

Roof Framing with *Cantilever (Gerber) Girders & Open Web Steel Joists*

Stiffeners



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& Open Web Steel Joists

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- *ERRATA* -

Page 2 Reference no. 8 for CSSBI publication should read 9.

Page 12 The word "top-left" in item e. should read "mid-left"

Page 19 Factor of 2 in required spring stiffness, $2\,k_r$, has been omitted.
Replacement calculations are shown below........

Using design expressions as illustrated above, check stiffness requirement:

$$\text{Actual spring stiffness} = k = \frac{3\,(200\,000)\,(91.8 \times 10^6)}{10\,500\ \ (570^2)} = 16.1 \text{ kN/mm}$$

$$\text{Required spring stiffness} = 2\,k_r = \frac{2\,(433\,500)\,(8540{+}570)}{570\,(8540)} = 1.67 \text{ kN/mm}$$

There is sufficient bracing stiffness in a single joist bottom chord extension connection.

Extension Δ, under full C_f and using the one sided joist member stiffness,

$$\Delta = \frac{\delta}{\left(\frac{k}{k_r} - 1\right)} = \frac{15}{\left(\frac{16.1}{1.67/2} - 1\right)} = 0.821 \text{ mm}$$

Determine minimum connection force for joist bottom chord extension:

a. Stability force as $F = 2\,k_r\,\Delta = (1.67)(0.821) = 1.37$ kN
b. 1% of compression force in bottom flange of girder ≈ 8 kN
Therefore, total connection force = 1.37 + 8 = 9.37 kN

Two 5/8" diameter A307 bolts - single shear (threads excluded) = 65.8 kN
(greater than 9.37 kN of required resistance, OK)

Page 21
(1) In "Proposed Design Steps" , item 7: $k_a > k_r$ should read $k_a > 2k_r$
(2) Factor of 2 for required stiffness has been omitted in Example Design Checks.
Replacement calculations are shown below........

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6. $2k_r$ as per P. 19 ≈ 2.00 kN/mm (substituting d for d_j)
$\Delta = 15 / [(10/1.00)-1] = 1.67$ mm ($\delta = 15$ mm assumed)
Stability force F at connection ≈ (2.00)1.67 = 3.34 kN
7. ($k_a = 10$ kN/mm) >> ($2k_r = 2.00$ kN/mm)
Girder to column joint should be designed to carry
moment about "a-a" axis $F_H \cdot d \approx (3.34{+}8)\,0.457 = 5.18$ kN·m

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Preface

This publication has been prepared to explain the philosophy of the concept, the design, and the function of the structural steel "cantilever girder" or "Gerber" roof framing system. Although comments are restricted to roof framing applications, the concept may also be found in floor framing applications, and some of the comments may be equally applicable to such uses. Judgement in this regard is left to the discretion of the reader. A number of references, combined with good engineering and construction practice, form the basis for this document.

Preparation of this publication by the Canadian Institute of Steel Construction has been carried out with the financial assistance of the Steel Structures Education Foundation. Although no effort has been spared to ensure that all data in this publication are factual and that numerical values are accurate to a degree consistent with current structural design practice, the Canadian Institute of Steel Construction and the Steel Structures Education Foundation do not assume any responsibility for errors or oversights resulting from use of the information contained herein.

The Institute recognizes the contribution of a task force of the Association of Professional Engineers of British Columbia, under the chairmanship of C. Peter Jones. Specific analytical studies by Professors Noel Nathan and Roy Hooley of the University of British Columbia deserve specific mention. The authors are also indebted to Messrs. J. Springfield, T. V. Galambos and several engineers from the Canadian steel fabricating industry for their technical review and suggestions.

As with the preparation of any document on a technical subject where there may be several solutions to a problem, it is expected that opinions may differ on the approach taken. Suggestions for improvement of this publication should be forwarded to the publisher for consideration in future printings.

Roof Framing with Cantilever (Gerber) Girders and Open Web Steel Joists

Introduction

The structural steel roof framing described in this publication and commonly called the cantilever girder or Gerber system has been used successfully for many years throughout North America. The economy of the Gerber system is obtained from simple, repetitive framing in stabilized, relatively uniformly loaded structures. Its primary use is to resist gravity loads. It should be noted that Gerber girder roof framing is relatively inefficient for supporting moving loads e.g. vehicular parking or heavy mono-rail systems. A typical configuration for use in a single storey building roof framing system is illustrated in Fig. 1.

The system illustrated and discussed consists of open web steel joists (OWSJ's) supported by cantilever and suspended span W-shape girders. The suspended segments are assumed to be "pin" connected to the ends of cantilevers formed by the cantilever segments. Each cantilever segment is supported by HSS or W-shape columns connected to simple base plates. Gravity loads at column bases are generally moderate and foundation type will depend upon specific loads and soil conditions. Base fixity of columns is usually not assumed in design.

The inter-dependency of structural members in providing structural capacity and both local and overall structural stability of the vertical load resisting framing is very important. These aspects are covered in the following paragraphs, with the function of each component described. All the conditions discussed in this publication are applicable to usual Gerber roof system design. Design and construction guidelines are presented based on current practice and available structural research information. Example design calculations and references for supplementary reading are also provided. When unusual conditions occur, the designer must be prepared to investigate all of the conditions applicable, for all possible loading combinations.

In single storey buildings using this roof framing system, lateral loads caused by wind or earthquake are collected by in-plane roof bracing or an engineered roof deck diaphragm, and are distributed to lateral load resisting elements or systems. These may include interior or exterior braced frames, masonry or concrete shear walls, or a steel rigid frame using components not illustrated. Provision of lateral load resistance is an essential structural consideration and design examples are readily found in steel, concrete and masonry technical publications. Therefore, the following text will address only the important strength and stability related design criteria for the vertical load resisting system, leaving lateral load resistance issues to other publications.

Design and Construction Considerations

Buildings should be designed to provide sufficient structural capacity to resist safely and efficiently all loads and effects of loads that may reasonably be expected, with adequate consideration given to construction procedures and the anticipated service life of the building. Live loads due to occupancy, snow, rain, wind and earthquakes, etc. are generally computed using rules prescribed in Part 4 of the National Building Code of Canada (NBCC). Dead loads can vary significantly from light built-up roof systems to heavy "inverted" roof or "protected membrane" systems, ballasted with crushed stone or concrete pavers to prevent insulation flotation. Therefore, loads must be accurately computed for each project.

Figure 2 provides a simplified flow chart for the Gerber girder design process. This flow chart is intended to assist a designer to quickly configure a Gerber roof system and to achieve structural economy without compromising structural safety. The analysis-design process is usually simple, and manual structural analysis is adequate for joist and girder member design. The steel design standard CAN3-S16.1[1] and several design aids[2,3] provide guidelines which produce adequate designs. For member selection and code check, an automated procedure using Canadian computer software[4] is available. Use of a simple analysis-design process will be most appropriate when:

i. column spacing is relatively uniform
ii. roof loading is basically uniform
iii. cantilever length is equal to or less than that giving approximately equal positive and negative moments under maximum uniformly distributed loads on all spans
iv. suspended span members are shallower than cantilever members

v. girder web stiffeners are used at column girder joints
vi. girder is torsionally restrained about its longitudinal axis at supports
vii. top of each column is laterally supported
viii. W-shape or WWF sections are used for cantilever sections

Roof Deck

The primary role of steel roof deck is to serve as a base for weatherproof and waterproof roof construction materials. Its primary structural function is to carry gravity loads, and wind loads normal to its plane. Although ponded rainfall and drifted snow are the usual governing roof load conditions, special note should be made of any additional loads due to other uses that may be made of the roof.

In addition to its primary structural function, a steel roof deck, attached to the structural steel framing, is frequently designed to act as a horizontal shear diaphragm, with the steel deck forming the web, interior roof purlins or OWSJ's forming the web stiffeners, and the perimeter or panel boundary structural members on all four sides forming the flanges of the diaphragm. This shear diaphragm may be used to transfer wind and seismic loads to lateral load resisting components.

The design, fabrication and erection considerations for steel roof deck intended for use with conventional roofing systems are described in Reference 5. In using this standard, it should be noted that minimum structural connections are supplied unless special connection requirements are specified. The most common form of deck fastening to steel framing is by means of welding (Fig. 3), although mechanical fasteners (Fig. 4) are rapidly gaining acceptance as an alternative. A review of fastening methods for steel deck is provided in Reference 6. The type and size of fastener should be matched to connecting members. For example, arc spot weld diameters proposed must be compatible with the width of OWSJ top chord members.

These deck-to-roof-framing connections permit steel deck to provide lateral support to roof purlins (Fig. 5) which in turn provide wind-uplift resistance to the roof deck. Design standard CAN3-S136[7,8] provides shear and tensile capacities for arc spot weld design. Spacing of fastenings to supports, diameter of arc spot welds, and side lap and end lap fastening rules can affect uplift resistance, the ability of steel deck to provide lateral support to the connected steel members, and the ability of steel deck to perform as a lateral load resisting diaphragm.

When a steel roof deck is designed to act as a roof diaphragm, connection requirements are usually increased, particularly where local diaphragm stresses are high. It follows that deck gauge may also be governed by shear stresses in the diaphragm. Designers are referred to a CSSBI publication[8] , steel deck manufacturers' design aids[10,11] as well as other design publications[12,13] for guidance on roof diaphragm design.

OWSJ Roof Purlins

Open web steel joists (OWSJ's or joists) are usually proprietary products whose design, manufacture, transport, erection and connection are governed by the requirements of Clause 16 of S16.1. The Standard and its Commentary[2] specify the information to be provided by the building designer and the joist manufacturer. A CISC publication[3] provides recommended practice to assist in the use of OWSJ's in construction.

In providing a joist manufacturer with design information, the building designer should specify on the drawings design loading conditions, including dead load, live load, wind uplift, point load and/or uniformly distributed loading, extent and intensity of snow pile-up etc.. A joist schedule, see Reference 3, prepared by the building designer, prescribing all design loads, web opening dimensions, shoe depth, bottom chord extensions etc., is recommended to convey structural design, detailing and special manufacturing criteria to the OWSJ manufacturer. Data which describes the detailed OWSJ's, their lateral bridging or lateral supports and end connections, etc., provided by the joist manufacturer on shop drawings must be reviewed and the adequacy of the structural design confirmed by the building designer before joist fabrication.

Open web steel joist roof purlins provide direct support to steel roof deck to carry gravity loads and wind uplift forces. Joist loads are transferred through the joist shoes, field welded or bolted to girder members. In checking overall building design, the designer must verify that these connections meet all design criteria, including wind uplift.

Lateral support to joist top and bottom chords is necessary to provide stability during construction and, in some cases, to the bottom chord under design criteria stipulated by the building designer. This is accomplished by the use of horizontal or x-bridging (or a combination) normally placed to meet specified slenderness requirements for tension and compression chords. Since the steel roof deck is supported directly on OWSJ's and is connected to their top chords by welds or mechanical fasteners, the top chords are laterally supported by the steel deck in the completed structure. It follows that OWSJ's, laterally stiffened by steel roof deck provide lateral support to the top flange of supporting girders (Fig. 6). For net wind uplift design conditions which induce compression force in the bottom chord, permanent lateral supports to the bottom chord, spaced at less than code limiting l/r criteria for "tension" chords, may be necessary to provide stability. All bridging lines should be permanently anchored to provide adequate support to the joists under construction and all other loading conditions. Removal of, or alteration to anchorage and bridging members during or after construction should not be permitted without the engineer's review and approval.

In some circumstances, tension chord lateral support at the first bottom chord panel point may be necessary to stabilize end compression diagonals. For example, when net uplift conditions produce compressive stress in the bottom chord or when sloped bottom chord extensions are needed because of depth differentials between girders and OWSJ's, lateral support at the intersection of the joist bottom chord and the sloped chord extension may be used to provide out-of-plane stability[14,15] (Fig. 7).

Joist top chord connections to a girder provide lateral support at intervals along the length of the girder top flange. Joist bottom chords are generally stopped short of their end supports for ease of erection and saving of structural material. However, joist bottom chord extensions are usually added at supports to provide lateral/torsional support to girder and overall stability to the girder-column assembly (Fig. 8). Frequently these joists are assumed to act as tie joists as per S16.1, to assist in the erection and plumbing of the steel frame. Also, bottom chord extensions may be used between column lines to enhance girder uplift resistance or to stabilize the tips of long cantilevers.

Erection and plumbing of the steel frame may be facilitated by bolting either the top or bottom chord of a tie joist, and after plumbing the columns, the other chord is then welded. Tie joists are normally designed on a simple span basis without applied end moments. OWSJ's used in this configuration, but which are expected to carry end moments[16] due to lateral forces on the building should be designated "special joists", and the appropriate end moments must be provided to the joist manufacturer by the building designer. Further discussion on the use of tie joists is provided under the heading "Special Construction Considerations". Design considerations relating to stability of the "tie joist - Gerber girder - column" assembly are provided under the heading "*Axially Loaded Columns*".

Gerber Girders

The principle of cantilever and suspended span construction developed by Gerber about a century ago, was chosen to produce a statically determinate structure with an even distribution of girder design moments under uniform loading. Although this system is also used in multi-storey construction as a primary girder system and as secondary framing members in the stub-girder floor framing system, all further reference in this publication will be to roof construction. Being statically determinate, girder bending moments are easily evaluated by hand which in turn facilitates design review. Gerber girder roof members using W-shapes are shallower and lighter than equivalent simply supported design alternatives, and simpler connection details for fabrication and erection result in increased economy.

Gerber girder construction is most commonly used in conjunction with OWSJ secondary framing. End reactions from suspended segments of the Gerber framing system are transferred to ends of cantilever members through simple shear connections, which are treated as "pinned" or "hinged" connections for analysis purposes. The cantilever members rest on columns, and due to continuity over the columns, these become points of maximum negative bending moment. These column-to-girder joints must, therefore, be carefully examined to avoid girder cross sectional instability and to provide column stability transverse to the longitudinal axis of the girder. The girder must also be checked for web crippling and web buckling at these locations.

A ***suspended span*** girder member (Fig. 1) is designed considering girder ends to be simply supported. Under gravity loading, the top flange of this portion of girder is in compression, and lateral support is provided by ends of joists framing onto it (Fig. 6). Under net wind uplift loading, the bottom flange of the girder can go into compression. In such cases, the girder must be investigated to determine if torsional support is required. Joist bottom chord extensions or other positive means may be used to provide such support (Fig. 7). More detail is provided under the heading "Special Construction Considerations".

A suspended span girder is design checked using
- Cl. 13.6 for moment resistance of girder members
- Cl. 13.4.1 for shear resistance (since the analysis is elastic)

Girder torque caused by unbalanced or eccentric joist reactions on the girder can normally be resisted by the bending resistance of the joist top chord and girder top flange as well as tensile-shear resistance of the joist connection. Under conditions such as one-sided joist spans, or unequal joist spans on opposite sides of the girder, girder torque due to eccentric loading or unbalanced loading should be investigated.

The design of a ***cantilever span*** girder is affected by the selection of various construction details incorporated in a framing assembly. Figure 9 illustrates the major strength and stability considerations at or near column supports, as follows:

a. girder section laterally and torsionally restrained at column supports by joists with top and bottom chord connections, or by creating column continuity through the girder. (S16.1 Cl. 15.2)
b. top of column laterally supported by joist bottom chord extensions, unless column continuity through the girder is achieved
c. girder web crippling and buckling, check need for web stiffeners
d. girder bearing at column
e. top flange laterally supported at joist connections
f. torsional support to tip of cantilever (top/bottom flange connections) if necessary
g. minimize moment restraint at cantilever-tip "cantilever to suspended span member connection", unless additional negative moment at column support is considered in the analysis. Single-web-plate, double-angle and end-plate connections are all commonly used.

For cost effectiveness reasons only some of the illustrated construction details are incorporated in each design. Three design approaches are thus possible...

i. When girder web stiffeners are omitted at supports:
   - girder member must be lateral-torsionally restrained about its longitudinal axis by bracing
   - top of column must be laterally supported by bracing supplied for girder bottom flange
   - web crippling and buckling are prevented by ensuring appropriate web thickness and slenderness limitations
ii. When girder web stiffeners are used at column supports:
   a. size stiffeners for strength and stability of web under concentrated reaction at column, as in Example 2, and provide lateral-torsional restraint to girder member at column by specifying direct support to girder bottom flange or top of column by joist bottom chord extensions as in Example 4
   b. provide lateral restraint to girder and lateral support to column by extending an appropriate portion of column's stiffness to top of girder using full depth girder web stiffeners, as in Example 5, and by providing adequate strength/stiffness in girder-column connection

Cantilever girder member design process may include:

i. evaluate moment resistance of cantilevers and girder section between supports, for lateral-torsional buckling behaviour - assuming no distortion of beam cross section (Appendix "A", Refs.17-20, and Example 1)
ii. if required by design, provide lateral-torsional support to girder between column supports under net uplift force (e.g. connecting joist bottom chord extension to girder bottom flange)
iii. ensure net wind uplift resistance in girder-to-column connection, if appropriate
iv. for deep "I" shaped sections with narrow flange widths, check buckling resistance of laterally unsupported girder compression elements using Appendix "B". This design check is not needed for W or standard WWF girders. See Example 1.
v. prevent service load yielding of net girder section due to bolt hole details at column cap locations, and design to S16.1 - Cl. 15.1 when bolt holes occur in top flange above a column
iv. if long cantilevers are used, geometry of framing layout will usually result in OWSJ connection near tip of cantilever. Provide torsional restraint to cantilever tip with bottom chord extension, if required.

Axially Loaded Columns

The vertical reactions of cantilever girders (due to gravity loads or net wind uplift) are directly supported by relatively slender columns. To evaluate compressive resistance of a column, the top of the column is assumed "pinned" in both directions to simulate the lack of moment restraint. Lateral translation at top of the column, in an out-of-plane direction, can create an unstable structural configuration and must be prevented. Therefore, either column continuity through the girder, or tying of the columns in the out of plane direction must be addressed. It should be noted that column continuity through a girder may be achieved by appropriate sizing of full depth girder web stiffeners and selection of girder-column connection (see Example 5). As illustrated in Fig. 8, an OWSJ bottom chord extension may be designed to provide lateral support to the top of a column. The selected column shafts are usually shop welded to simple base plates with nominal connections. Thus, to facilitate computation of column capacity, bases are generally assumed as "pinned".

The column length, L , for column buckling in the plane of girder framing may be assumed conservatively as the length measured from column base to the under-side of the girder, and its effective length factor for design may be assumed as 1.0, thus, L_x=1.0(L). To simplify structural design, effective column length, L_y (product of column length and effective length factor), for column buckling out of plane, i.e. perpendicular to the girder framing, and L_x for column buckling in the plane of girder-column framing, are proposed in Fig.10. It should be noted that these effective length measurements differ slightly from S16.1 rules to account for the stiff-girder and slender-column arrangement usually encountered in high roof single storey buildings using simple column to girder connections.

Figure 10 also describes overall stability conditions for the "joist - Gerber girder - column" assembly:

Case 1 Joist depths and girder depth are similar. Joist bottom chord extension is used to support top of column and provide lateral-torsional support to girder. Girder web crippling and buckling are prevented through the use of girder web stiffeners. Column selection is based on axially loaded member design using effective length in both directions, $L_x = L_y$.

Case 2 Same as Case 1, except that joists are deeper than the girder. Column selection is based on axially loaded member design using effective lengths L_x and L_y as illustrated.

Case 3 Same as Case 1, except that joists are shallower than the girder. By appropriately sizing a column cap plate, girder web stiffeners, and the girder-column connection, column continuity may be assumed for column stability purposes. Column selection is based on axially loaded member design using effective lengths L_x and L_y as illustrated. Alternatively, sloped joist bottom chord extensions may be used to provide direct support to girder-column joint. See also Case 6.

Case 4 A joist bottom chord extension is not used to support top of column at a column line. By appropriately sizing a column cap plate, girder web stiffeners, and the girder-column connection, column continuity and lateral-torsional stability of the girder are provided. See Example 5. Column axial resistance is computed using effective lengths L_x & L_y as illustrated.

Case 5 Steel joist and the girder depths are similar. Girder lateral-torsional support at column is provided by joist bottom chord connection. Joist bottom chord extension is also used to support top of the column. Crippling and buckling resistances of unstiffened girder web at columns are design checked and stiffened if required. Column selection is based on axial-load member design using effective length L in both directions. $L_x = L_y$.

Case 6 Same as Case 5, except that joists are shallower than the girder member. Girder lateral-torsional support at column is provided by joist bottom chord framing. A sloped joist bottom chord extension is used to support top of the column. Lateral support to joist bottom chord may be required at point "p". Column selection is based on axial-load member design using effective length L in both directions. $L_x = L_y$.

Case 7 Same as Case 5, except that joists are deeper than the girder member. Girder lateral-torsional support at column is provided by joist bottom chord framing to column. Joist bottom chord extension is also used to support column. Column selection is based on axial-load member design using effective lengths L_x and L_y as illustrated.

Case 8 Girder web stiffeners are omitted at column. Girder section is not restrained against rotation about its longitudinal axis at points of support. Sidesway web buckling is not prevented. Top of column is not laterally supported. **This is considered to be an instability condition**[21 to 24] , see also S16.1 Cl. 15.2

Figure 11 illustrates an unstable framing assembly which may be viewed as a potentially more severe case of instability than the structural arrangement of Case 8. Reasonable remedial solutions may include the following:

i. use structural bracing from bottom chord level of a pair of joists to top of column. Similar bracing at top chord level to the top flange of the girder may be necessary to provide torsional restraint to the girder at column, depending on L_u of the specific girder section, or

ii. specify girder web stiffeners, and stiff girder to column connections so as to create continuity of each column through the girder, and specify structural bracing as noted above either to the top or to the bottom flange of the girder.

Note: a "maximum" cusp in the girder bending moment diagram occurs at this point.

Columns must be properly connected to girders and base plates to resist net wind uplift when condition exists. Column base to footing connection resistance and footing pull-out resistance must also be addressed, although recent research tests[25] indicate that pull-out resistance of footings and slab-on-grade is rarely critical.

Special Construction Considerations

To assist in the erection and plumbing of a steel frame during construction, tie joists with top and bottom chords connected to at least one side of a column/girder joint are frequently used as noted earlier. It has been demonstrated by research tests[26] and theoretical analysis that column-joist framing with opposing tie joists, utilizing both top and bottom chord connections, can cause an accumulation of significant joist bottom chord compression and top chord tension due to end moments under gravity roof loading. Theoretically, the connected bottom chords and the first compression diagonals could be the most critically loaded members. However, a redistribution of forces probably occurs in many cases, due to joint slippage at bolted joist chord connections, inelastic action in steel material as well as a minor amount of out-of-plane buckling. For these reasons, most OWSJ's designed on a simple span basis perform satisfactorily in such applications.

A joist bottom chord extension is usually added at a column to provide torsional stability to the girder, and to provide overall stability to the girder-column assembly. Using the design information provided by Reference 21, a simplified design process is proposed in Example 4, demonstrating the calculation required in providing overall stability to a girder-column assembly by prescribing supporting members of sufficient strength and stiffness.

Design Example Problems

The following five design examples illustrate major design considerations in roof framing. In many ways, they also numerically demonstrate the fact that a simple analysis-design procedure can be used to produce adequate Gerber roof framing members.

Example 1 is intended to show trial member selection and detailed evaluation of moment resistance for a cantilever girder using proposed design rules as described in Appendices "A" and "B".

THE EXAMPLE BUILDING : Single storey, Cantilever and suspended span roof framing with OWSJ purlins (as per Reference 27) Column spacing : 12 m in the girder direction, 10.5 m in the joist direction
Dead load (excluding steel weight) = 0.7 kPa, Ground snow = 2 kPa
Lateral load (wind) was design checked, but not covered by the following examples

- Simple design steps illustrating trial girder size selection are shown in Part 1 of the calculations.
- Detailed design checks are performed in Part 2 of the calculations.

Example 2 validates the need for web stiffeners for the Example 1 cantilever girder - illustrating girder web perpendicular to W-shape column web configuration - considered to be a more severe case of column-to-girder connection. Following design checks showing that web stiffeners are required, stiffeners are then selected and welds are sized. Note: similar design computations should also be provided for situations when girder and column webs are parallel as in normal applications.

Example 3 illustrates the design of an interior W-shaped column. A simple column selection procedure is proposed, followed by a more detailed design check taking into account effective length factor calculations, end moment effects, and axial load amplification effect, U_y.

Example 4 illustrates proposed calculation to evaluate minimum member and connection design forces for tie joist or joist with bottom chord extension. Also illustrated is the stiffness of lateral support responsible for overall girder-column assembly stability. Girder and column members used for this design example are obtained from Example 3.

Example 5 "Part 1" illustrates proposed cantilever girder to column connection design checks using the selected girder and column members as per design example in Reference 27. In "Part 2" a tentative design procedure for column to stiffened-girder connection, with only joist top chords connected at the column line, is proposed and illustrated with a design calculation. Research results obtained from simple beam tests, as in Reference 28, are used to justify this procedure.

Closure

The primary objective of this document has been to describe the cantilever girder or Gerber system used in single storey roof framing systems. The examples, appendices, and references provide further information on appropriate methods for in-depth analysis of special applications and more detailed understanding of the performance of major components and individual sub-components . It is hoped that the illustrations and examples will aid in understanding of the concept. It is acknowledged that linear elastic analysis of this concept will not always provide theorists with clear-cut answers. Nevertheless, use in many millions of square metres of structure has proven the concept to be functional, safe and cost effective.

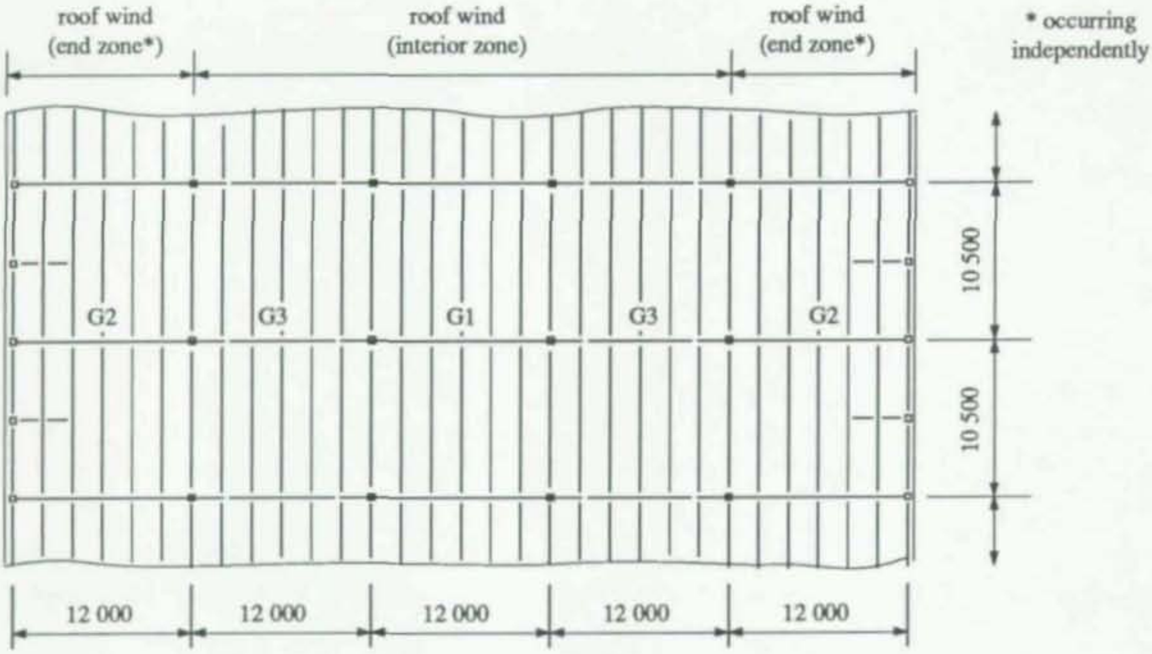
In addition to the analytical research referred to earlier which will be on-going after publication, laboratory research sponsored by the Steel Structures Education Foundation will be carried out to refine some of the analytical techniques and the design parameters suggested in this document. A full-scale laboratory research programme will be conducted. The principle objective is to correlate back-span and cantilever interactions. Also, the magnitude of stability forces required at the girder column joints will be evaluated. It is hoped that a report on this project, scheduled to begin in September 1989 will form a valuable sequel to this publication.

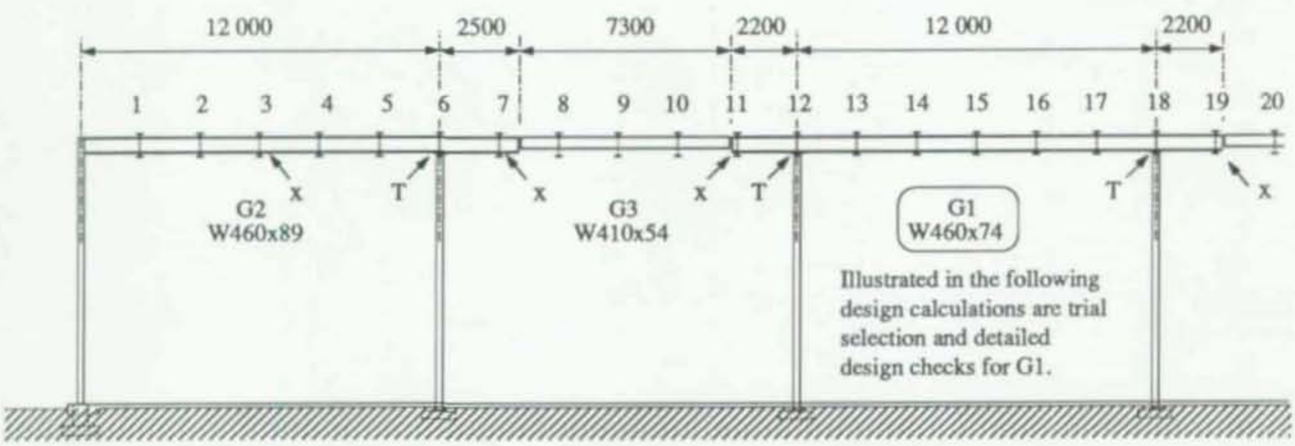
Symbols

Only CAN3-S16.1 defined symbols are used in this text, unless otherwise noted.

d'	depth of girder (centre to centre of flanges), mm
L_c	actual length of cantilever, mm
I_j, I_{j1}, I_{j2}	moment of inertia of joist top chord, mm^4
I_{yf}	I_y of compression flange, mm^4
k_g	distance measured from top of flange to fillet, mm
s, s_1, s_2	joist spacing, mm
t_{cap}	thickness of cap plate, mm
U_y	moment amplification factor
w_g	girder web thickness, mm
w_c	column web thickness, mm
x_1, x_2	joist end panel length, mm
κ	spring constant contributed by girder web and joist framing, N/mm per mm of a long strut
υ	Poisson's ratio = 0.3

Design Example 1 Cantilever Girder (G1) Design





Lateral support conditions:
x = bottom chord extension (BCE)
T = BCE and acting as tie joist

Unfactored girder loads from joist lines:

Joist line location	Dead load [1] kN	100% snow [2] kN	Wind uplift [3] kN
1-5, 25-29	18.1	33.6	27.6
8 - 10	18.1	33.6	22.4
6, 24	18.1	33.6	23.9
7, 11-23	18.1	33.6	20.3

Notes:
1. including steel weight
2. roof snow
3. wind loads for strength design are computed using Commentary B, Supplement to the National Building Code of Canada, 1985.

Design Example 1 Part 1 Trial Selection of Girder G1

Total factored joist reaction for loading case : -

(i) $\alpha_D D + \alpha_L L = 1.25\,(18.1) + 1.5\,(33.6) = 73$ kN

(ii) $\alpha_D D + \alpha_L (50\%\,L) = 1.25\,(18.1) + 1.5\,(50\%$ of $33.6) = 47.8$ kN

(iii) $\alpha_D D + \frac{1}{2}\alpha_L L + \frac{1}{2}\alpha_L\,(50\%\,L) = 1.25\,(18.1) + (0.5)(1.5)(33.6) + (0.5)(1.5)(0.5)(33.6) = 60.4$ kN

Total factored load, computed from reactions of G3, acting at end of cantilever due to suspended span member : -

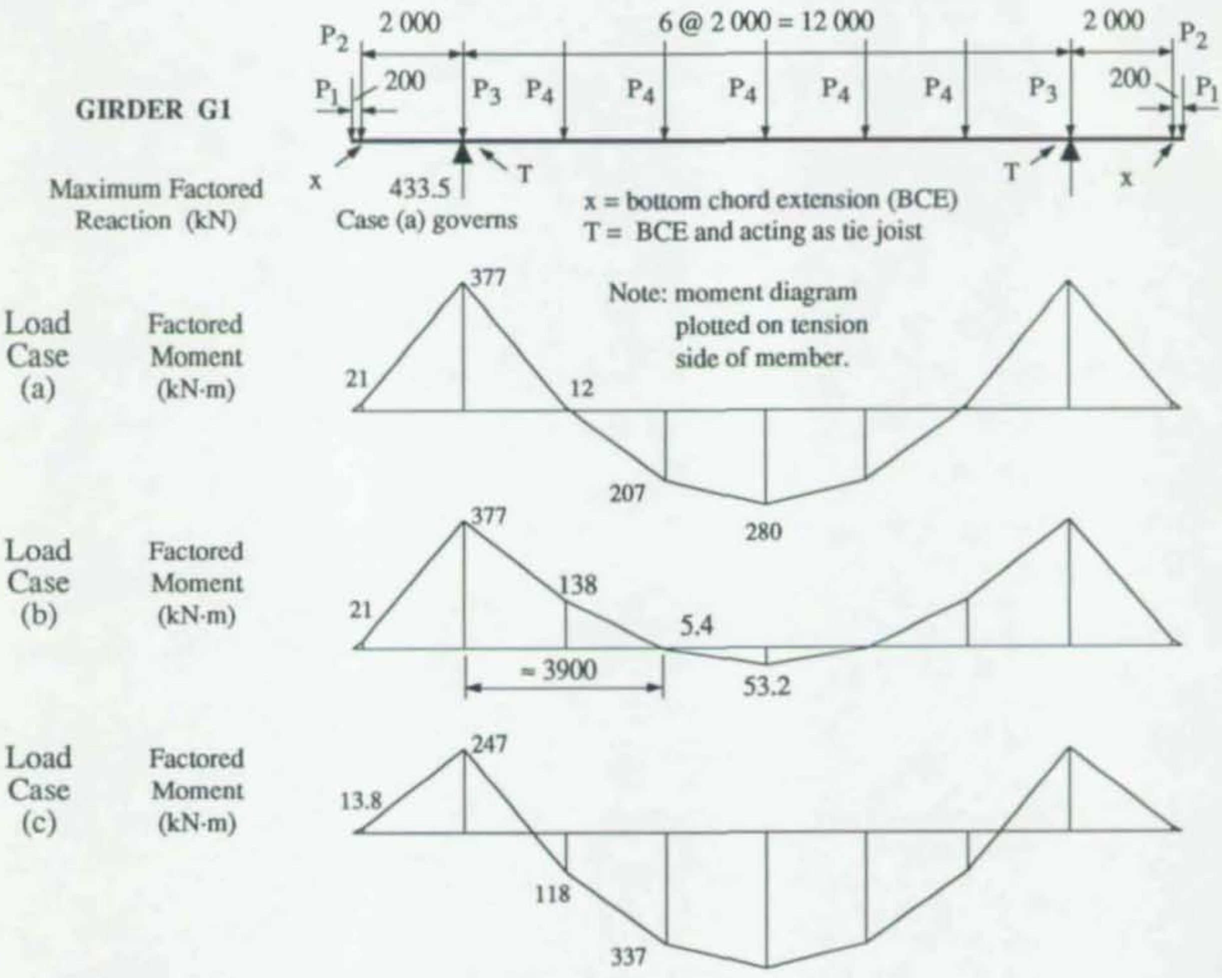
(i) $\alpha_D D + \alpha_L L = 105$ kN

(ii) $\alpha_D D + \alpha_L (50\%\,L) = 68.8$ kN

Cantilever girder G1 gravity load design loading cases : -

Load Case	Loading Condition		Factored Point Loads (kN)			
	Cantilevers & Drop-in	Centre Span	P_1	P_2	P_3	P_4
(a)	Dead plus Full Snow	Dead plus Full Snow	105	73.0	73.0	73.0
(b)*	Dead plus Full Snow	Dead plus Half Snow	105	73.0	60.4	47.8
(c)	Dead plus Half Snow	Dead plus Full Snow	68.8	47.8	60.4	73.0

* may be more severe than the unbalanced load called for by the National Building Code of Canada.



Design Example 1 Part 1 Trial Selection of Girder G1 (Continued .....)

(A) Select trial girder section for gravity load design moments .....

(1) Check negative moment at supports (bottom flange in compression)
Load cases (a) and (b) give maximum negative moment, M_f = 377 kN·m

Let us assume effective lengths of girder member as follows: -
(i) cantilever, KL= 1.0(2200) = 2200 mm (assume cantilever lateral-torsionally braced near tip)
cantilever, KL= 1.5(2200) = 3300 mm (assume cantilever lateraly braced near tip)
(ii) maximum interior unsupported length ≈ 3900 mm (column support to point of zero moment)
- governed by case (b) moment diagram
Longest effective length is 3900 mm. Joist bottom chord extension not needed at cantilever tip.
Using Beam Selection Table from CISC Handbook of Steel Construction, the factored moment resistance, M_r of W460x74 for unsupported length, L' of 4000 mm is given as 380 kN·m and M_r of W460x74 for L' = 3500 mm is given as 407 kN·m. By interpolation, we obtain M_r at 3900 mm ≈ 385 kN·m.

Since (M_r = 385 kN·m) > (M_f = 377 kN·m) the trial section of W460x74 is OK

Note: In this case, several approximate assumptions are made. CAN3-S16.1 Cl. 13.6 is used for M_r computation, ω is assumed as 1.0 and the unsupported length L is assumed as illustrated above.

(2) Check maximum positive moment at mid span (top flange in compression)
Load case (c) gives maximum negative moment, M_f = 410 kN·m

Let us assume unsupported lengths of girder member as 2000 mm
- joists provide lateral support to compression flange at 2 m intervals

Using Beam Selection Table from CISC Handbook of Steel Construction, the factored moment resistance M_r of W460x74 for unsupported length, L' of 2000 mm is given as 445 kN·m, since L_u is given as 2730 mm and greater than 2000 mm of joist spacing. (ω, in this case, is also assumed as 1.0)

Since (M_r = 445 kN·m) > (M_f = 410 kN·m) the trial section of W460x74 is OK

(B) Check trial girder section for net wind uplift design moments .....

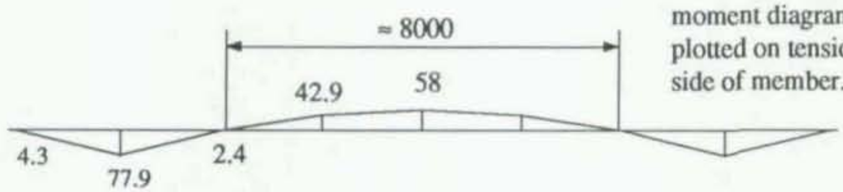
Total factored joist reaction for dead and wind loads = $\alpha'_D D + \alpha_w W$ = 0.85 (18.1) + 1.5 (-20.3) = -15.1 kN
(for all values of P_2 to P_4)

Total factored load at end of cantilever (from G3) = $\alpha'_D D + \alpha_w W$ = -21.7 kN
(for values of P_1)

Note:
moment diagram plotted on tension side of member.

Load Case (d)

Factored Moment (kN·m)

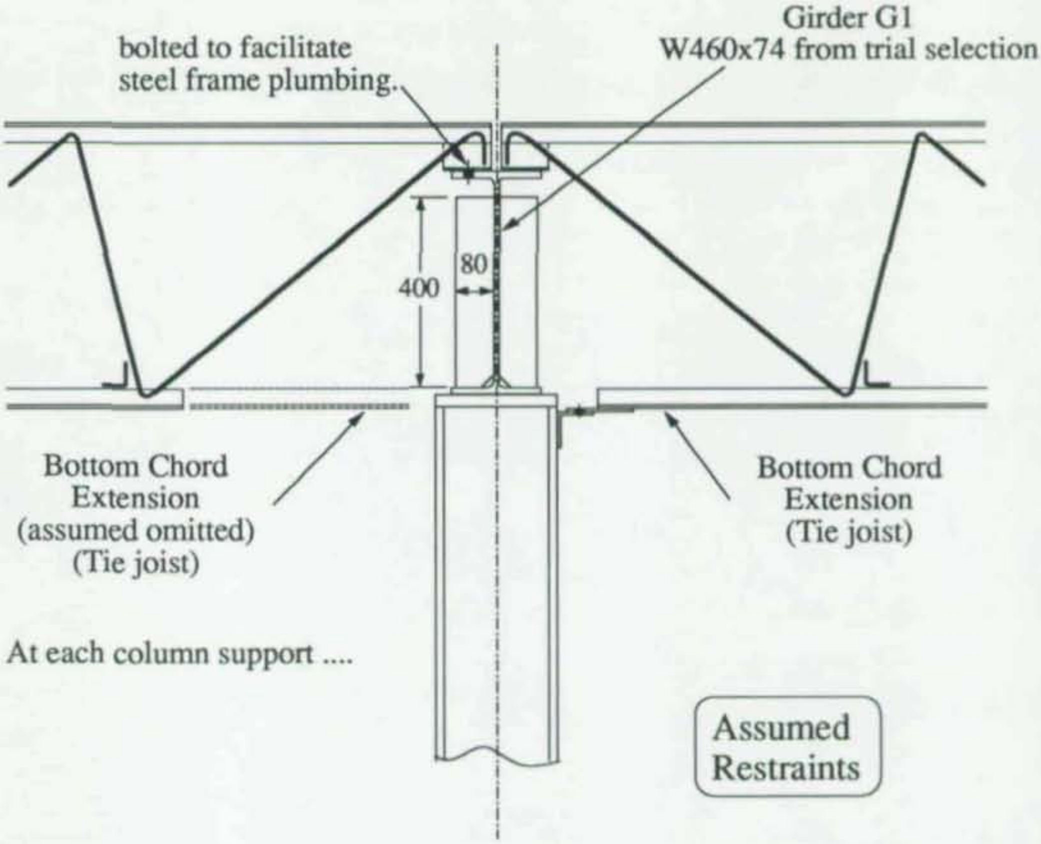


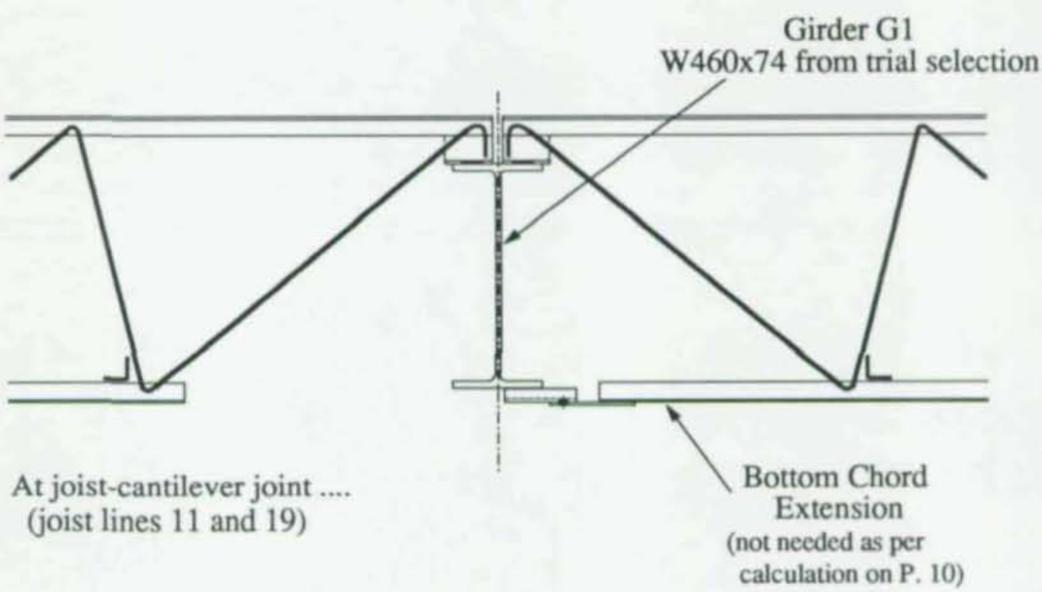
Let us assume unsupported length of girder member as 8000 mm (length of compression flange)

M_r of W460x74 for unsupported length of 8000 mm = 164 kN·m >> (M_f = 58 kN·m) OK

Trial Girder Section W460x74 is OK for approximate moment resistance design checks.

Design Example 1 Part 1 Trial Selection of Girder G1 (Continued .....)





Girder section W460x74 (Class 1 section in bending - see Table 5-1 of Handbook)

d = 457 mm	$I_y = 16.6 \times 10^6$ mm^4	$J = 517 \times 10^3$ mm^4	b = 190 mm	w = 9 mm
t = 14.5 mm	$Z_x = 1650 \times 10^3$ mm^3	$C_w = 813 \times 10^9$ mm^6	k = 31 mm	$S_x = 1460 \times 10^3$ mm^3

Design Example 1 Part 2 Detailed Design Check of Girder G1 for Moment Resistance

(1) Design Check Cantilever Girder for Moment Resistance using Appendix "A"

a. Assume web stiffeners are used as illustrated in Figure 9.

b. Girder is torsionally supported by joist top & bottom chord connections at each column line.

d. Assume cantilever tips are laterally supported by joist top chord connections as noted in figure on P.11. (73 kN joist reaction is loaded at top flange level)

c. End shear from suspended span girder member of W410x54 is transferred through the use of double angle connection, as illustrated in Figure 9. (105 kN transferred through web connection)

e. Using the top-left detail of Figure A2 , K may be estimated as within the range of 1.0 to 1.5. Thus, let us assume K = 1.5. Let us also use cantilever length $L_c = 2200$ mm.

(i) Check cantilevers:

Using Equation [A.1] in Appendix "A", compute elastic buckling moment resistance of cantilever,

$$M_u = \frac{\pi}{K L_c}\sqrt{E I_y G J + \frac{\pi^2 E^2}{(K L_c)^2} I_y C_W} = \frac{\pi}{3300}\sqrt{1.32 \times 10^{23} + 4.89 \times 10^{23}}$$

$$= 745 \times 10^6 \text{ N·mm}$$

where, E = 200 000 MPa and G = 77 000 MPa

$M_p = F_y Z_x = 0.3\,(1650) = 495$ kN·m and $M_u > (2/3)\,M_p$

$$M_r = 1.15\,\phi\,M_p\left(1 - \frac{0.28\,M_p}{M_u}\right) = 1.15\,(0.9)\,(495)\left[1 - \frac{0.28\,(495)}{745}\right]$$

$= 417$ kN.m $< \phi M_p = 445$ kN·m thus $M_r = 417$ kN·m or $>$ $M_f = 377$ kN·m OK

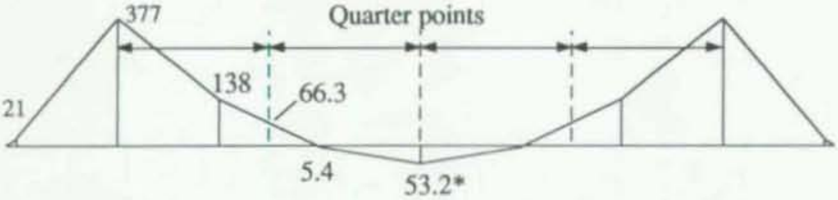
(ii) Check girder between column supports:

The girder is lateral-torsionally restrained at column supports. Cases (a) & (b) produce maximum negative moments. The maximum positive moment, in load case (b) is less than that of Case (a). Load case (b) is more critical (largest negative moment and smallest positive moment).
Using Equation [A.5] in Appendix "A", compute elastic moment resistance of girder (between column supports) by assuming continuous lateral support for girder tension (top) flange thru' evenly spaced joist end connections.

$$M_u = \frac{1}{\omega\, d'}\left[G J + \frac{\pi^2 E I_y d'^2}{2 L^2}\right] = \frac{1}{0.255\,(442.5)}\left[77\,000\,(517\,000) + \frac{\pi^2 (200\,000)\,(16.6 \times 10^6)\,442.5^2}{2\,(12\,000)^2}\right]$$

$= 550 \times 10^6$ N·mm where, $d' = d - t = 457 - 14.5 = 442.5$ mm L = 12 000 mm

and where, $$\omega = \frac{3M_2 + 4M_3 + 3M_4 + 2M_{max}}{12M_{max}} = \frac{3\,(66.3) + 4\,(0) + 3\,(66.3) + 2\,(377)}{12\,(377)} = 0.255$$ See Eqn. [A.4], Appendix "A"



* M_3 is zero because compression top-flange is laterally supported by joist seat.

Load case (b) is considered to be more critical

$$M_r = 1.15\,\phi\,M_p\left(1 - \frac{0.28\,M_p}{M_u}\right) = 1.15\,(0.9)\,(495)\left[1 - \frac{0.28\,(495)}{550}\right] = 383 \text{ kN·m} > M_f = 377 \text{ kN·m OK}$$

Design Example 1 Part 2 Detailed Design Check of Girder G1 (Continued .....)

(2) Design Check Cantilever Girder for Moment Resistance using Appendix "B"

Design of cantilever girder - bottom flange lateral buckling resistance:

Assume $I_{J1} = I_{J2} = 0.091 \times 10^6\ mm^4$ for 2L40x40x4 top chord using design criteria in Appendix "B"

End panel joist chord length, $x_1 = x_2 = 850$ mm; joist spacing, $s_1 = s_2 = 2\ 000$ mm

$$\kappa = \frac{E}{\dfrac{4\left(1-\upsilon^2\right)d'^3}{w^3} + \dfrac{d'^2 x_1 s_1}{12 I_{j1}} + \dfrac{d'^2 x_2 s_2}{12 I_{j2}}}$$

$$= \frac{200\ 000}{\dfrac{4\,(1-0.3^2)\,442.5^3}{9.0^3} + \dfrac{442.5^2\ (850)\,(2\ 000)}{12\,(0.091 \times 10^6)} + \dfrac{442.5^2\ (850)\,(2\ 000)}{12\,(0.091 \times 10^6)}}$$

$= 0.19$ N/mm/mm of girder length

Critical buckling load of compression flange, P_{cr} as in Equation [B.1] in Appendix "B"

$P_{cr} = 2\sqrt{\kappa\, E\, I_{yf}}$ critical load for an infinitely long compression member

$= 2\sqrt{0.19\,(200\ 000)\,8.29 \times 10^6} = 1120 \times 10^3$ N

where, $I_{yf} = 190^3\,(14.5)/12 = 8.29 \times 10^6\ mm^4$

Effective bottom chord force at factored load, P_f for load case (a) or (b)

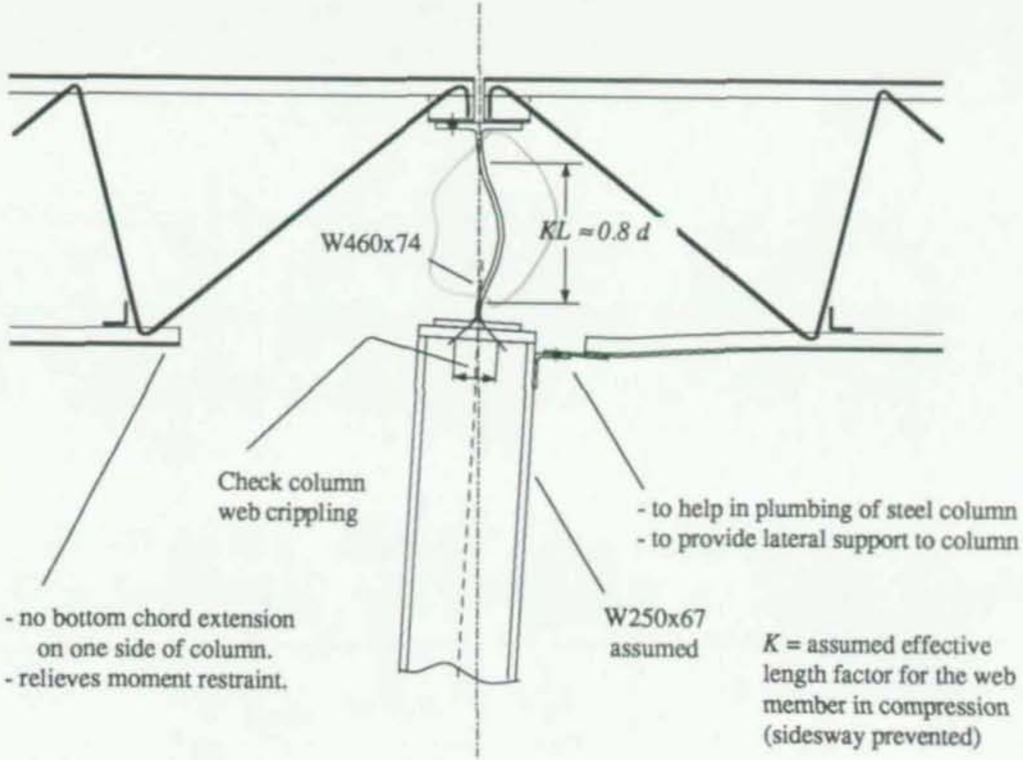
$$P_{flange} = 50\%\ \text{of}\ \frac{M_f\, b\, t}{S_x} = 50\%\ \text{of}\ \frac{377\,(190)\,(14.5)}{1460} = 356\ \text{kN}$$

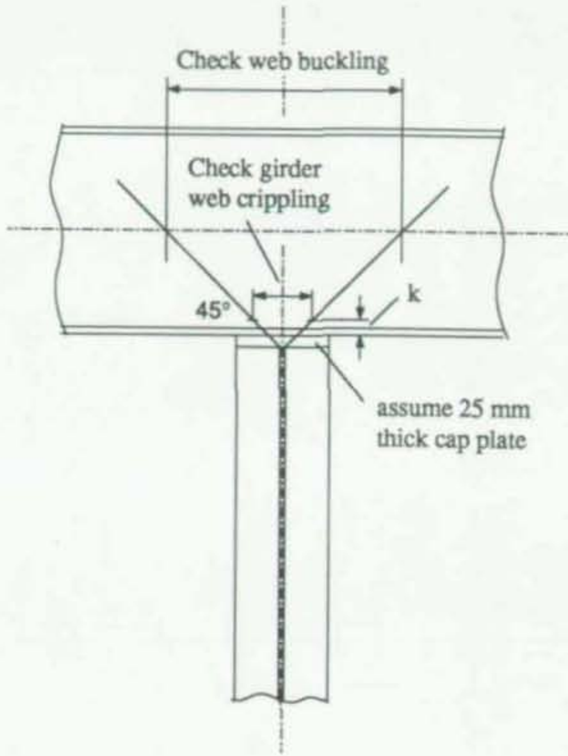
$$P_{web} = 50\%\ \text{of}\ (0.15\, h)\, w \left(\frac{M_f}{S_x}\right) = 50\%\ \text{of}\ 0.15\,(457 - 2 \times 14.5)\,9\left(\frac{377}{1460}\right) = 74.6\ \text{kN}$$ (15% of web area)

$P_f = P_{flange} + P_{web} = 356 + 74.6 = 431$ kN $\ll \phi P_{cr} = 1008$ kN OK (this design criteria is generally not critical for W- and WWF- shapes)

Note: 50% maximum flange force and 50% of partial web force is proposed to be effective, to simulate the effect of variation in axial load along the length of the bottom flange.

For a deep girder with narrow flanges carrying joists with small top chords spaced relatively far apart, the value of P_{cr} could be quite small; and this mode of failure may become critical.





Girder web buckling resistance, C_r, design check

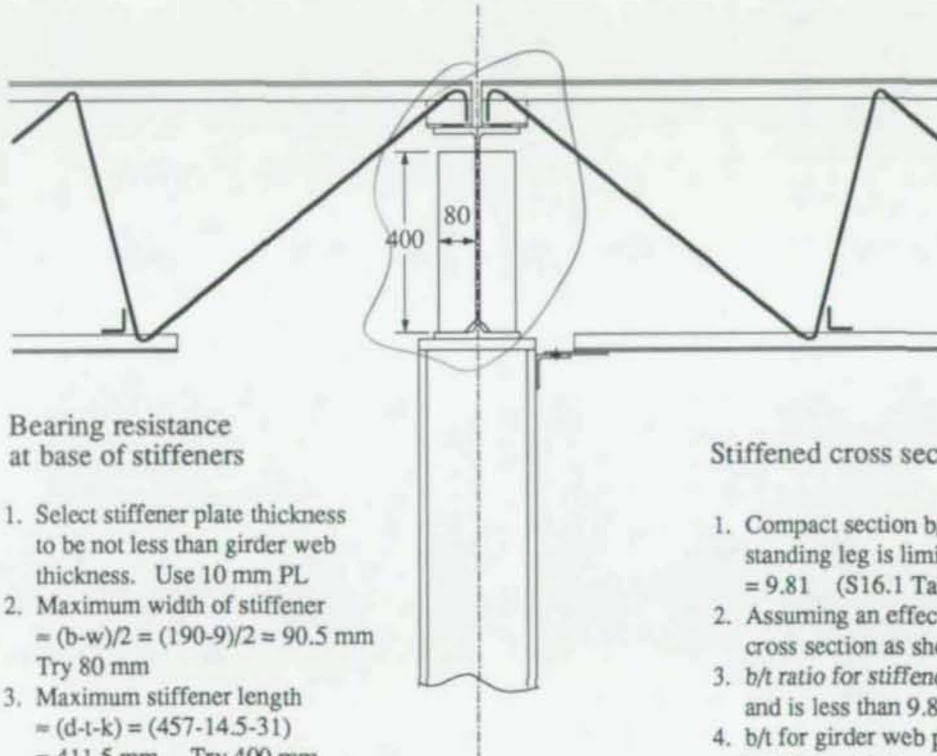
Effective girder web area for compression resistance,
$\approx w_g\, d = 9(457) = 4110\ mm^2$
r_y of web plate $= w_g / \sqrt{(12)} = 9/\sqrt{12} = 2.6$ mm
KL / r_y of web plate $= 0.8(457)/2.6 = 141$
$C_r/A = 76.6$ MPa --- Handbook Table 4-3

$C_r = 76.6(4110)/1000 = 315$ kN $<$ 433.5 kN
Web stiffeners required

*** Determine if girder web stiffeners are required

Column web crippling resistance, B_{rc}
$= 1.25\, \phi\, w_c\, (N + 2k)\, F_y$ (S16.1 Cl. 15.8)
$= 1.25\, \phi\, w_c\, [\, w_g + 2\, (k_g + t_{cap}\,)\,]\, F_y$
$= 1.25\, (0.9)(8.9)\, [\, 9.0 + 2\, (\, 31 + 25)]\, (0.3)$
$= 363$ kN $<$ 433.5 kN (max. fac. reaction)
Web stiffeners required

Girder web crippling resistance, B_{rg}
$= 1.25\, \phi\, w_g\, (N + 2k)\, F_y$
$= 1.25\, \phi\, w_g\, [\, w_c + 2\, (k_g + t_{cap}\,)\,]\, F_y$
$= 1.25\, (0.9)(9.0)\, [\, 8.9 + 2\, (\, 31 + 25)]\, (0.3)$
$= 367$ kN $<$ 433.5 kN (max. fac. reaction)
Web stiffeners required

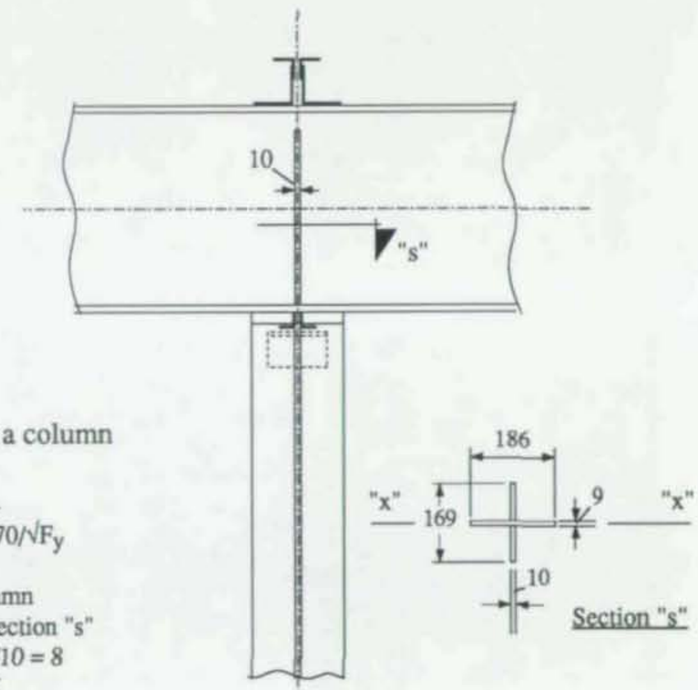


Bearing resistance at base of stiffeners

1. Select stiffener plate thickness to be not less than girder web thickness. Use 10 mm PL
2. Maximum width of stiffener $\approx$ (b-w)/2 = (190-9)/2 = 90.5 mm
   Try 80 mm
3. Maximum stiffener length $\approx$ (d-t-k) = (457-14.5-31)
   = 411.5 mm   Try 400 mm
4. Bearing area of two stiffeners (assuming 25 mm clipped for clearance at fillets of girder)
   A $\approx$ 2(80-25)(10) = 1100 mm^2
5. Bearing resistance at base of stiffeners $\approx 1.50\ \phi\ F_y$ A
   = 1.50(0.9)(0.300)(1100)
   = 446 kN > 433.5 kN
   Girder reaction

Stiffened cross section as a column

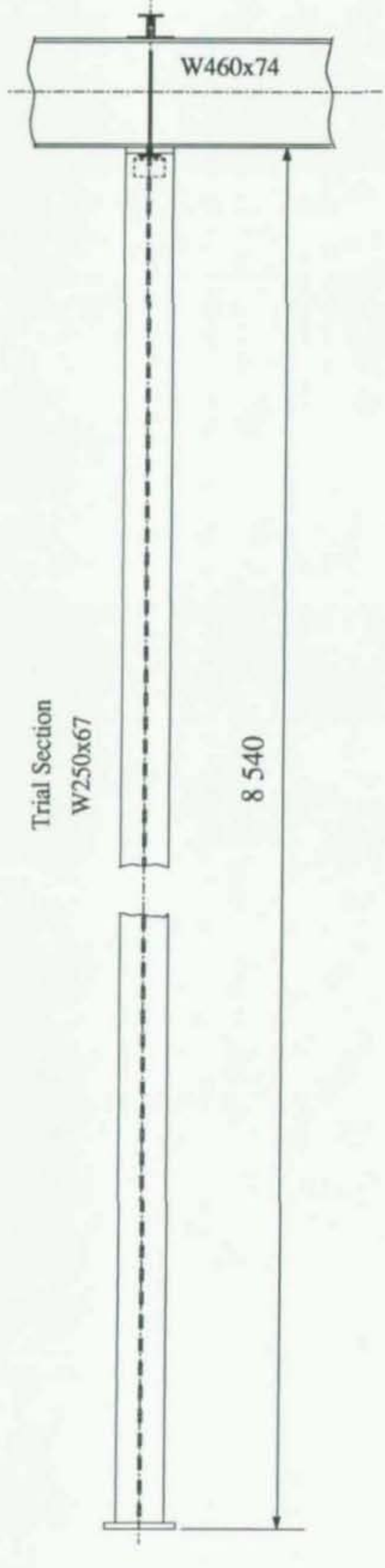
1. Compact section b/t of outstanding leg is limited to $170/\sqrt{F_y}$
   = 9.81   (S16.1 Table 1)
2. Assuming an effective column cross section as shown in section "s"
3. *b/t ratio for stiffeners = 80/10 = 8* and is less than 9.81   OK
4. b/t for girder web plate $\approx$ (186-10)/2/9
   = 9.78 and is less than 9.81   OK
5. Cross section area = 3274 mm^2
6. Radius of gyration about x axis = 35 mm
7. Assume column effective length = depth girder. Slenderness ratio = 457/35 = 13
8. Using Table 4-3 of Handbook, C_r /A for slenderness ratio of 14 is 269 MPa
9. C_r = 0.269(3274) = 881 kN >> 433.5 kN
   Girder web buckling cannot occur.



Stiffener to web welds

1. Using 5 mm fillet welds (minimum size), weld resistance = 0.765 kN/mm. Table 3-24 of Handbook
2. Required weld length per stiffener $\approx$ (433.5/2)/0.765 = 283 mm << available length for welding   OK
3. Maximum total factored uplift (as on P. 28 of Single Storey Building Design Aid - Ref. 27) = 132 kN
4. Factored resistance of flange to stiffener welds $\approx$ 4(80-25)(0.765) = 168 kN   OK

Designing Girder Stiffeners and Welds



Design Example 3 Interior Column Design

Using column-girder arrangement in Design Example 2, evaluate selected column:-
by procedure a) simple column selection, and
procedure b) detailed design computation

a) Design check for axial resistance

Try W250x67
$A = 8550\ mm^2$ $r_x = 110\ mm$
$I_y = 22.2 \times 10^6 mm^4$ $r_y = 51\ mm$ $M_{ry} = 89.6\ kN{\cdot}m$

1. Assume effective length $(K\,L)_x = (K\,L)_y = 8540$ mm
   See Case 1, Figure 10, where $L_x = (KL)_x$ and $L_y = (KL)_y$
2. $(K\,L)_x / r_x = (8540)/110 = 78$
3. $(K\,L)_y / r_y = (8540)/51 = 167$
4. $C_r / A = 58.3$ MPa (governed by y-axis)
   Using Handbook Table 4-3
5. $C_r = 58.3(8550)/1000 = 498$ kN
   > 433.5 kN W250x67 is OK

In this simple column selection procedure, only axial loads are considered and any induced column moment due to girder-column frame action is totally ignored. A more refined design procedure, in part b) of this example, illustrates that this simple column selection procedure yields conservative column member.

b) More detailed design check for axial-flexural resistance

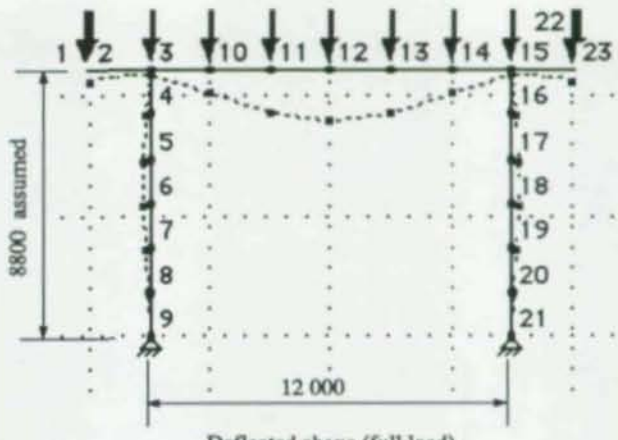
Using W460x74 - Gerber girder and
W250x67 - column (y-axis bending)

1. Frame deflected shape and column factored design forces are obtained from plane frame analysis.
2. Sway prevented case is assumed - roof is braced (through diaphragm design)
3. Using Appendix C of S16.1
   $G_L = 10$ for non-rigid or simple base detail
   $G_U = (I_c/L_c)/(I_g/L_g)$
   $= (22.2/8\ 800) / (333/12\ 000) = 0.09$
4. Using sidesway prevented alignment chart, S16.1 Appendix C,
   $K_y = 0.72$ Thus $(KL)_y = 0.72(8800) = 6340$ mm
5. $(KL)_x = 8540$ mm ; does not govern design
   $(KL)_y / r_y = 6340/51 = 124$
   $C_r / A = 92.4$ MPa Handbook Table 4-3
   $C_r = 92.4(8550)/1000 = 790$ kN
6. $C_e /A = 128$ MPa Handbook Table 4-8
   $C_e = 128(8550)/1000 = 1094$ kN
7. $C_f / C_e = 433.5/1094 = 0.40$
   $U_y = 1.67$ Handbook Table 4-9
8. Assuming $\omega = 1.0$, the following interaction expressions are design checked....

$$\frac{C_f}{C_{ro}} + \frac{0.6\,M_{fy}}{M_{ry}} = \frac{433.5}{2310} + \frac{0.6\,(5.86)}{89.6} = 0.23$$

$$\frac{C_f}{C_r} + \frac{U_y M_{fy}}{M_{ry}} = \frac{433.5}{790} + \frac{1.67\,(5.86)}{89.6} = 0.66$$

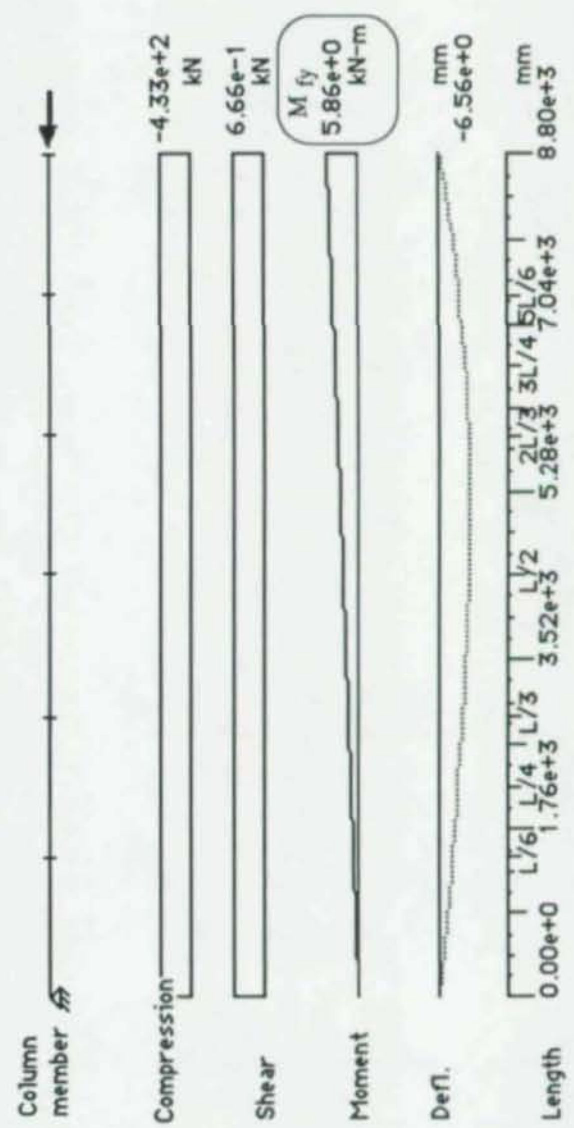
Column section W250x67 is OK



Deflected shape (full load)

When a cantilever roof girder is subjected to full and/or partial loading, girder-end rotations can induce moments to the supporting columns. Unequal roof bays of joist framing on either side of the girder can also induce out of plane moments to the supporting columns.

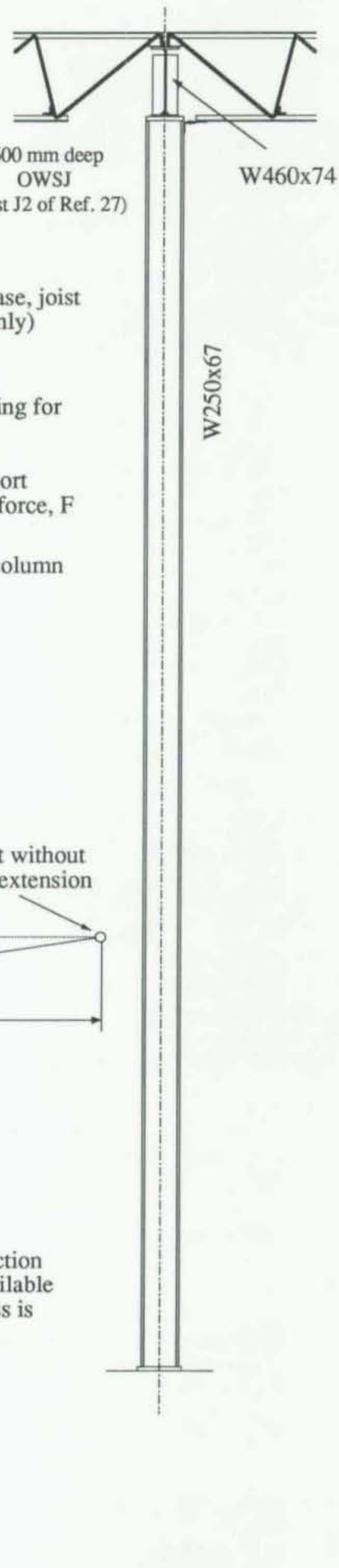
It may be demonstrated that the supporting columns, selected using design procedure a), are capable of resisting additional moments, provided that the girder to column and the tie-joist to column connections are designed to resist the entire induced moments. Design procedure b) illustrates the design check for one of the many critical load combinations.

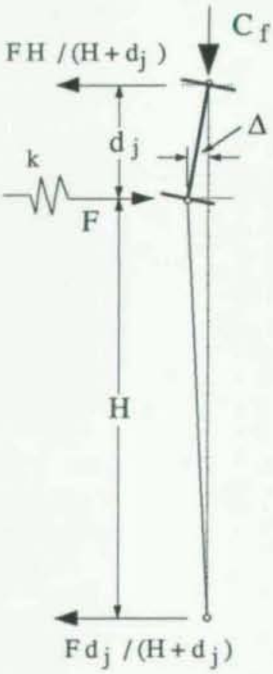


Column design foeces (full load)*

* Output from Frame Mac ™ on Macintosh ™, programmed by Erez Anzel, 1986

Design Example 4 Stability Design for Girder-Column Assembly
(supported by bottom chord extension of joists)





Assumptions:

F = joist chord restraining force (in this case, joist bottom chord extension on one side only)

k = spring stiffness = F/Δ

d_j = effective joist depth

I = moment of inertia of joist (after allowing for flexibility of joist web members)

L = joist span

$M = F \cdot d_j$ = end moment of joist at support connection due to stability force, F

θ = joist end rotation = Δ/d_j

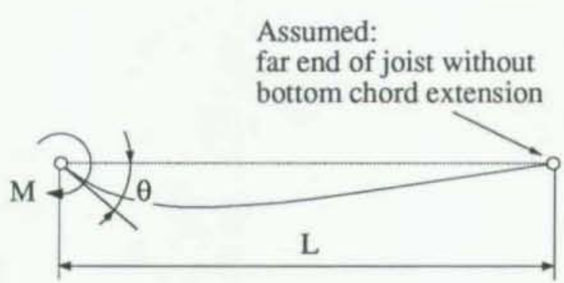
C_f = maximum factored load carried by column

Assuming the column is subjected to small lateral displacement, Δ

$M = F \cdot d_j = 3\,E\,I\,\theta / L$

substituting, $\theta = \Delta / d_j$, thus stiffness provided by joist connection,

$$F/\Delta = k = 3\,E\,I / (L\,d_j^2)$$



Summing moments about the cap plate,

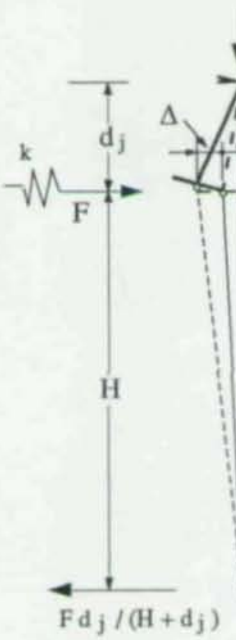
$$C_f\,\Delta = F\,d_j\,H / (H + d_j)$$

substituting $F = k_r\,\Delta$ into above yields the stiffness required to brace the assembly,

$$k_r = C_f\,(H + d_j)/(d_j \cdot H)$$

Note:
This example illustrates one sided joist bottom chord extension connection detail. For two sided joist bottom chord extension connection, the available stiffness from joists is $4\,E\,I / (L \cdot d_j^2)$ and the required bracing stiffness is $C_f\,(H+d_j)/(2\,d_j \cdot H)$

Design Example 4 (Continued.......)



Initial assembly out of straightness may be assumed from out-of-square of girder as permitted by CAN3-G40.20 "General Requirements for Rolled or Welded Structural Quality Steel" Thus initial deflection, $\delta \approx (T+T')d/b = 6(457)/190 \approx 15$ mm for W460x74

The growth in deflection, Δ, is determined by summing moments about the cap plate, $C_f(\delta+\Delta) = F\, d_j\, H / (H + d_j)$.

Substituting $F = k\Delta$ and $k_r = C_f (H+d_j)/(d_j \cdot H)$ into the moment equilibrium equation and solving for Δ,

$$\Delta = \frac{\delta}{\left(\frac{k}{k_r} - 1\right)}$$

If the additional deflection Δ at incipient buckling is equal to the initial δ, then the actual spring stiffness must be at least equal to twice the spring stiffness, $k = 2\,k_r$, or

$$\frac{3\,E\,I}{L\,d_j^2} = \frac{2\,C_f(H + d_j)}{d_j \cdot H}$$

and the force that must be resisted by the joist bottom chord is

$$F = k\,\Delta = 2\,k_r\,\Delta$$

Note: For two sided joist bottom chord extension connection formula (b) becomes $\dfrac{4\,E\,I}{L\,d_j^2} = \dfrac{C_f(H + d_j)}{d_j \cdot H}$

In this example, $C_f = 433.5$ kN*, $H = 8540$ mm, $L = 10500$ mm, $d_j = 570$ mm (600 mm overall depth),
$I \approx 101\,000\,000 / 1.1 = 91.8 \times 10^6 \text{mm}^4$ (allowing 10% loss of inertia for web deflection)

* for columns on joist lines 12 and 18 only.

Using design expressions as illustrated above, check stiffness requirement:

Actual spring stiffness $= k = \dfrac{3\,(200\,000)(91.8 \times 10^6)}{10\,500\ (570^2)} = 16.1$ kN/mm

Required spring stiffness $= k_r = \dfrac{2\,(433\,500)(8540+570)}{570\,(8540)} = 1.67$ kN/mm

There is sufficient bracing stiffness in a single joist bottom chord extension connection.

Extension Δ, under full C_f and using the one sided joist member stiffness,

$$\Delta = \frac{\delta}{\left(\frac{k}{k_r} - 1\right)} = \frac{15}{\left(\frac{16.1}{1.67} - 1\right)} = 1.74 \text{ mm}$$

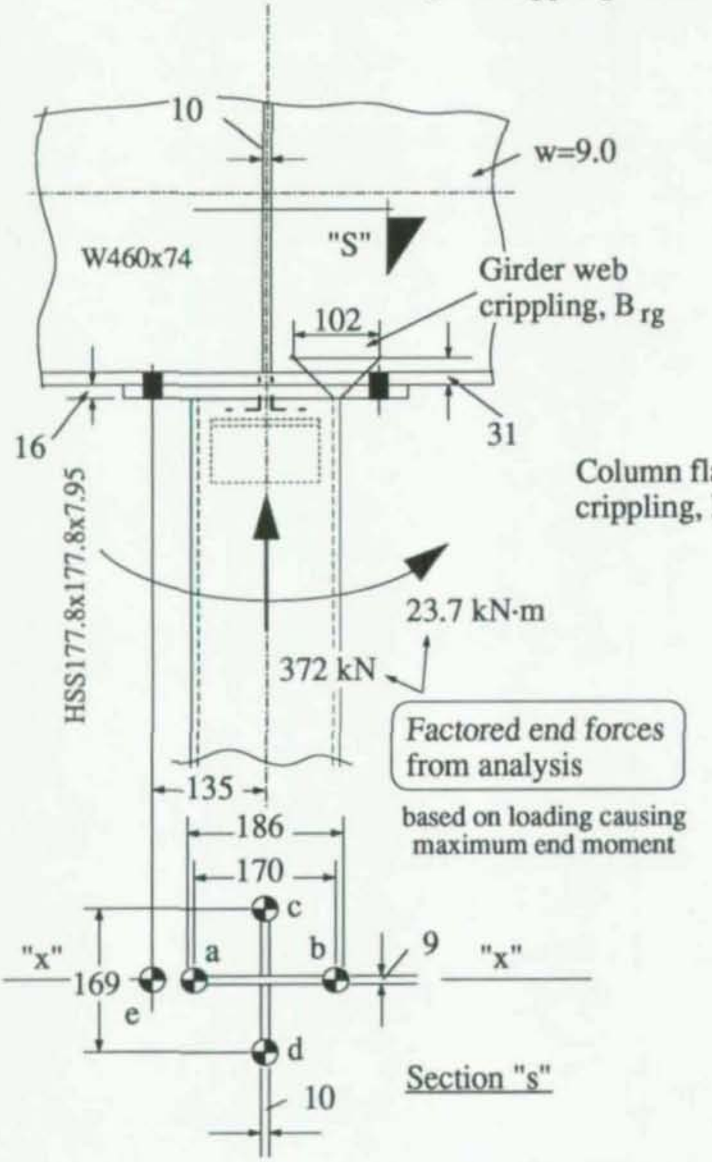
Determine minimum connection force for joist bottom chord extension:

a. Stability force as $F = 2\,k_r\,\Delta = 2\,(1.67)(1.74) = 5.81$ kN
b. 1% of compression force in bottom flange of girder ≈ 8 kN
Therefore, total connection force $= 5.81 + 8 = 13.8$ kN

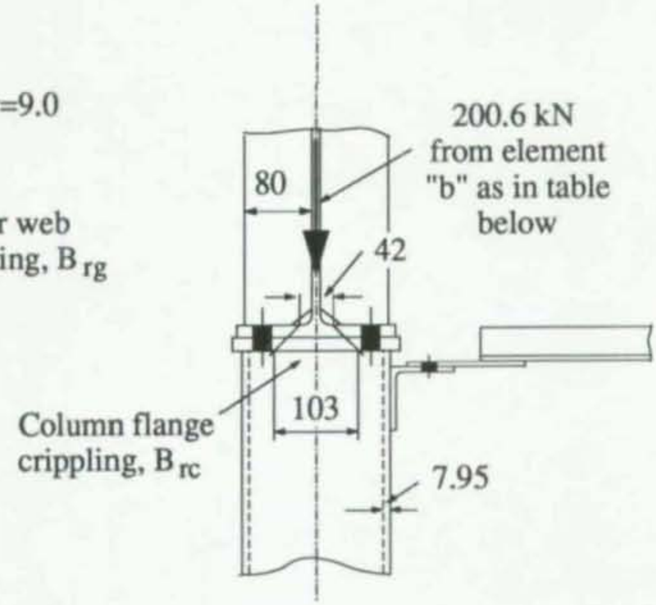
Two 5/8" diameter A307 bolts - single shear (threads excluded) = 65.8 kN
(greater than 13.8 kN of required resistance, OK)

Note: See Reference 22 for formulation of basic concept and other explanation

Design Example 5 (Part 1) Cantilever Girder to Column Connection Design Checks
(web crippling checks for gravity loads)



Section "s"



Girder web crippling resistance, B_{rg}
$= 1.25\ (0.9)\ w_g (102)\ (0.3)$
$= 310\ \text{kN} \quad > \quad B_f = 200.6\ \text{kN}$

Col. flange crippling resistance, B_{rc}
$= 1.25\ (0.9)\ w_c\ (103)\ (0.35)$
$= 322\ \text{kN} \quad > \quad B_f = 200.6\ \text{kN}$
(force at location b)

Connection OK for crippling resistance checks.

Analysis Assumption

Connection Design Forces	Factored compression (kN) at location				
	a	b	c	d	e
due to 372 kN	93	93	93	93	0
due to 15.8 kN·m *	-93	93	0	0	0
due to (23.7-15.8) kN·m	14.6	14.6	14.6	14.6	-58.4
Total Effect	14.6	200.6	107.6	107.6	-58.4

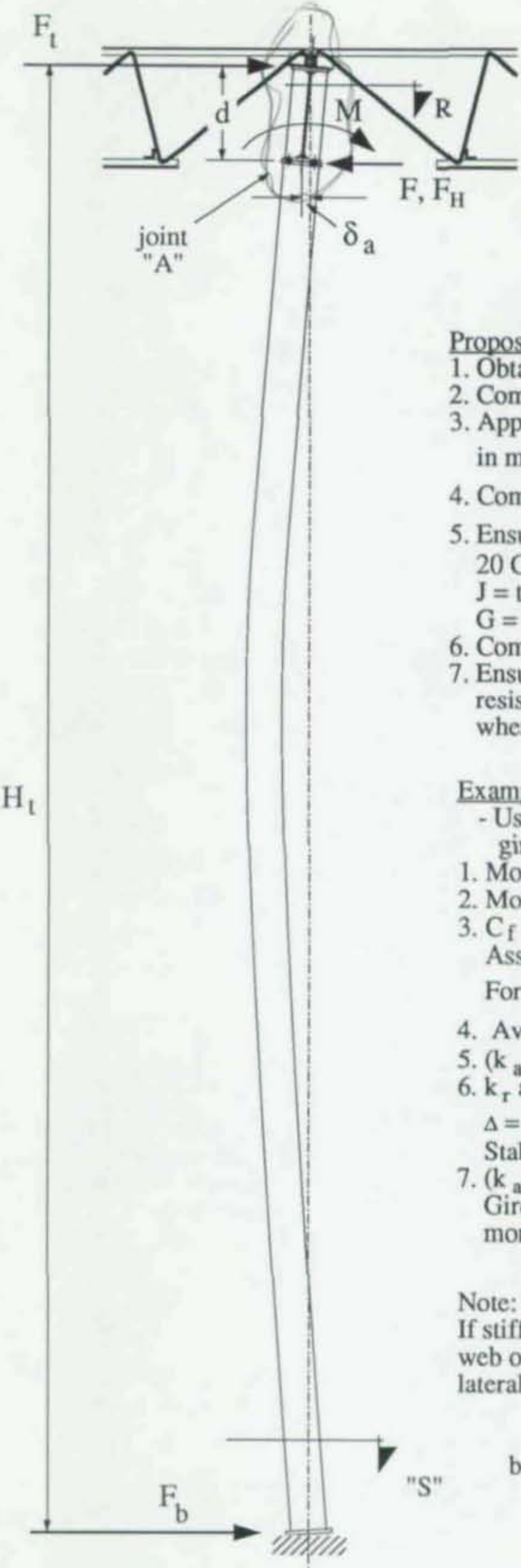
Verification of cap plate moment resistance and bolt tensile resistance (not illustrated with this example) should be carried out.

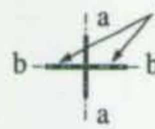
* part of 23.7 kN·m to reduce compression at point a to zero.

Most critical compression force

Tension carried by two bolts

Design Example 5 (Part 2) Column-girder connection to achieve column continuity and to provide lateral-torsional support to girder





Section "R"

Tentative Design Procedure:
Column to Stiffened-Girder Connection
(Tie-joist not used)

Proposed Design Steps: - Intended to give conservative results

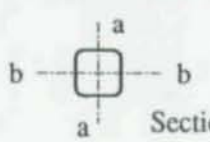
1. Obtain moment of inertia of selected column about axis "a-a"
2. Compute moment of inertia of stiffeners about axis "a-a"
3. Apply unit force (F) at joint A to beam of 2 segments, differing in moment of inertia. Member length = H_t. Obtain δ_a at A.
4. Compute available stiffness, $k_a = F/\delta_a$
5. Ensure available stiffness ($k_a = F/\delta_a$) > the required stiffness of $20\,GJ/(L\,d^2)$, where L= spacing between column supports, J = torsional constant of the girder, d = depth of girder and G = shear modulus 77 000 MPa. (See Reference 28)
6. Compute k_r, Δ and F, using expressions on P. 19.
7. Ensure $k_a > k_r$, and available girder-column joint moment resistance to be not less than approximately the product $F_H \cdot d$, where $F_H = F + 1\%$ flange force.

Example Design Checks

- Using selected members as in Reference 27:- W460x74 girder, HSS177.8x177.8x7.95 column, stiff gdr./col. joint.

1. Moment of inertia of column selected = $24.8 \times 10^6\,mm^4$
2. Moment of inertia of a pair of stiffeners ≈ $4.02 \times 10^6\,mm^4$
3. $C_f = 433.5$ kN (as in Example 4)
   Assume 1 kN horizl. force at girder-column joint as shown
   For F = 1 kN, the value $\delta_a = 0.1$ mm (by stiffness analysis)
4. Available stiffness = $k_a = F/\delta_a = 10$ kN/mm
5. ($k_a = 10$ kN/mm) >> { $20\,GJ/(L\,d^2) = 0.36$ kN/mm}
6. k_r as per P. 19 ≈ 2.00 kN/mm (substituting d for d_j)
   $\Delta = 15/[(10/2.00)-1] = 3.75$ mm ($\delta = 15$ mm assumed)
   Stability force F at connection ≈ 2(2.00)3.75 = 15 kN
7. ($k_a = 10$ kN/mm) >> ($k_r = 2.00$ kN/mm)
   Girder to column joint should be designed to carry moment about "a-a" axis $F_H \cdot d$ ≈ (15+8) 0.457 = 10.5 kN·m

Note:
If stiffeners were not used, the assembly would fail by girder web out-of-plane bending, lateral buckling of bottom flange or lateral-torsional buckling of girder.



Section "S"

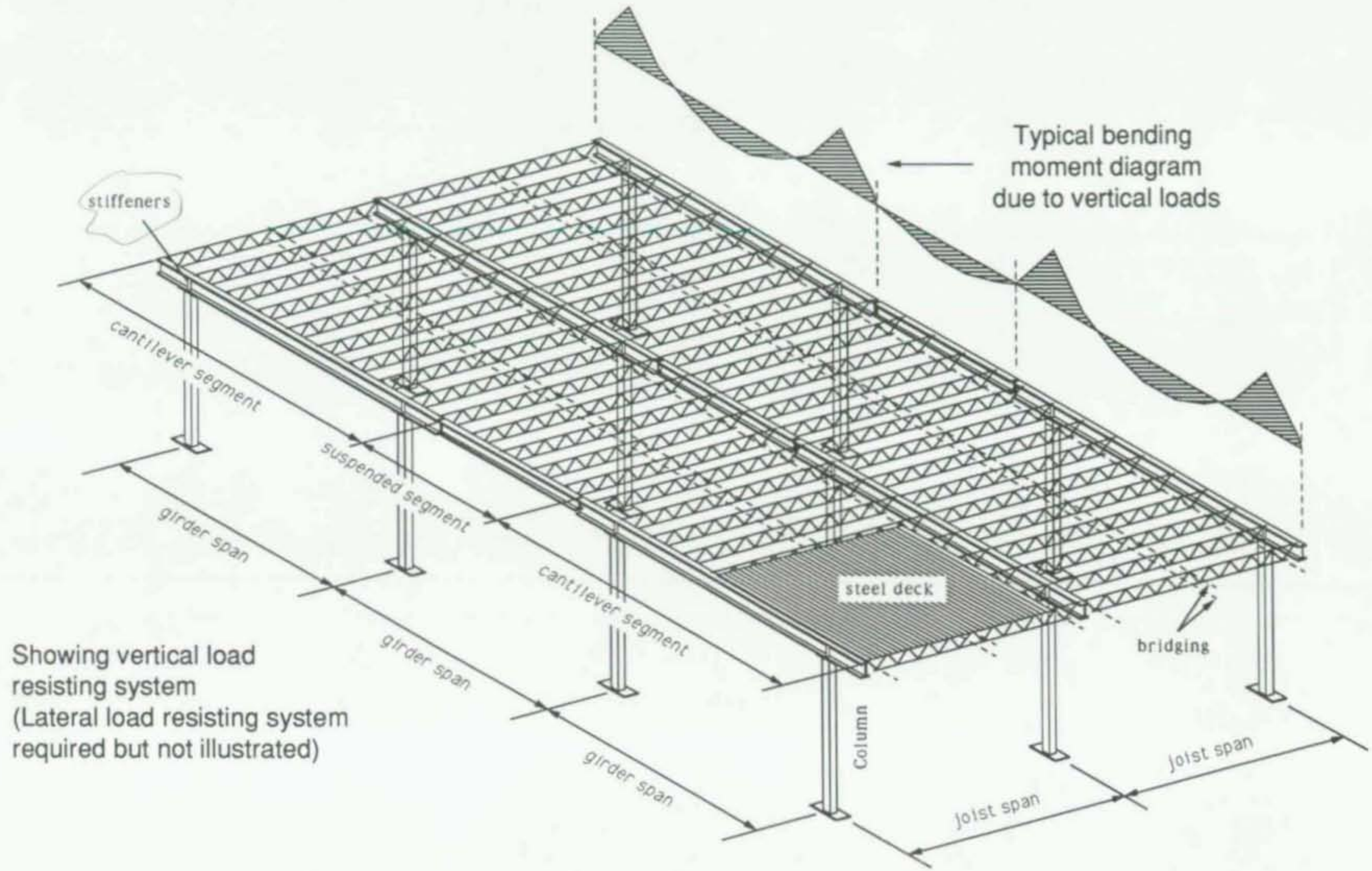


Figure 1 Roof Construction using Cantilever (Gerber) Girder and OWSJ Framing

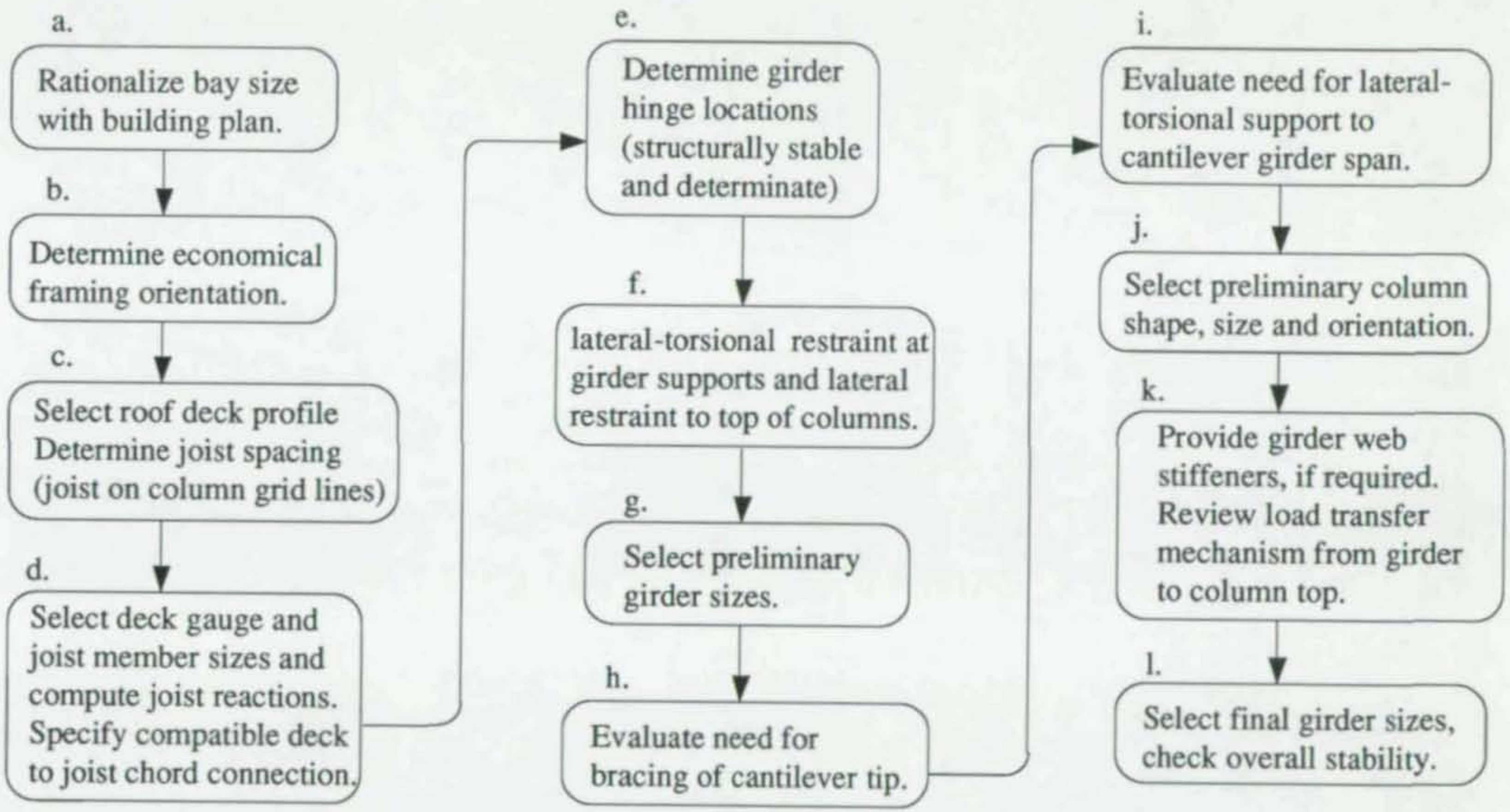
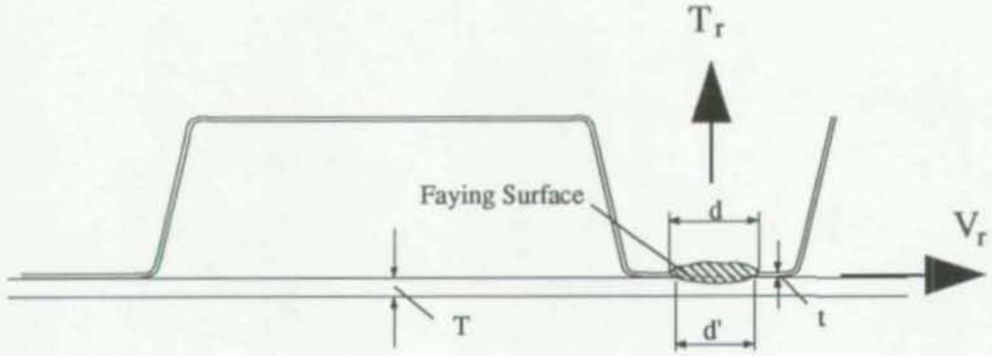


Figure 2 Cantilever/Gerber Girder Design
- Simplified Flow Chart

planning stages - steps a & b
preliminary design - steps c to j
final design checks - steps k & l
stability checks - steps c to l



Design Formulae

Factored shear resistance,
$V_r \le \phi_c 10^3\ (20t - 5)$ newtons
$\le \phi_c\, d\, F_y\ (80t - d) / 18.4$

Factored tensile resistance,
$T_r \le \phi_c 10^3\ (5.6t - 1)$ newtons
$\le \phi_c\, d\, F_y\ (112t - d) / 92$

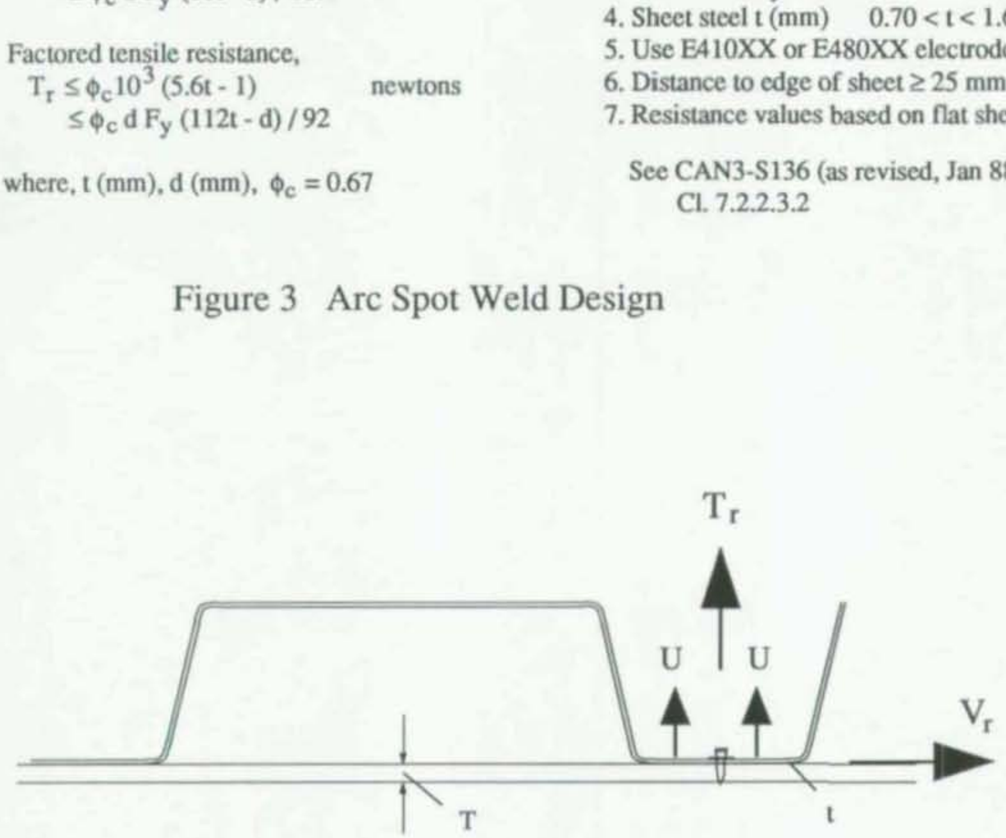
where, t (mm), d (mm), $\phi_c = 0.67$

Limitations

1. Visible nominal diameter, $d \ge 20$ mm
2. Thickness of supporting steel, $T \ge 2.5t$
3. Sheet steel $F_y \ge 230$ MPa
4. Sheet steel t (mm) $0.70 < t < 1.67$
5. Use E410XX or E480XX electrodes
6. Distance to edge of sheet ≥ 25 mm
7. Resistance values based on flat sheets

See CAN3-S136 (as revised, Jan 88)
Cl. 7.2.2.3.2

Figure 3 Arc Spot Weld Design



Values of T_r, V_r and U for flat sheet connection to be obtained from manufacturer.

Figure 4 An Example of Field-Applied Sheeting Fastener

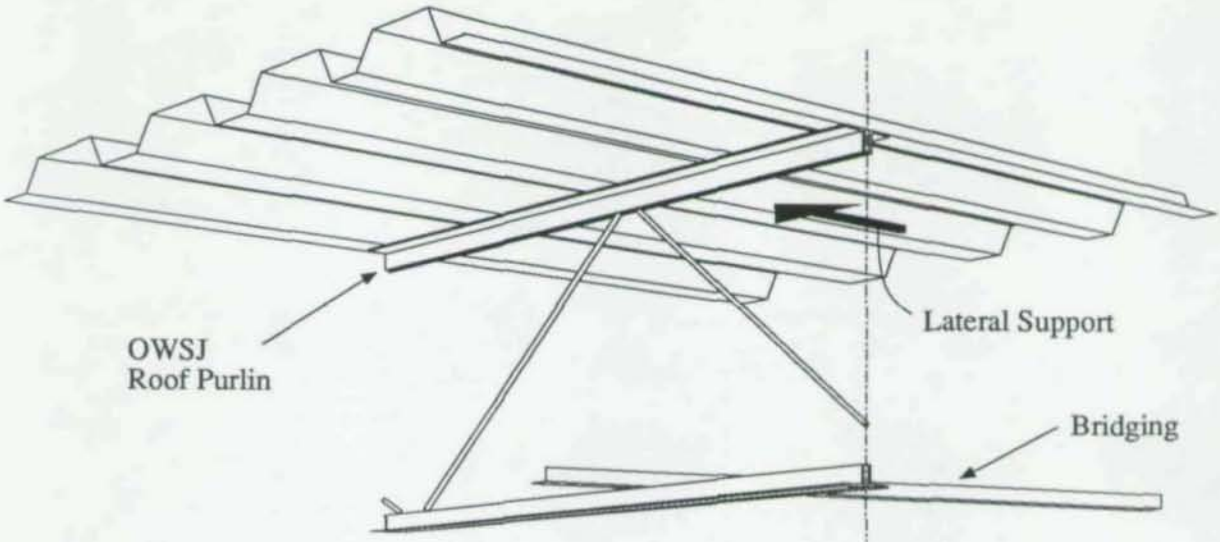


Figure 5 Lateral Support to Top of Joist by Steel Deck

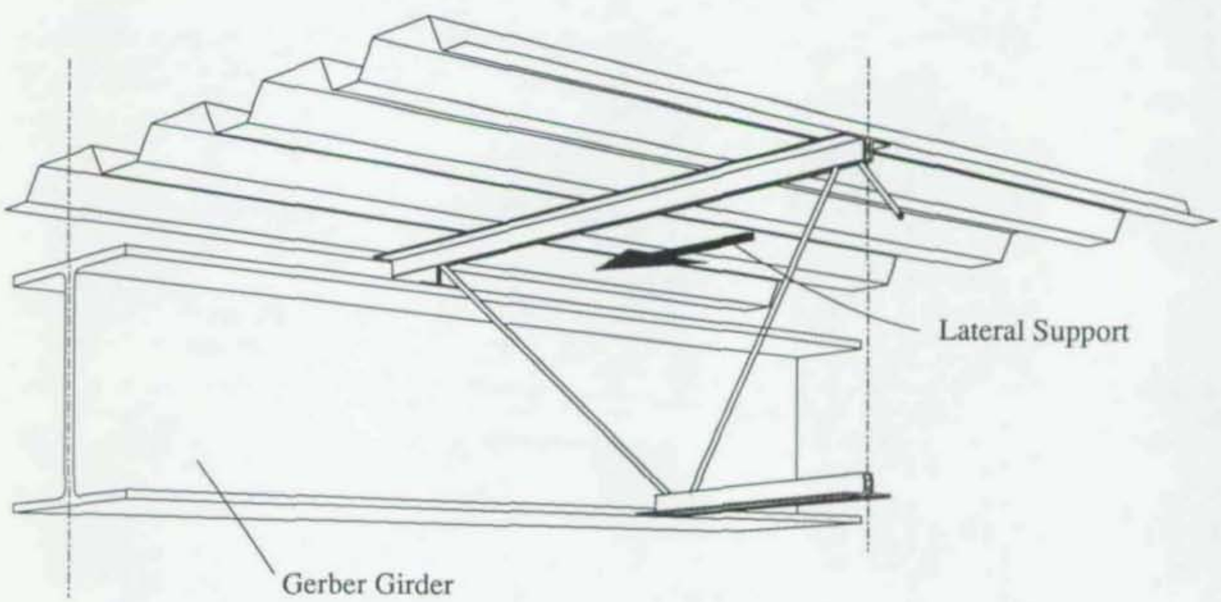


Figure 6 Lateral Support to Top Flange of Girder

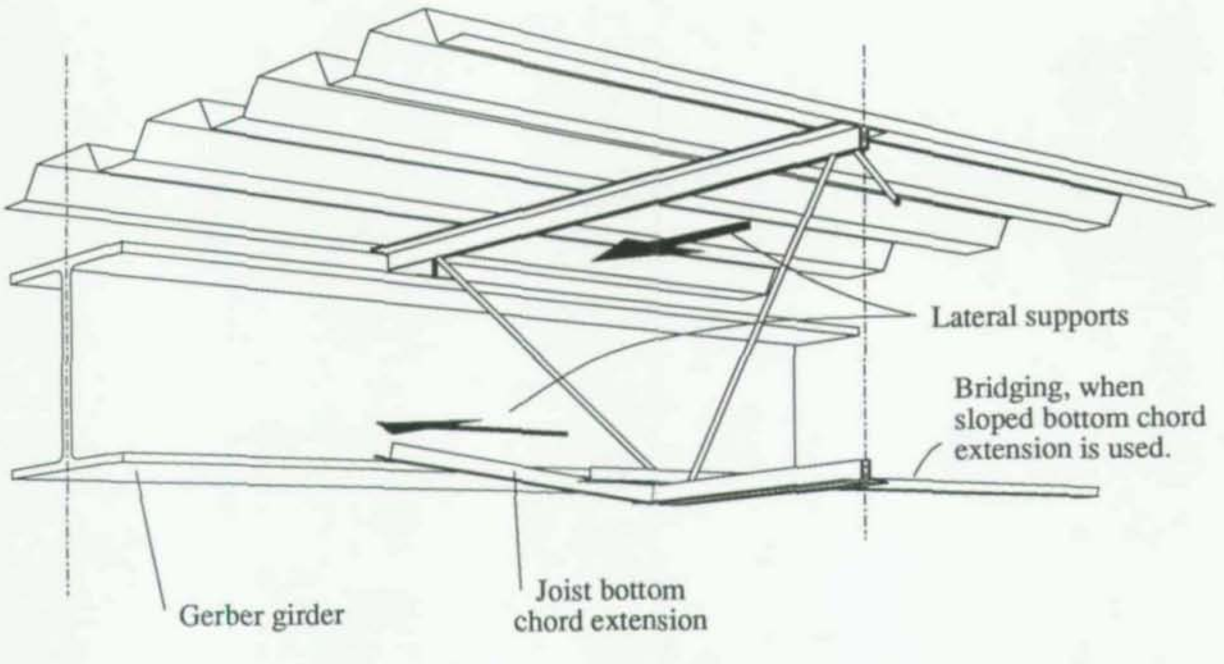


Figure 7 Lateral Support to Bottom Flange of Girder

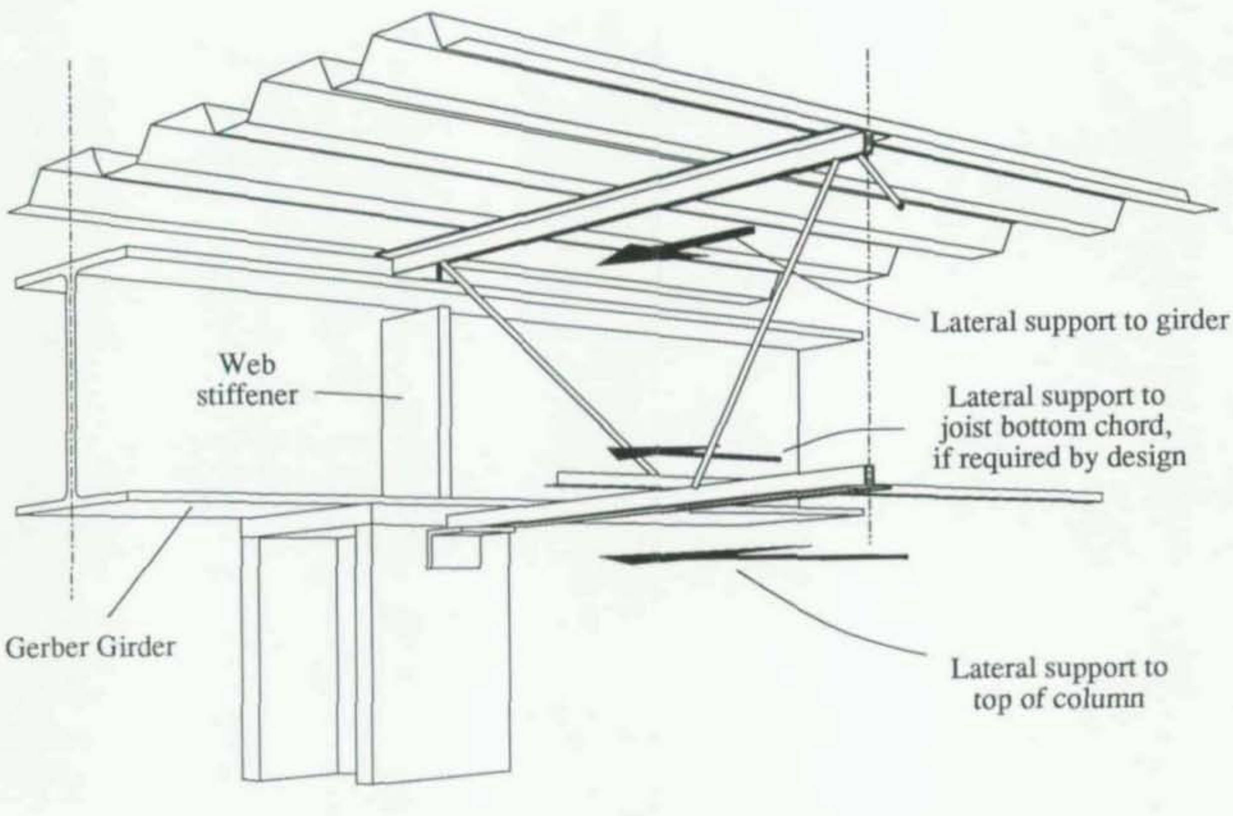


Figure 8 Lateral Supports at Joist-Girder-Column Joint

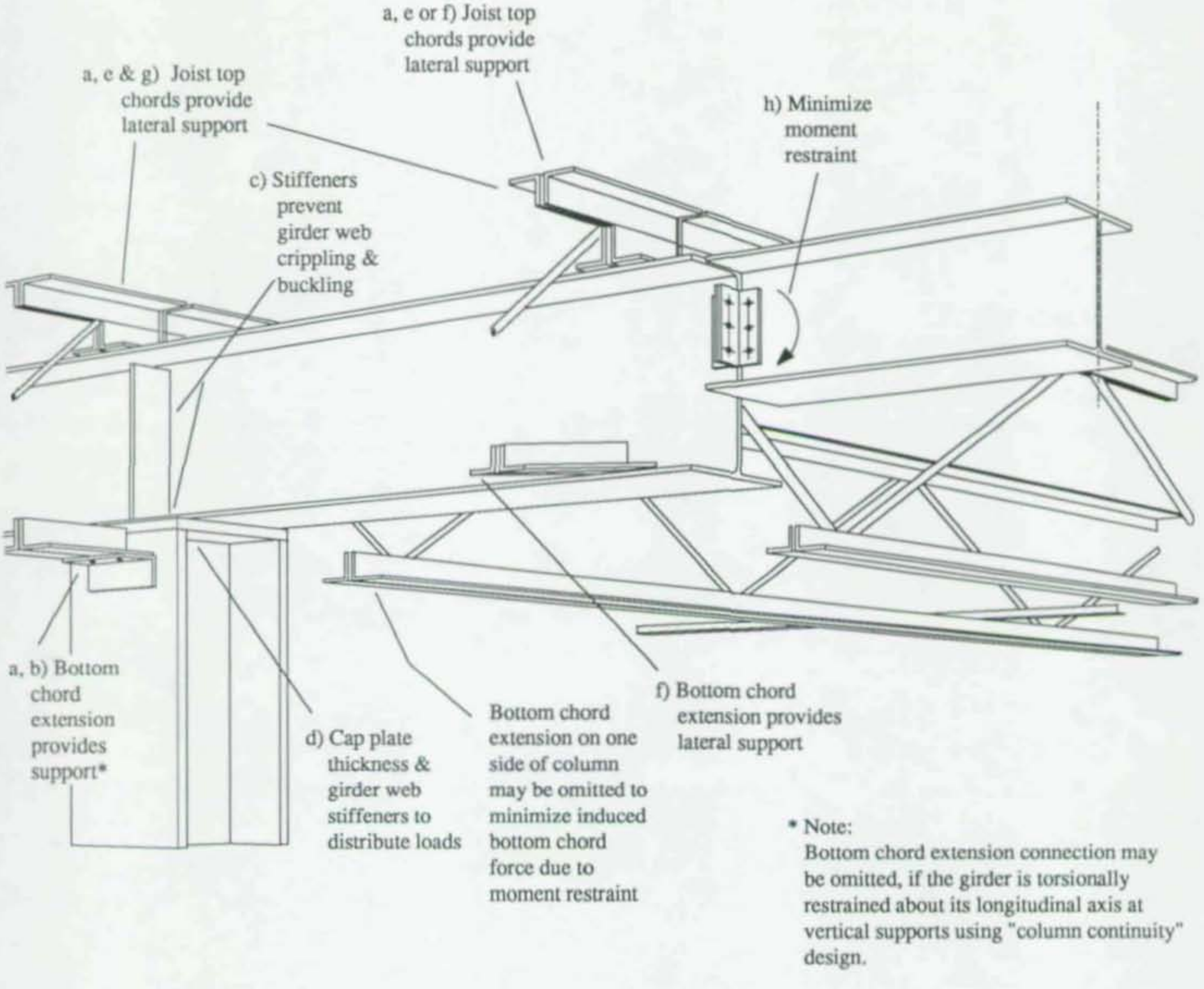
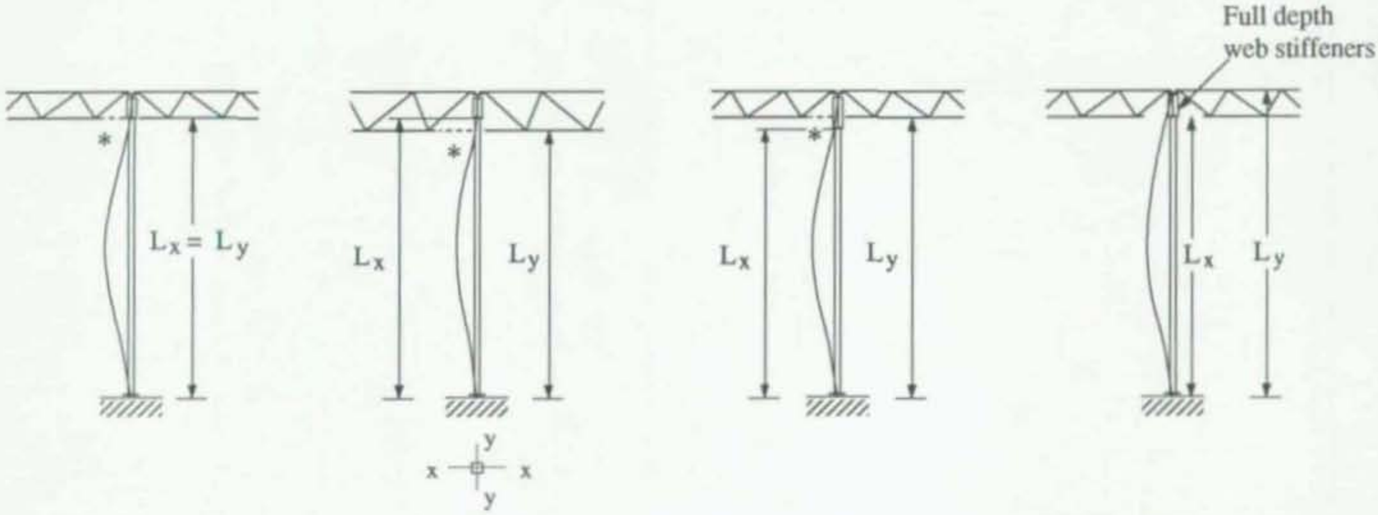


Figure 9 Gerber Construction Details

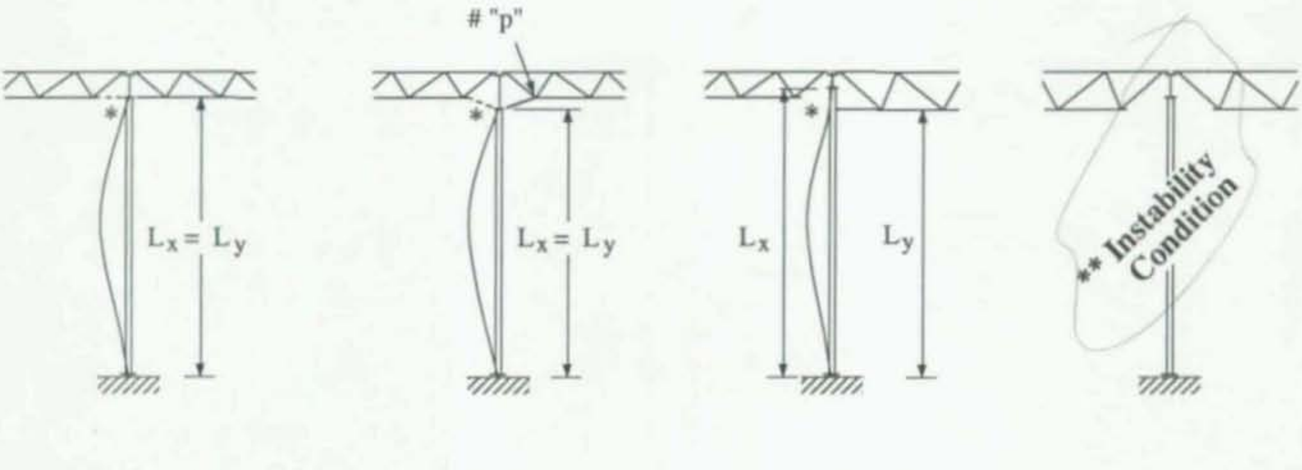


Case 1
Joist & girder similar depth ; girder web stiffened ; joist bottom chord provides lateral support to column

Case 2
Joist deeper than girder ; girder web stiffened ; joist bottom chord provides lateral support to column

Case 3
Girder deeper than joist ; girder web stiffened ; joist bottom chord provides lateral support to column

Case 4
Column continuity, created by stiffened girder web and stiff column-to-girder joint, provides lateral-torsional support to girder and lateral support to column.



Case 5
Joist & girder similar depth; *girder web not stiffened* ; joist bottom chord provides lateral support to column

Case 6
Girder deeper than joist; *girder web not stiffened* ; joist bottom chord provides lateral support to column

Case 7
Joist deeper than girder ; *girder web not stiffened* ; joist bottom chord provides lateral support to column

Case 8
Girder web not stiffened; *column top laterally* unsupported

Figure 10 Stability Considerations for Gerber Girder - Column Assembly

Notes:
Bridging line at "p" is proposed.
* Bottom chord extension on one side of girder-column joint may be omitted to minimize induced bottom chord force due to joist end moment restraint.
** See References 21 to 24.

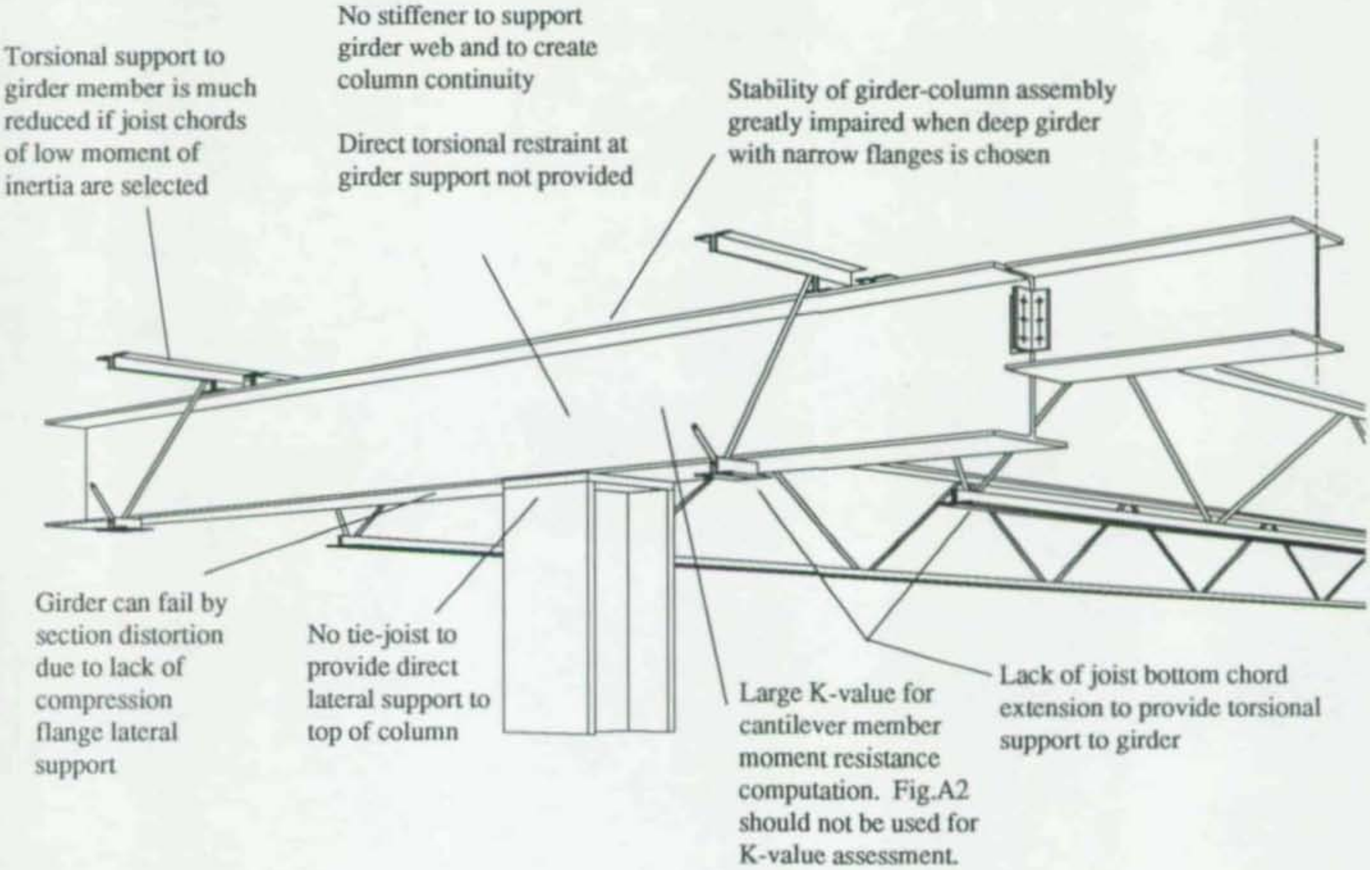


Figure 11 Stability Considerations - if joist framing NOT on column line

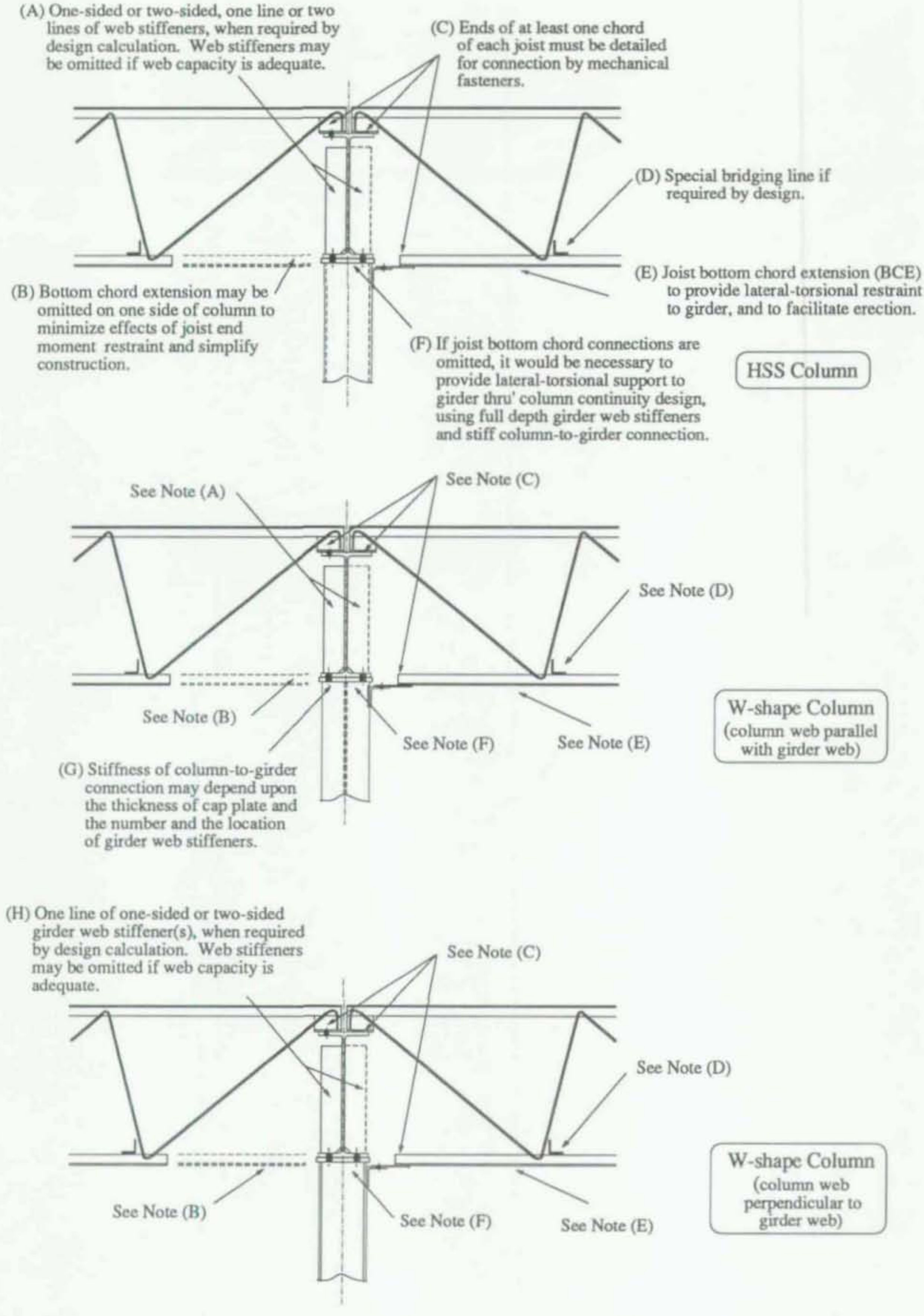


Figure 12 Structural Details at Joist-Girder-Column Joint
(Girder-Column Assembly Stability provided by Joist BCE)

Appendix "A" Proposed Moment Resistance Evaluation Rules for Cantilever Girders

While CAN3-S16.1 provides basic expression on critical elastic buckling moment resistance, M_u for simply supported beams with equal end moments, and equivalent moment factor, ω, for some non-uniform moments. Several references should be consulted for M_u and ω computation to reflect other support and loading conditions.

1. The following design steps are proposed in evaluating moment resistance of cantilevers:
   a. compute critical elastic buckling moment resistance, M_u using Nethercot[17] expression and as in Fig. A1

$$M_u = \frac{\pi}{K L_c}\sqrt{E I_y G J + \frac{\pi^2 E^2}{(K L_c)^2} I_y C_w} \qquad [A.1]$$

   where K = effective length factor for cantilever.
   Kirby-Nethercot[18] and the SSRC Guide[20] suggest the use of several effective lengths for cantilevers, as in Figure A1. Also see Fig. A2 for guidance in selecting appropriate K value for Gerber-cantilever application.

   b. compute factored moment resistance, M_r for girder of class 1 & 2 sections using CAN3-S16.1 Cl. 13.6 as

$$M_r = 1.15\,\phi\, M_p\left(1 - \frac{0.28\, M_p}{M_u}\right) \quad \text{but not greater than } \phi M_p \qquad [A.2]$$

$$\text{when} \quad M_u > \frac{2}{3} M_p \text{, or}$$

$$M_r = \phi\, M_u$$

$$\text{when} \quad M_u \le \frac{2}{3} M_p \qquad \text{where } M_p \text{ is the product } F_y Z_x \qquad [A.3]$$

   Note: For class 3 and 4 sections, step 1.b. may be used by replacing M_p with M_y (= $F_y S_x$)

2. When the girder bottom flange between vertical supports is in compression, the critical moment resistance against elastic lateral-torsional buckling is increased by tension (top) flange uniform lateral support through generally equal spaced joist framing. The CAN3-S16.1 rule for M_u computation may be modified as follows:
   a. obtain equivalent moment factor[18] for M_u expression as

$$\omega = \frac{3M_2 + 4M_3 + 3M_4 + 2M_{max}}{12M_{max}} \qquad [A.4]$$

   Note: see Figure A3 for correct use of this design expression.

   b. compute critical elastic buckling moment resistance, M_u using Roeder-Assadi[19,20] expression for beams with tension flange laterally supported along its full length. Also see Fig. A3.

$$M_u = \frac{1}{\omega\, d'}\left[G J + \frac{\pi^2 E I_y d'^2}{2 L^2}\right] \qquad [A.5]$$

   where, L = length between vertical supports at which the member is lateral-torsionally restrained.

   c. compute M_r for girder of class 1 & 2 sections using CAN3-S16.1 Cl. 13.6 as

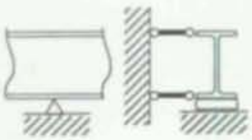
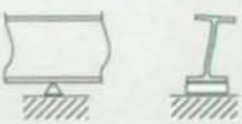
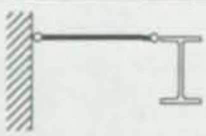
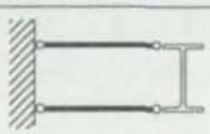
$$M_r = 1.15\,\phi\, M_p\left(1 - \frac{0.28\, M_p}{M_u}\right) \quad \text{but not greater than } \phi M_p$$

$$\text{when} \quad M_u > \frac{2}{3} M_p \text{, or}$$

$$M_r = \phi\, M_u$$

$$\text{when} \quad M_u \le \frac{2}{3} M_p \qquad \text{where } M_p \text{ is the product } F_y Z_x$$

   Note: For class 3 and 4 sections, step 2.c. may be used by replacing M_p with M_y (= $F_y S_x$)

Restraint at Root *	Restraint at Tip	Value of K for load at	
		Top Flange 	Other Part 
Top flange laterally supported  bottom flange laterally supported and section torsionally restrained about its longitudinal axis		2.5	1.0
		2.5	0.9
		1.2	0.7
Top flange laterally unsupported  bottom flange lateral-torsionally restrained about its longitudinal axis		7.5	3.0
		7.5	2.7
		3.6	2.1

Details illustrated within the shaded area are **Not Recommended** for Gerber-cantilever design. However, all values listed within this table are used in assessing the proposed design K-values as shown in Fig. A2.

* - section free to rotate about weak axis.
- design cases represent continuous girder in which length of the back span is longer than the cantilever length.
- Kirby-Nethercot diagram (Ref. 18) of restraint at root has been modified to better illustrate the structural restraint assumptions.

$$M_u = \frac{\pi}{K L_c}\sqrt{E I_y G J + \frac{\pi^2 E^2}{(K L_c)^2} I_y C_w}$$

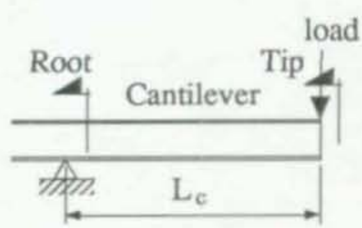
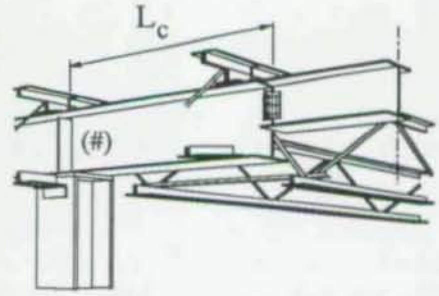
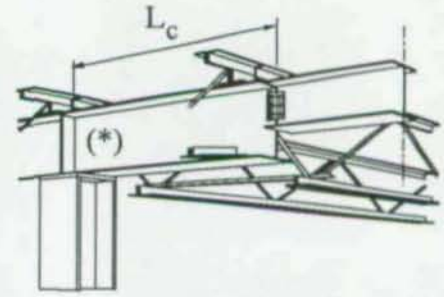
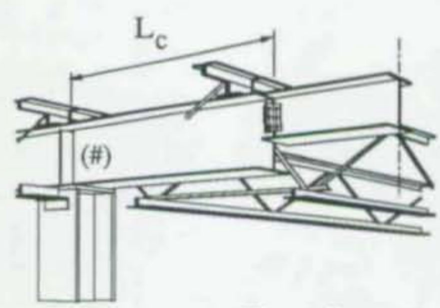
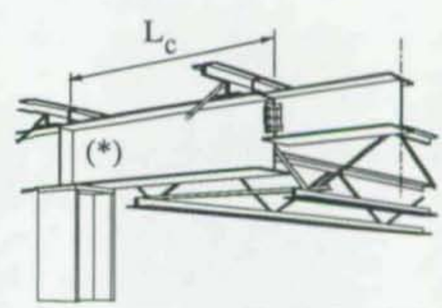
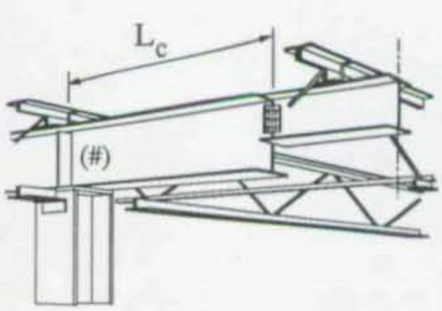
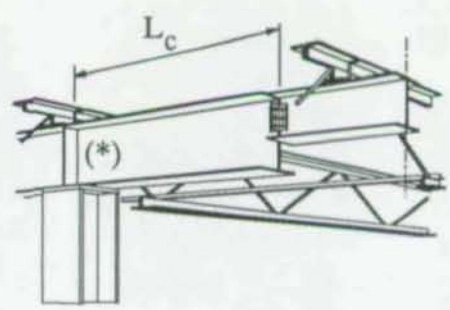
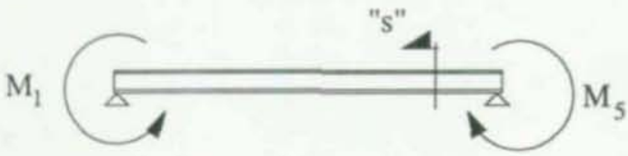


Fig. A1 Kirby-Nethercot Proposed Effective Length Factors (K)

	ROOT - lateral-torsionally supported Stability of girder-column assembly provided by joist bottom chord extension connection(s)	ROOT - lateral-torsionally supported Stability of girder-column assembly provided by column continuity design.
TIP - Lateral-torsionally supported	 K = 0.8 to 1.0	 K = 1.0 to 1.5
TIP - Laterally supported	 K = 1.0 to 1.5	 K = 1.5 to 2.0
TIP - Lack of support	 K = 1.5 to 2.5	 K = 2.0 to 3.0

Note: (#) Girder web stiffeners may be omitted, if web capacity is adequate.
(*) Full depth web stiffeners are used to create column continuity.

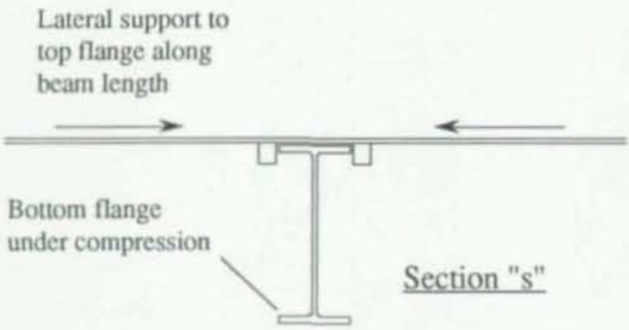
Fig. A2 Proposed Effective Length Factors (K) for Gerber-Cantilever Design



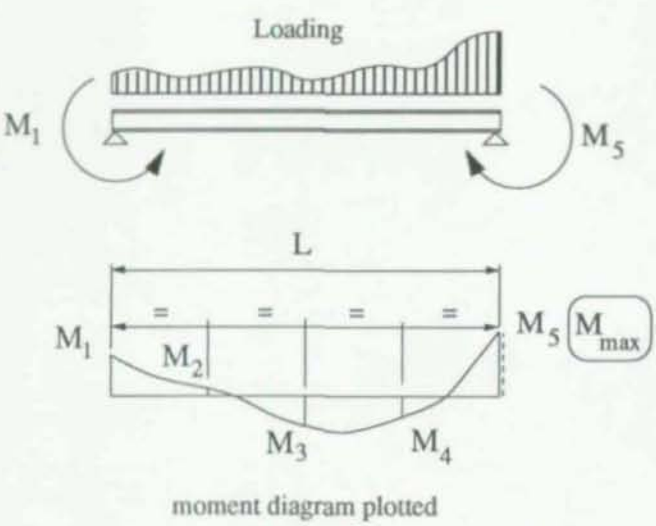
Roeder - Assadi Expression
Moment Resistance at Critical Elastic Buckling

$$M_u = \frac{1}{\omega d'}\left[G J + \frac{\pi^2 E I_y d'^2}{2 L^2} \right]$$

d' = depth of girder centre to centre of flange



Note:
- torsionally restrained about its longitudinal axis at vertical supports, but torsionally unrestrained between end supports
- section warping unrestrained
- top flange laterally supported
- bottom flange laterally unsupported



Kirby - Nethercot Expression
Equivalent Moment Factor

$$\omega = \frac{3M_2 + 4M_3 + 3M_4 + 2M_{max}}{12M_{max}}$$

Note:
All moment values are to be absolute values. For M_2 to M_4, only include moment values at locations where compression flange is laterally unrestrained, in other words, M = 0 where compression flange is laterally supported.

Fig. A3 Computing Equivalent Uniform Moment
(Cantilever Girder Between Supports)

Appendix "B" Cantilever Girder Cross-Section Stability Check

Svensson[29] proposes a method for evaluating flexural critical buckling stress for a class of beams for which the assumption of undistorted cross sections (as in Appendix "A") is not appropriate. Williams and Jemah[30] provides design curves which are more comprehensive covering many possible combinations of free, simply supported and built-in ends for a steel beam connected to a rigid floor slab. For Gerber-cantilever girder and OWSJ roof framing, a similar mode of girder failure may be described. The top flange of a Gerber girder is laterally and torsionally restrained by the connected joist members. The lower flange together with a portion of the web of the Gerber girder is prevented from lateral buckling by the bending stiffness of the web plate and the bending stiffness of the joist chords connected to the girder top flange (Fig. B1). Girder instability through loss of moment resistance by section distortion due to web bending should be design checked.

Design procedure proposed:

Using Engesser formula (Bleich[31]),

$P_{cr} = 2\sqrt{\kappa E I_{yf}}$ critical end-load for an infinitely long strut [B.1]

where, $I_{yf} = \left(\frac{b^3 t}{12}\right)$ = moment of inertia of compression flange about y-y axis

κ = spring constant contributed by joist chord and girder web bending

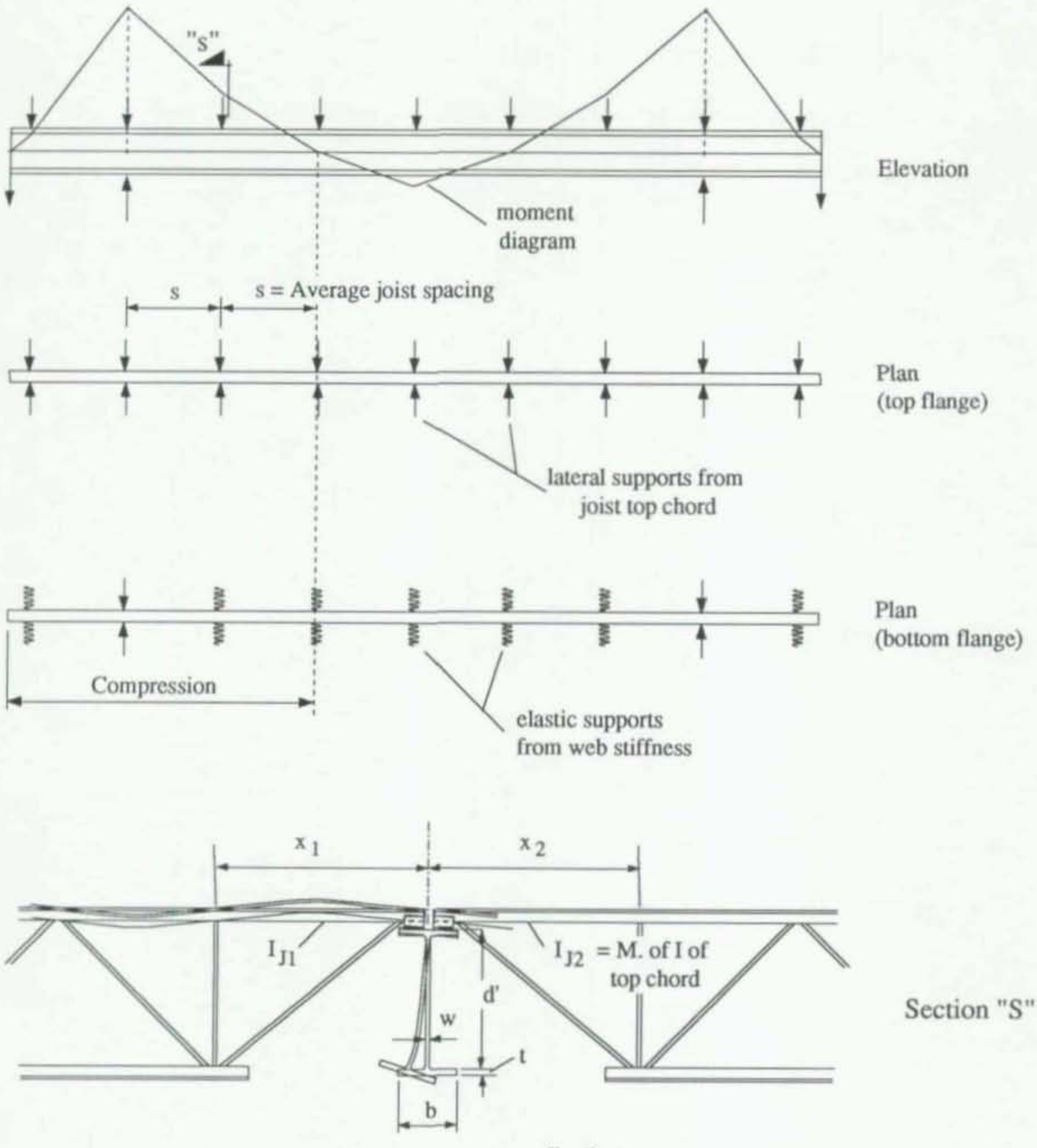
and, $$\kappa = \frac{E}{\frac{4\left(1-\upsilon^2\right)d'^3}{w^3} + \frac{d'^2 x_1 s_1}{3 I_J}}$$ .... if joist framing on one side

or, $$\kappa = \frac{E}{\frac{4\,(1-\upsilon^2)\,d'^3}{w^3} + \frac{d'^2 x_1 s_1}{12 I_{J1}} + \frac{d'^2 x_2 s_2}{12 I_{J2}}}$$ .... if joist framing on two sides

taking into account the flexural stiffness of girder web and of the end-panel joist top chords.

For symbols, see Fig. B1. Also see Example 1 provided.

Since a Gerber girder section is prismatic throughout its entire length, the induced axial flange stress should also vary proportionally with the bending moment diagram along the girder span (i.e. zero stress when moment is zero and maximum when moment is at maximum). A segment of girder bottom flange-web, loaded with zero compression at one end and a maximum compression at the other, may be considered less severely loaded than an end-loaded strut of similar length, cross section and restraint conditions, because an end loaded strut is subjected to uniform compression. To obtain effective design compression to simulate an equivalent end-loaded strut, as used with design expression [B.1], it is proposed that the actual compression, as obtained from the cross-sectional area of the bottom flange including about 15% of the girder web using the maximum support moment, be multiplied by 0.5.



s_1 and s_2 are spacings for side-1 and side-2 joists.

$d' = d - t$
= depth of girder, centre to centre of flange (mm)

Fig. B1 Cantilever Girder Stability Check for Slender Girders

Note: This design check need not be performed, if a W-shape or a WWF-shape is selected for the girder.

01227

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