DESIGN AND BEHAVIOR OF A REAL PR BUILDING

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ABSTRACT

The use of PR-Connections in steel buildings can result in very economical designs. In addition, because the fabrication details are not complicated and most welding is eliminated, PR buildings are fast and simple to erect. The writer's firm has designed several constructed buildings that utilize PR-Connections. The purpose of this paper is to present an analytical study of one such building. This study considers the design and behavior of the PR connections, columns, beams, and resulting frames. The study also considers the effect of connection modeling and connection shakedown on the final design.

INTRODUCTION

The writer's firm has been associated with the design and construction of a variety of PR buildings. The fabricators and erectors that have worked on these buildings find that they are fast and simple to erect in comparison to more traditional rigid frame (FR) buildings that typically require a substantial amount of field welding. In addition, in certain circumstances, using PR connections has reduced the overall steel weight for the building.

Despite these obvious advantages, there are very few if any other firms in the United States designing PR buildings. There are most likely a variety of reasons for this; however, in the writer's opinion there are three major reasons. First, the current literature does not provide clear guidance as to when or if PR buildings are more economical than FR buildings. This type of literature has to come from an authoritative body such as The American Institute of Steel Construction (AISC) or from designers and builders of real projects. Second, there is currently not a single authoritative guide to designing PR buildings. The writer's firm has had to put together a design procedure that is based on a stack of journal papers, research reports, and design guides. In addition, after all the literature has been reviewed, there are still a variety of gaps and problems with the design guidance. Third, there is a lack of appropriate computer software tools commercially available to the designer which incorporate the design guidance in the literature. In the writer's opinion, hand methods for the design of PR buildings are not and will not be used by practicing engineers. Reliable and well-documented design software must be available.

This paper presents an analytical study of a real PR building. In this study, specifics regarding the design of the connections, beams, and columns are presented. In addition, two specific design considerations are examined. The first consideration is what impact the analytical representation of the moment-rotation curve has on the building design. The second consideration is how connection shakedown, resulting from transient loads, influences the ability to reduce beam sizes in PR moment frames compared to beam sizes determined by assuming simple supports.

STUDY BUILDING

The study building is a four-story office building in Louisville, Kentucky. The floor system is constructed of four-inches of normal weight concrete on 9/16-inch permanent form deck. The deck sits on steel bar joists that rest on wide-flange steel girders. The building cladding consists of pre-cast concrete panels. The lateral system consists of PR frames in the North-South direction and FR frames in the East-West direction. A schematic of the typical floor plan for the building is presented in Fig. 1.

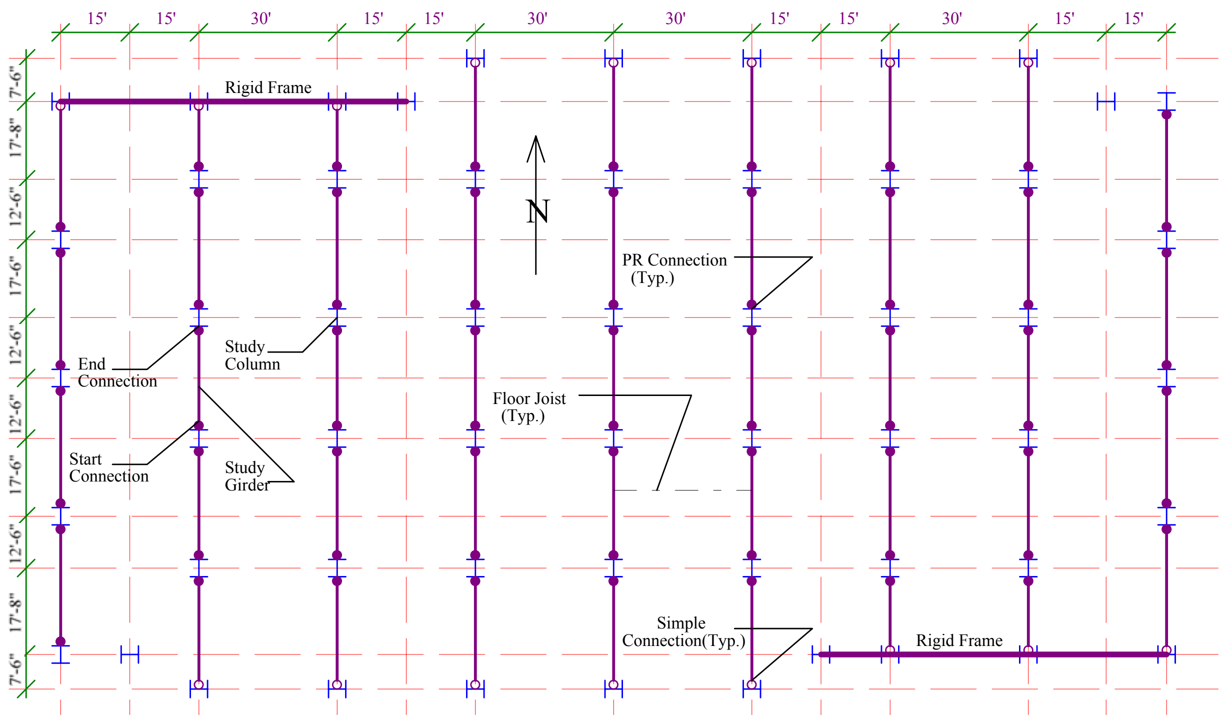

Figure 1 – Study Building Level 2 Floor Plan

BUILDING LOADS

Typical floor loads included a dead load (DL) of 56 psf, a superimposed dead load (SD) of 30 psf and a reducible live load (LL) of 80 psf. Floor loading within the core of the building included a dead load of 56 psf, a superimposed dead load of 10 psf and a non-reducible live load (NRL) of 125 psf. Typical roof loading included a dead load of 20 psf, a superimposed dead load of 10 psf, and a non-reducible live load of 20 psf. Within the mechanical penthouse (located on the roof) a dead load of 113 psf, a superimposed dead load of 10 psf, and a non-reducible live load of 50 psf were used. A superimposed dead load of approximately 720 plf was assumed for the pre-cast panels.

The wind (WL) and earthquake (EQ) lateral loads are summarized in Table 1 below. A basic wind speed of 70 miles per hour (mph) and an A_v of 0.07 and an A_a of 0.05 were used to calculate the lateral loads. In the North-South (N-S) direction the wind forces exceed the EQ forces and control the design of the PR-frames. In the East-West (E-W) direction the EQ forces exceed the wind forces and control the design of the rigid-frames. This combination of wind forces controlling the design in one direction while EQ forces control the design in the opposite direction is typical for the design of many office buildings along the east coast of the USA.

Table 1 – Summary of Building Lateral Loads

Level	Wind E-W (Kips)	Wind N-S (Kips)	EQ E-W (Kips)	EQ N-S (Kips)
Roof	32	81	60	60
Floor 4	24	47	64	64
Floor 3	21	43	35	35
Floor 2	20	40	17	17
Base Shear	97	211	176	176

BUILDING ANALYSIS

A three-dimensional model of the entire building including all PR frames, FR frames, and leaner columns was used. Gravity loads were applied to frame beams with appropriate live load reductions. Wind loads were applied as point loads to master nodes at each floor. The master nodes were located at the center of wind force and a rigid diaphragm was assumed at each floor to distribute the wind load. All the analysis was conducting using an in-house program.

A Stage I analysis of the building was conducted first. This was a first-order (no P-Δ or P-δ) non-linear connection, path independent analysis of the building considering only gravity loads. Non-linear connection analysis simply means that the full non-linear connection moment-rotation behavior was considered in the analysis. This is done by using a secant stiffness that is based on the current moment and rotation at the connection which follows the moment-rotation behavior input for the connection. Path independent means that the connection is assumed to load and unload along the non-linear connection curve. Consequently, the sequence of loading does not influence the result. The purpose of this analysis is to determine the column loads that result from the applied beam loads. The resulting column loads are then corrected to reflect the fact that not all of the floor members that frame into the columns were present in the model and that there is a difference in live load reductions for beams and columns. This is done using a method similar to that described by Ziemian (1).

A Stage II analysis was then conducted. This analysis was a second-order (with P-Δ and P-δ), non-linear connection, path independent analysis. The second-order behavior is incorporated into the analysis by use of a stability function stiffness matrix for the column elements, which is described in Chapter 8 of Beaufait et al. (2). In addition, load combinations are prescribed such that a single analysis is done for each load combination rather than superimposing the analysis results from the primary loads (DL, SD, LL, NRL, WL) that make up the load combination. This is the typical analysis conducted by the writer's firm for most PR building designs.

A Stage III analysis was then conducted. This analysis is the same as the Stage II analysis with the exception that the connections are represented with a fixed linear stiffness rather than the full non-linear moment-rotation behavior. The connection stiffness assumed was the secant stiffness associated with 0.0025 radians as recommended in ASCE (3).

A Stage IV analysis was the last analysis conducted in the study. This analysis was a second-order (with P-Δ and P-δ), non-linear connection, path dependent analysis. The path dependence is incorporated into the analysis by assuming a connection behavior with more realistic loading and unloading assumptions. This behavior is shown graphically in Fig. 2 and is described in more detail later. In this analysis, a number of load cases are considered. Each load case is made up of a series of load steps. Each load step is one of the primary loads multiplied by a load factor. The load cases considered in this analysis are presented in Table 2 below. The 0.4 live load in combination with wind is based on Ellingwood (4). Because the PR frames are of interest in this study the WL in the load cases is the WL in the N-S direction. The Stage IV analysis is considered to be the most exact analysis; however, it should be noted that it is impractical to use on a daily design basis.

Table 2 – Load Cases For Stage IV Analysis

Analysis Step	Case S1	Case S2	Case S3	Case S4	Case S5	Case S6	Case S7
Step 1	+1.0 DL	+1.0 DL	+1.0 DL	+1.0 DL	+1.0 DL	+1.0 DL	+1.0 DL
Step 2		+1.0 SD	+1.0 SD	+1.0 SD	+1.0 SD	+1.0 SD	+1.0 SD
Step 3			+1.0 LL	+1.0 LL	+1.0 LL	+1.0 LL	+1.0 WL
Step 4			+1.0 NRL	+1.0 NRL	+1.0 NRL	+1.0 NRL	-2.0 WL
Step 5				-1.0 LL	-0.6 LL	-0.6 LL	+2.0 WL
Step 6				-1.0 NRL	-0.6 NRL	-0.6 NRL	-1.0 WL
Step 7					+1.0 WL	+1.0 WL	+1.0 LL
Step 8					-2.0 WL	-2.0 WL	+1.0 NRL
Step 9					+2.0 WL	+2.0 WL	
Step 10						-1.0 WL	
Step 11						+0.6 LL	
Step 12						+0.6 NRL	

CONNECTIONS

Steel PR connections were used. Composite connections could not be used because of the bar joist floor framing. The steel connections were top and bottom seat-angle with web angle connections. The typical connection detail used on the project is shown in Fig 2. It should be noted that only one connection type was used on the entire job. The connections angle sizes were not adjusted to try to "tune" the building as has been suggested in past literature. Such variations in angle sizes throughout the job results in increased complexity and installation problems.

The moment-rotation behavior for the connection was determined using Eq 1 below and is shown graphically in Fig 2. M_{cu} is the ultimate moment capacity of the connection and θ_0 is a reference rotation take as M_{cu} / K_{ci} where K_{ci} is the initial stiffness of the connection. The parameter n is a shape parameter. A method for calculating values for each of these variables is given in Mayangarum (5). M_c and θ_c are the connection moment and rotation respectively.

$$\frac{M_c}{M_{cu}} = \frac{\dfrac{\theta_c}{\theta_0}}{\left[1+\left(\dfrac{\theta_c}{\theta_0}\right)^n\right]^{\frac{1}{n}}} \qquad \text{(Eq 1)}$$

In the Stage I and II analysis, the connection is assumed to load and unload along the non-linear connection curve shown in Fig. 2. In the Stage III analysis the connection is assumed to load and unload along the secant stiffness shown in Fig. 2. In the Stage IV analysis the connection is assumed to initially load along the non-linear connection curve; however, subsequent unloading and reloading is assumed to occur along a line with a slope similar to the initial connection stiffness.

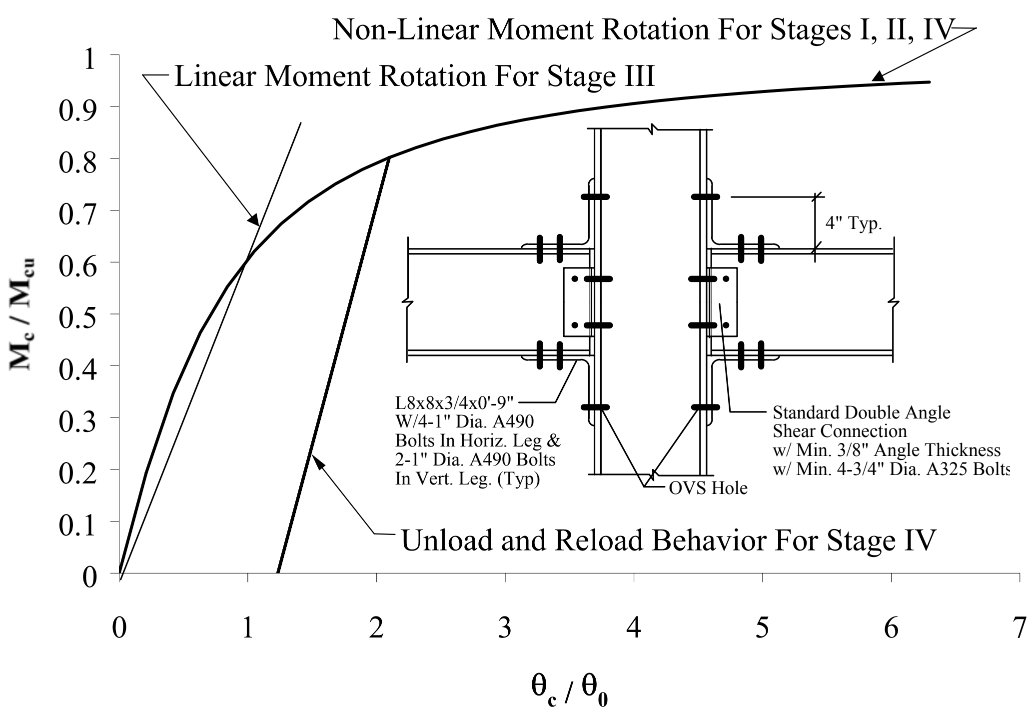

Figure 2 – Typical PR Connection

In the event of a moment reversal, the connection behavior was assumed to be anti-symmetric about the abscissa except that the origin of the non-linear behavior was shifted to coincide with the rotation at the point of moment reversal. This is essentially the hysteretic behavior recommended by Sourochnikoff (6); however, the writers recognize that this is most likely not an appropriate hysteretic behavior for the top and bottom angle connections used. Despite this understanding, the hysteretic behavior was chosen because the study results showed that the connections did not go through a moment reversal. Consequently, the first quadrant of the hysteretic behavior was believed to be the most important and is believed to be valid for this connection type. However, it should be noted that future PR connection research should include cycles in the positive rotation quadrants to better define this behavior.

The typical beam sizes on the job were W21 and W24 beams. Values of the connection and beam parameters are shown in Table 3 below. The quantity M_{cu}/M_p is the ratio of connection strength over the plastic moment strength of the bare steel beam. This range of connection to beam strengths is typical for the PR buildings designed by the writer's firm. This ratio is notably less than the 0.75 ratio recommended by ASCE (3).

Table 3 – Summary of Connection and Typical Beam Properties

Beam	M_p (K-in)	M_{cu} (K-in)	M_{cu}/M_p	K_{ci} (K-in/rad)	n
W21x44	4296	2127	0.50	777,461	1.05
W21x68	7200	2127	0.30	777,461	1.05
W24x68	7968	2417	0.30	1,005,000	0.97

Summary plots of the connection moment-rotation behavior for the start and end connections on the study girder (shown in Fig. 1) are presented in Fig. 3 and 4. As can be seen in Fig. 3 and 4, the connections do not go through a moment reversal for Load Cases S4 and S6. In both of these load cases the beams are fully loaded and then some portion of the live load is removed. Subsequent gravity loading and lateral loading result in the connections simply following up and down the linear unloading curve. This occurs for two reasons. First, there is no moment reversal. Second, the connection moments generated by the wind loading are less than the live load removed and are less than the connection moment that remains when the live load is removed.

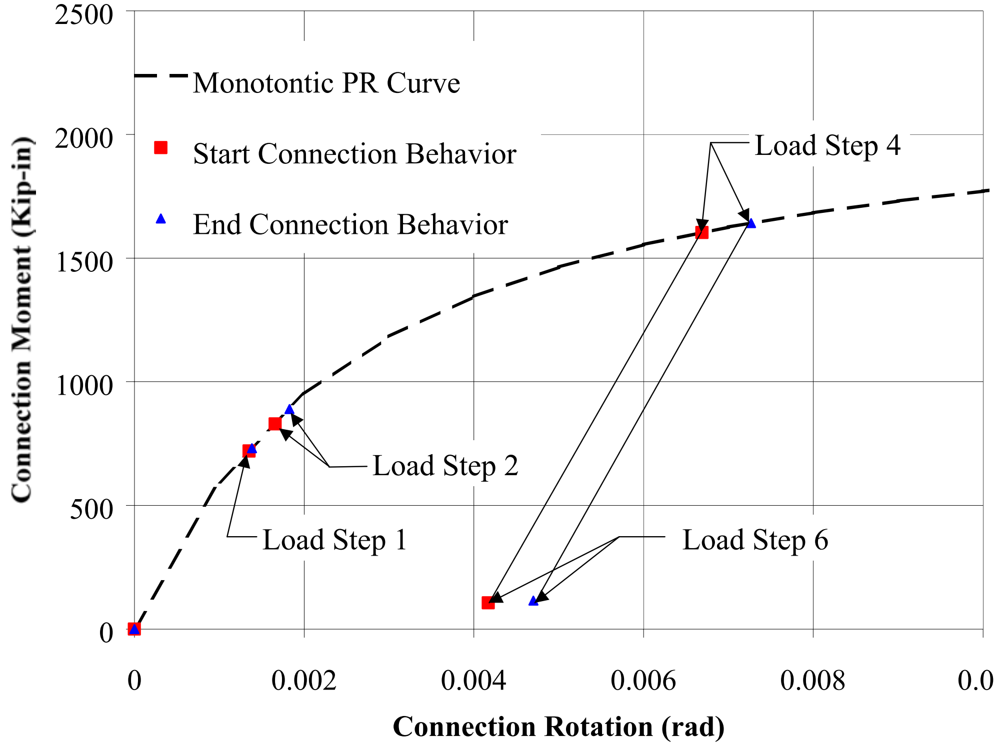

Figure 3 – PR Connection Behavior For Load Case S4

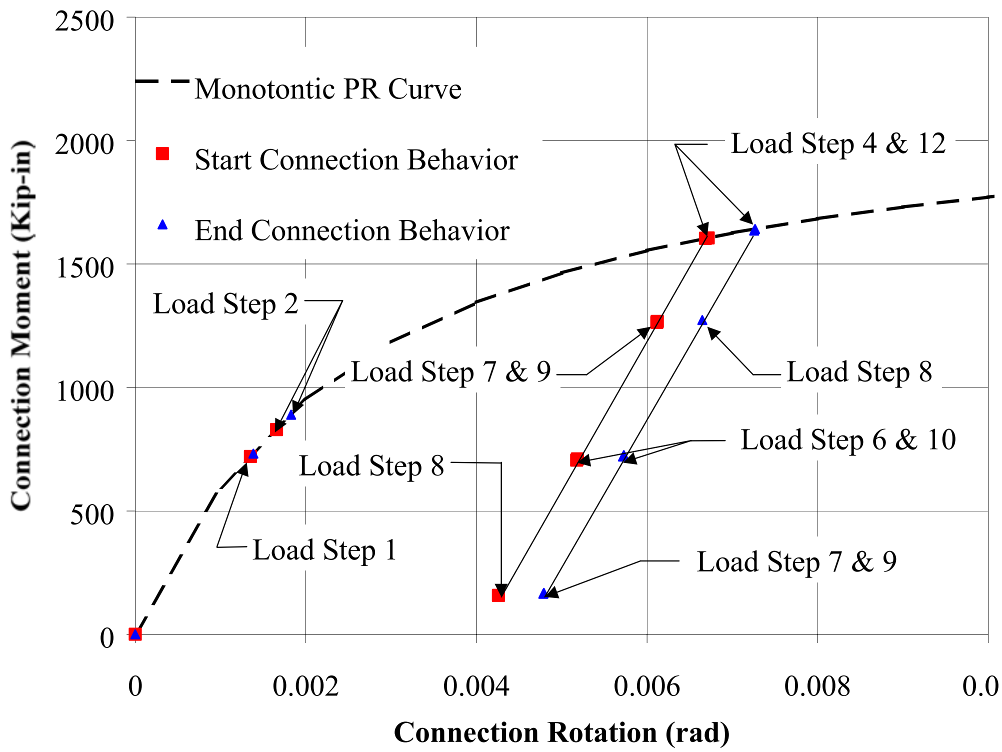

Figure 4 – PR Connection Behavior For Load Case S6

BUILDING BEHAVIOR

The period and story drifts are the primary building characteristics considered in design. A modal analysis was used to determine the frame period. Because this is an elastic analysis, a constant connection stiffness must be used (i.e. not the non-linear connection behavior). In design the frame period is typically used to determine seismic forces. The lower the period the higher the seismic forces. Consequently, the initial stiffness of the connection was used in the modal analysis. This results in the lowest frame period and conservative seismic forces. The building period in the N-S direction (PR frame direction) was calculated as 1.83 seconds. The building period in the E-W direction (FR frame direction) was calculated as 2.55 seconds.

As discussed above, the wind lateral forces in the direction of the PR frames were higher than the seismic lateral forces. Consequently, the drift resulting from wind loads is the only drift of interest in this design. The study frame drifts for the Stage II, III, and IV analysis are summarized in Table 4.

Table 4 – Summary of Frame Drift Resulting From Wind (in)

Level	Roof	4	3	2
Level Height (H) (Ft)	58.1	43.7	30.1	16.6
H/400 (in)	1.74	1.31	0.90	0.50
Stage II Analysis				
S5 +/- WL	2.29	1.94	1.33	0.63
Stage III Analysis				
S5 +/- WL	1.38	1.15	0.81	0.43
Stage IV Analysis				
S5 +WL	1.25	1.05	0.75	0.40
S5 -WL	-1.25	-1.05	-0.75	-0.40
S6 +WL	1.25	1.05	0.75	0.40
S6 -WL	-1.25	-1.05	-0.75	-0.40
S7 +WL	1.53/1.32	1.29/1.10	0.9/0.77	0.46/0.41
S7 -WL	-1.12	-0.93	-0.65	-0.37

Consider the results of the Stage IV Analysis first. For Load Case S5 and S6, the full live load was applied and then 0.6 of the live load was removed before wind loads were applied. As shown in Fig.4, this type of loading resulted in the connections behaving elastically for subsequent wind loading and unloading. Consequently, the building drifts are the same in each direction and for both of the load combinations. In Load Case 7 the connections were not unloaded prior to the wind loading occurring. Consequently, when wind load was applied in the positive direction (South), the start connection loaded along the non-linear connection curve while the end connection unloaded along the elastic unloading curve. This is shown in Fig. 5. In the subsequent negative loading (North), the start connection re-loaded along the elastic curve until it hit the original non-linear curve and then started loading along the non-linear curve. The end connection simply unloaded along the elastic unloading curve. After this point, the connections follow the elastic loading and unloading curve during any further wind loading in either direction. The result of this loading sequence is that the building develops a permanent set in the direction of the first applied wind loading and that after one full load reversal the wind deflections in the direction of the first loading will reduce. This is shown for the S7+WL entry in Table 4. The first value is the building drift resulting from the first wind load. The second value is the building drift that results from any subsequent wind load after a full wind load reversal. Because the connections behave elastically after the first full wind loading in the negative direction, the story drift in the negative direction does not exhibit this reduction in drift. The permanent set in the building is one-half the difference between the final drift in the positive direction and the drift in the negative direction. For Load Case 7 the permanent set is one-half of 1.32 inches – 1.12 inches, or 0.1 inches.

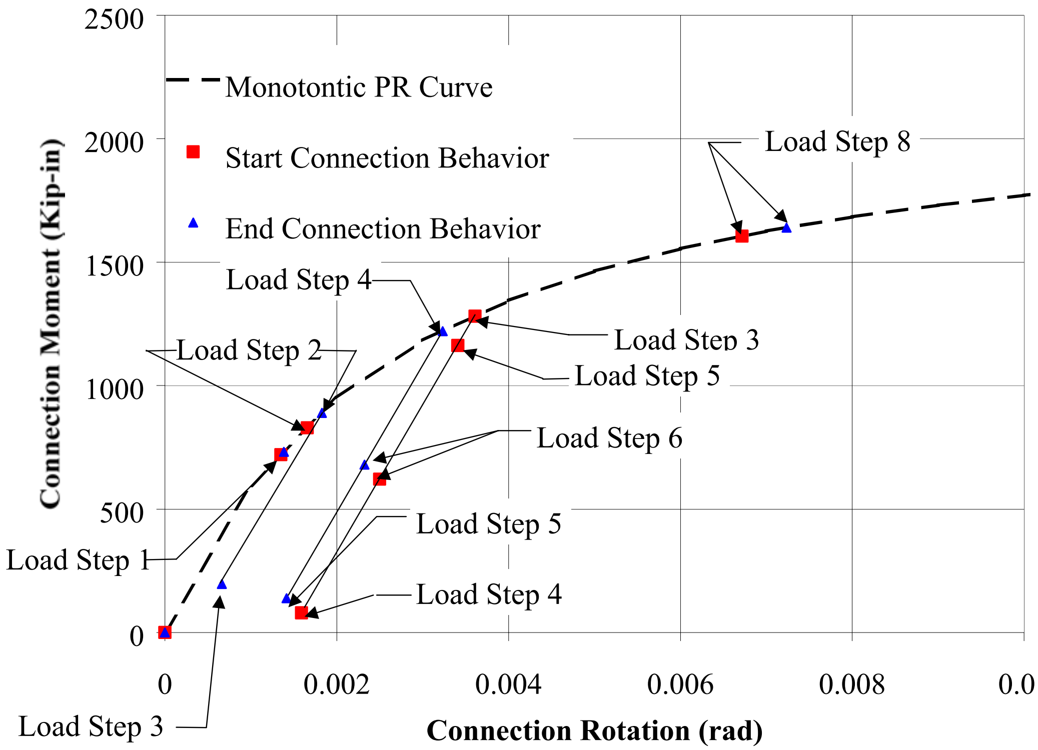

Figure 5 – PR Connection Behavior For Load Case S7

Next, consider the differences between the Stage II, III, and IV Analysis results. Because of the path independent nature of the Stage II and III Analysis, the lateral drift results are the same for Load Cases 5, 6 and 7. Consequently, the results for Load Case 5 are the only results presented. Review of Table 4 shows that the Stage II analysis provides a very conservative estimate of building drift compared to the Stage IV analysis. The Stage III analysis provides a much better estimate of the building drifts. The reason for the large discrepancy between the Stage II and Stage IV analysis can be attributed to the resulting elastic behavior the connections attain after an unloading cycle. The unloading can occur during removal of live load and / or wind load cycles. The resulting elastic behavior means that less connection rotation is required to develop the connection moments required to resist the wind forces. Less rotation results in less building drift.

COLUMN BEHAVIOR

The study column location is shown in Fig. 1. A gravity analysis for each column was done including live load reductions. The ultimate axial column load (P_u) was determined to be 651 Kips. A W12X72 was selected as a preliminary column size. This column has an axial load strength of 655 Kips assuming an effective length factor of 1.0. A Stage II building analysis was done using the standard ultimate strength load combinations prescribed by the building code. The combination of gravity plus lateral ended up controlling the design with design forces of P_u = 524 Kips and M_u = 171 Kip-ft. A W12X87 column size was required for these design forces. The effective length factor was calculated as 1.65 using the method outlined by Driscoll (7).

BEAM BEHAVIOR

The study beam location is shown in Fig.1. The loads on this beam were 1.686 Kips/ft DL, 0.301 Kips/ft SD, and 3.763 Kips/ft NRL. Beam depth within the floor plate was limited to 21-inches to minimize the floor-to-floor height. A preliminary design based on a beam with simple connections resulted in a W21x83 preliminary size. The preliminary design was controlled by the moment strength limit state where M_u was approximately 7956 Kip-inches. After the building was analyzed and some design iterations were completed, a final beam size of a W21X68 was chosen. This reduction in beam size resulted in roughly a 7% reduction in steel weight for the typical floor.

Table 5 – Summary of Study Beam Deflections and Moments

Load Case	Deflections (in)			Positive Moments (Kip-in)		
	Stage II	Stage III	Stage IV	Stage II	Stage III	Stage IV
S1	0.15	0.17	0.15	854	908	854
S2	0.18	0.19	0.18	1003	1031	1003
S3	0.75	0.61	0.75	3767	3212	3767
S4	0.18	0.19	0.37	1003	1031	1752
S5	0.43	0.36	0.53	2163	1904	2557
S6	0.75	0.61	0.75	3767	3212	3767
S7	0.75	0.61	0.75	3767	3212	3767

The deflections and moments from the Stage II, III and IV analysis are presented in Table 5 below. First consider the results of the Stage IV analysis. The S1 deflection is the DL deflection. The S2 deflection is the combined DL and SD deflection. Consequently, the incremental deflection associated with adding the SD is the S2 deflection minus the S1 deflection. Similarly, the incremental deflection associated with the live load deflection is the S3 deflection minus the S2 deflection. For the study beam the incremental live load deflection is 0.57 in. When the live load is removed, as in Load Case 4, the connection behaves elastically and the resulting deflection is larger than the S2 deflection because of the plastic deformation that has taken place in the connection. From the design standpoint, this simply means that a

Stage II Analysis will provide accurate live load deflection estimates assuming the connection does not degrade because of moment reversals.

Now compare beam deflections considering Stage II, III, and IV results. The Stage II and IV results are identical except for Load Case S4 (the reasoning for this was discussed above) and for Load Case S5. The deflection for Load Case S5 is larger in the Stage IV analysis than in the Stage II analysis for the same reasons it is larger in Load Case S4. The plastic connection deformation caused by the full gravity loading and then unloading of a portion of the live load resulted in larger connection rotations than in the Stage II analysis. Again, from a design standpoint this means that as long as the combined wind, dead, and reduced live load moments don't exceed the dead and full live load moments then the full live load deflection determined from a Stage II analysis will be correct and can be used for design.

The Stage III results show that as the load is increased the calculated deflections go from conservative to un-conservative estimates. This is an obvious observation any time a constant stiffness is assumed for the connection. The ability of a Stage III analysis to predict beam deflections is very sensitive to the assumed stiffness. Because of this, a Stage III analysis is not recommended for beam deflection estimates. Review of the beam moments presented in Table 5 show the same type of relationships seen for beam deflections.

CONCLUSIONS

This paper has presented the design and analytical study of a real PR building. The following conclusions were determined for the building considered in this study:

1. Connection behavior stayed in the positive moment and positive rotation region of the moment rotation curve. Consequently, connection shakedown had no real effect on the design. Future research on PR connections should include cyclic testing of the connections in the positive rotation region to better define the connection behavior in this region.
2. Initial beam and column sizes can be estimated by assuming simply supported beams and braced columns.
3. A Stage II analysis results in overly conservative building drift estimates; while, a Stage III analysis provided much better drift estimates.
4. If wind, dead, and reduced live load connection moment does not exceed the dead and full live load connection moment then a Stage II analysis can be used to accurately predict the full live load beam deflections.
5. A Stage III analysis should not be used to determine beam deflections because of how sensitive the deflections are to the stiffness assumed in the analysis.

These conclusions are based on the design and analysis of a single building. The validity of these conclusions when considering other buildings will depend on a variety of factors including but not limited to the building geometry, the gravity and lateral loads and the number of PR frames.

REFERENCES

(1) Ziemian, R.D., and McGuire, W. (1992), “A Method for Incorporating Live Load Reduction Provisions in Frame Analysis,” *Engineering Journal* 29(1):1-3, American Institute of Steel Construction

(2) Beaufait, F. W., Rowan, W. H. Jr., Hoadley, P.G., Hackett, R. M (1975), Computer Methods of Structural Analysis, Third Edition, Prentice Hall.

(3) ASCE Task Committee on Design Criteria for Composite Structures in Steel and Concrete (1998) “Design Guide For Partially Restrained Composite Connections” *Journal of Structural Engineering*, 124(10): 1099-1113, American Society of Civil Engineers

(4) Ellingwood, B. (1989) “Serviceability Guidelines for Steel Structures,” *Engineering Journal* 26(1):1-8, American Institute of Steel Construction

(5) Mayangarum, A. (1996) “Design, Analysis, and Application of Bolted Semi-Rigid connections For Moment Resisting Frames,” MS Thesis, Lehigh University, Bethlehem, PA.

(6) Sourochnikoff, B. (1949) “Wind Stresses in Semi-Rigid Connections of Steel Framework,” Transactions, ASCE, Paper No. 2402, p. 382

(7) Driscoll, G.C. (1976), “Effective Lengths of Columns with Semi-Rigid Connections,” *Engineering Journal* 13(4):109-115, American Institute of Steel Construction