SINGLE SPAN RIGID FRAMES IN STEEL

AMERICAN INSTITUTE OF STEEL CONSTRUCTION, Inc.

FIRST EDITION

SINGLE SPAN
RIGID FRAMES IN STEEL

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AMERICAN INSTITUTE OF STEEL CONSTRUCTION, Inc.

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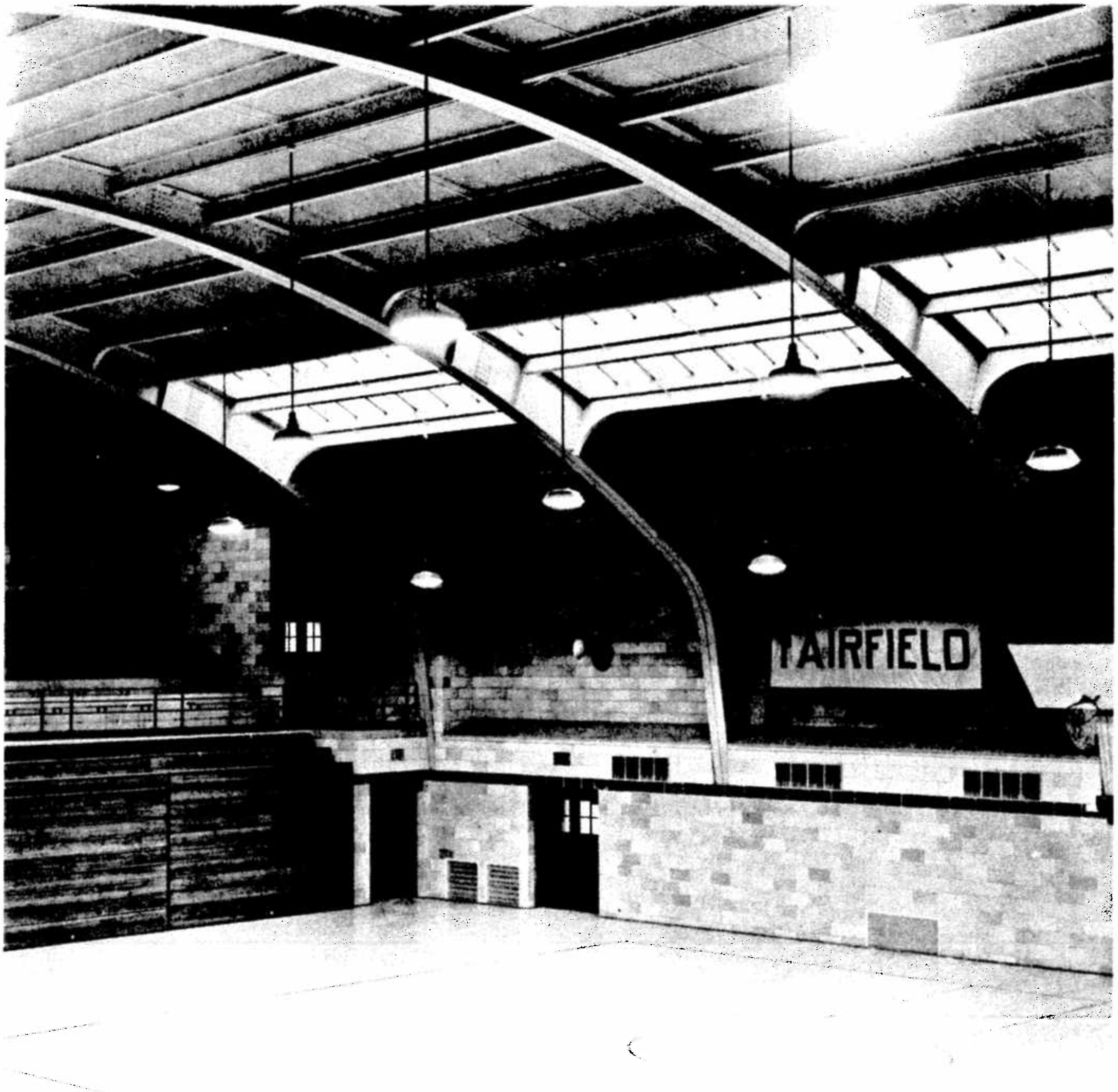


Figure 1

Gymnasium, Muscatine, Iowa, High School
Architects & Engineers: Keffer & Jones, Des Moines, Iowa

INTRODUCTION

The single span steel rigid frame has recently achieved wide popularity. Perhaps the most important factor contributing to this development has been the attractive appearance of these frames. Appearance alone, however, is but one of the advantages of this type of construction. It is equally adaptable to riveted or welded fabrication; it may be constructed of rolled beams, built-up members, or any combination thereof. These frames may be efficiently constructed for flat, gabled or curved roofs, and when so built of steel the economy and speed of erection is not surpassed with any other type of material or form of construction.

For identical loading it is possible that a greater weight of steel may be required for a rigid frame than for a truss-and-column design of similar span length. However, when consideration is given to the over-all cost of the framing, the ease and simplicity of its erection, increased clear headroom of the finished structure and saving in wall heights, it will be found that, even on a first cost basis, the rigid frame is highly competitive.

Rigid frames composed of rolled sections are commonly used for spans up to 100 feet in length; indeed, spans up to 197 feet have been built using rolled wide-flange beams. Built-up members have been utilized on spans of 250 feet. Welded fabrication offers particular advantages with short spans; with variable depth members; and on parabolic shape roofs.

The rigid frame does introduce certain problems in design which differ from those of the more common statically determinate structure. This fact often causes concern to the busy architect and engineer who have only limited time that may be allotted to the completion of any one set of plans. It is for this reason that the following material is presented as a guide to a simplified and practical method of design.

The distribution of moments in the statically indeterminate rigid frame is effected by its general shape (i.e., by the relationship of column-height-to-span and roof-rise-to-column-height) as well as by the relative stiffness of the several members. Since the general outline establishing the relationship of height and rise to span will be fixed for any given problem by architectural considerations, the only remaining variable influencing the distribution of moment will be the relative stiffness of the members themselves as finally proportioned. Within a wide range of cases it will be found that this variable has only a minor influence upon the moment distribution. Because of this fact, which will be discussed in greater detail on page 9, first approximations are generally possible, after a little experience, which will yield results sufficiently close as not to require later modification.

GENERAL CONSIDERATIONS

The following general considerations should be kept in mind during the early planning stage:

1. Spacing of Frames

The following suggested spacing of frames for various spans will provide good economy for average roof loads. Occasionally one of the many factors which affect the choice of spacing may be of unusual importance and, in such cases, it should be realized that some variation may be made in this suggested spacing with little effect on the total cost of the structure.

Span	*Spacing*
30′ to 40′	16′
40′ to 60′	18′
60′ to 100′	20′
Over 100′	1/5 to 1/6 of span.

2. Bracing at Knees

Tests indicate that adequate provisions should be made to prevent lateral movement of the compression (inner) flange of the knee—at or near the mid-point of the arc in the case of a curved inner flange; at their intersection in the case of straight column and girder flanges. Even though the intensity of stress in the compression flange may decrease rapidly a short distance from the knee, at least one additional point of lateral support on the girder in this vicinity is generally desirable, located so that ld/bt will not exceed 600 where l is measured along the inner (compression) flange and d is the greatest girder depth when this dimension is variable. Bracing to provide lateral support, in the form of haunched purlins, is illustrated in Fig. 1. In Fig. 2 the compression flange of the frame is braced to a line of purlins near the knee, and a sway frame is employed normal to, and located at, the deepest portion of the knee.

3. Wind Bracing

Each frame should be designed to resist by itself the portion of the transverse wind load to which it may be subjected. The deep connections required to brace the inner flange (see Item 2) sometimes provide a satisfactory portal system whereby it is possible to dispense entirely with diagonal bracing in the vertical longitudinal plane.

4. Column Bases

It is often impossible and nearly always uneconomical to provide footings which will completely "fix" the column bases against rotation. Therefore rigid frames are generally designed as if hinged at their column bases. Expensive pin-connected base details, however, are rarely required; in most cases an ordinary flat-ended base detail, with a single line of anchor bolts, placed perpendicular to the span at the column centerline, will suffice. If the anchor bolts are passed through lug angles attached to the column web, enough flexibility will usually be provided to allow

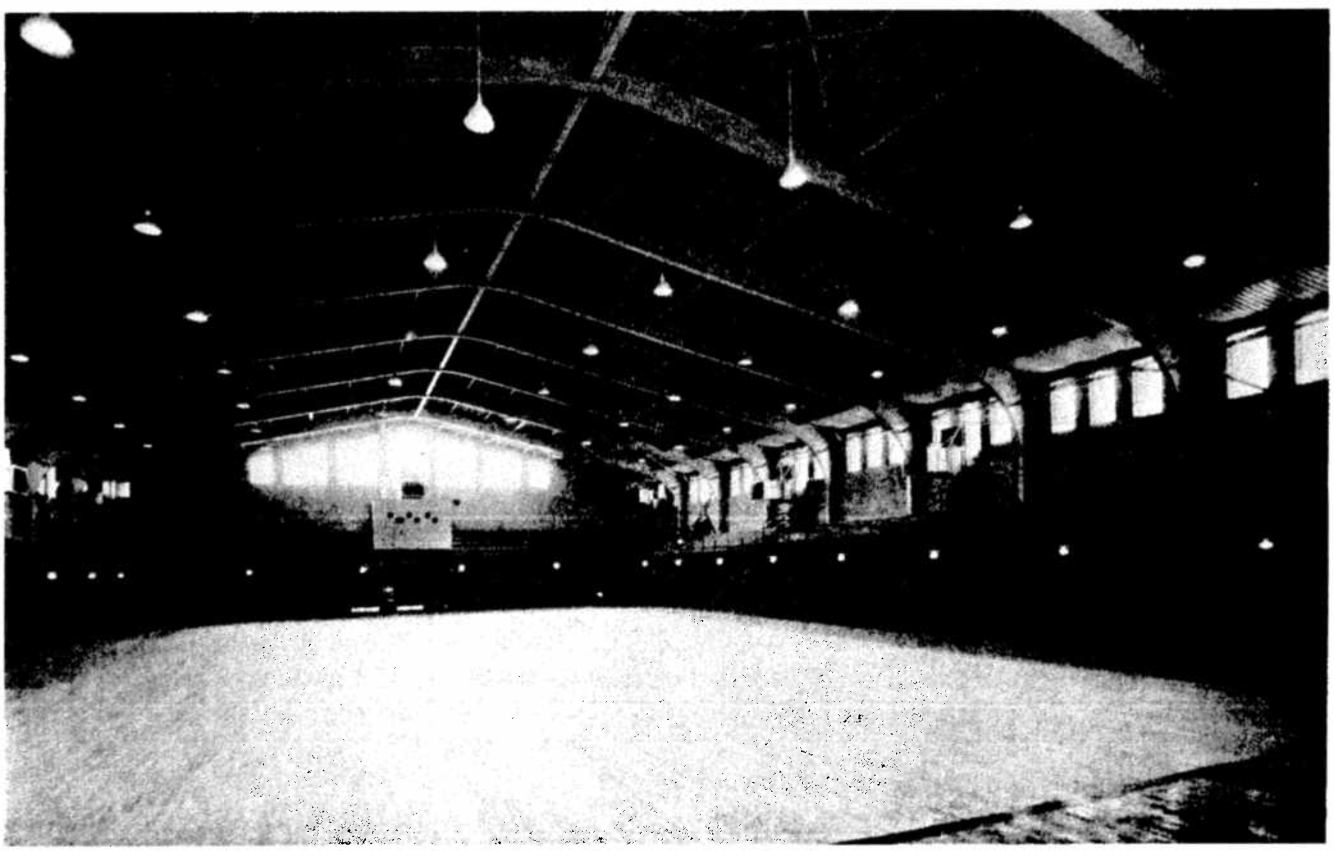


Figure 2

National Guard Armory, Rockford, Illinois
Architects & Engineers: Bradley & Bradley, Rockford, Illinois

for the very slight change in slope at the foot of the column associated with its rotation under pin-ended, rigid frame behavior. The maximum moment introduced by such flat-endedness would be limited at most to the product of the vertical reaction multiplied by one half the depth of the column section at the flat-ended base. Because of the adjustment in horizontal reaction effected by this "eccentricity" moment, any adjustments in the total moments at the knee and girder midspan would be but a small fraction of this "eccentricity" moment and may be neglected. In any case, positive provision should be made to transmit the horizontal shear, or thrust, from the column to its foundation, unless the columns themselves are tied together as discussed under Item 5.

5. Provisions for Ties

In order to reduce the foundation costs, which might become excessive if called upon to resist the frame thrusts, it is often desirable to provide a horizontal tie between the column bases of the frame. However, where foundation conditions are favorable such ties are sometimes omitted. Ties may be in the form of round bars, plates, or structural shapes; placed below the floor line and protected from contact with the earth.

6. Field Splices

The number and location of field splices must be largely determined by the size of the structure, weight of members and the facilities available for shipment and erection. Relatively small gable frames are often delivered in two sections and erected with a single splice at the peak of the gable. For larger frames the usual practice is to provide a field splice near each point of contraflexure in the girder, with additional splices as may be required by shipping restrictions.

7. Minor Stresses

Deformations due to shear, direct stress, temperature and tie-rod elongation are normally neglected in rigid frame analysis. This practice is usually entirely justifiable since, for the average frame, the stresses due to all of the foregoing rarely amount to more than 2 or 3 per cent of the total. (See page 23.)

DETERMINATION OF REACTIONS

The distinguishing characteristic of rigid frame design is the recognition and evaluation of stresses induced by the deformation of the frame when service loads are applied to it. Fundamental to the evaluation of these induced stresses is the requirement that all joints in the frame be truly rigid and that the maximum stress intensity along the component members be less than the elastic limit of the material. In the case of the single-span, single-story rigid frame with free ends ("hinged" column bases) the actual bending moment at any point along the frame may be readily computed in accordance with the ordinary principles of statics ($\Sigma M = 0$), once the horizontal thrusts at the column bases have been determined.

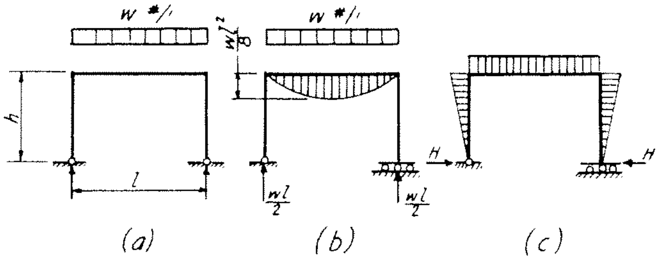


Figure 3

It is convenient to consider a rigid frame, such as shown in Fig. 3a, as a combination of the two statically determinate frames, Fig. 3b and 3c. The stress at any point in the given frame (Fig. 3a) is the algebraic sum of the stresses at the corresponding point in the other two frames. For example, the stress at the center of the girder (Fig. 3a) is $wl^2/8 - Hh$.

The charts on pages 14 to 17 provide a quick method for obtaining the horizontal reactions for a wide range of column height, roof rise, span and loading conditions. A sufficient number of curves have been provided to permit interpolation within the degree of accuracy required for design purposes. The following illustrative examples are indicative of the use of these charts.

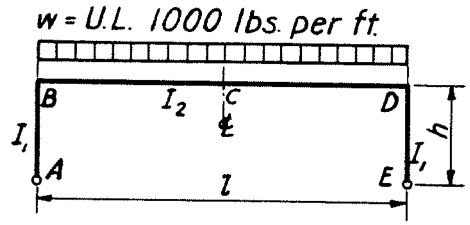
TABLE I

EFFECT OF RATIO $\frac{I_2}{I_1}$ ON MOMENT DISTRIBUTION

UNIFORM VERTICAL ROOF LOAD

RECTANGULAR FRAMES

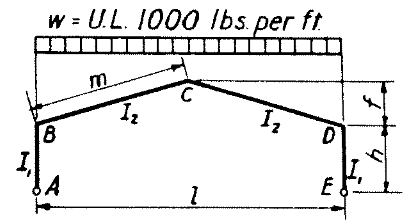
In the case of rectangular frames $m = \frac{1}{2} l$.



$\frac{h}{l}$	$Q = \frac{f}{h}$	$\frac{I_2}{I_1}$	l Feet	f Feet	h Feet	$K = \frac{I_2}{I_1}\frac{h}{m}$	Haunch Moment (Ft. Kips) M_D	Haunch Moment Variation	Mid-Span Moment (Ft. Kips) M_C	Mid-Span Moment Variation	Ratio $\frac{M_C}{M_D}$
1/8	0	0.5	80	0	10	.125	512.00	+ 4.0%	288.00	— 6.5%	.56
	0	1.0	80	0	10	.25	492.31	Base	307.69	Base	.63
	0	2.0	80	0	10	.50	457.14	— 7.2%	342.86	+ 11.4%	.75
1/4	0	0.5	80	0	20	.25	492.31	+ 7.7%	307.69	— 10.3%	.63
	0	1.0	80	0	20	.50	457.14	Base	342.86	Base	.75
	0	2.0	80	0	20	1.00	400.00	— 12.5%	400.00	+ 16.7%	1.00
1/2	0	0.5	80	0	40	.50	457.14	+ 14.3%	342.86	— 14.3%	.75
	0	1.0	80	0	40	1.00	400.00	Base	400.00	Base	1.00
	0	2.0	80	0	40	2.00	320.00	— 20.0%	480.00	+ 20.0%	1.50
1	0	0.5	80	0	80	1.00	400.00	+ 25.0%	400.00	— 16.7%	1.00
	0	1.0	80	0	80	2.00	320.00	Base	480.00	Base	1.50
	0	2.0	80	0	80	4.00	228.57	— 28.5%	571.43	+ 19.0%	2.50

UNIFORM VERTICAL ROOF LOAD

RIDGE FRAMES



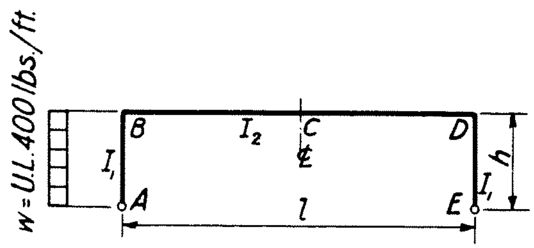
$\frac{h}{l}$	$Q = \frac{f}{h}$	$\frac{I_2}{I_1}$	l Feet	f Feet	h Feet	$K = \frac{I_2}{I_1}\frac{h}{m}$	Haunch Moment (Ft. Kips) M_D	Haunch Moment Variation	Mid-Span Moment (Ft. Kips) M_C	Mid-Span Moment Variation	Ratio $\frac{M_C}{M_D}$
1/8	2.0	0.5	80	20	10	.112	274.56	+ 0.9%	23.68	+ 41.8%	.087
	2.0	1.0	80	20	10	.224	272.23	Base	16.69	Base	.061
	2.0	2.0	80	20	10	.448	267.70	— 1.7%	3.10	— 81.3%	.012
1/4	1.0	0.5	80	20	20	.224	359.91	+ 3.1%	80.18	— 21.3%	.222
	1.0	1.0	80	20	20	.448	349.09	Base	101.82	Base	.292
	1.0	2.0	80	20	20	.896	329.28	— 5.7%	141.44	+ 39.0%	.43
1/2	0.5	0.5	80	20	40	.448	404.00	+ 8.6%	194.00	— 20.0%	.48
	0.5	1.0	80	20	40	.896	371.94	Base	242.09	Base	.65
	0.5	2.0	80	20	40	1.792	321.00	— 13.7%	318.50	+ 31.5%	.99
1	0.25	0.5	80	20	80	.896	392.91	+ 19.0%	308.86	— 20.3%	.79
	0.25	1.0	80	20	80	1.792	330.09	Base	387.39	Base	1.17
	0.25	2.0	80	20	80	3.584	250.12	— 24.2%	487.35	+ 25.8%	1.94

TABLE II

EFFECT OF RATIO $\frac{I_2}{I_1}$ ON MOMENT DISTRIBUTION

UNIFORM HORIZONTAL WIND LOAD

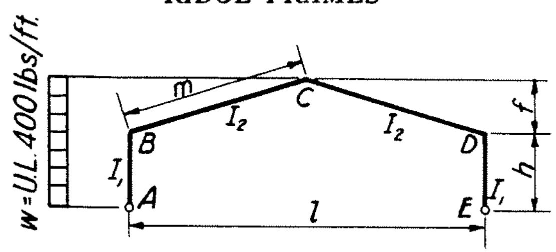
RECTANGULAR FRAMES



$\frac{h}{l}$	$Q = \frac{f}{h}$	$\frac{I_2}{I_1}$	l Feet	f Feet	h Feet	$K = \frac{I_2}{I_1}\frac{h}{m}$	Haunch Moment (Ft. Kips) M_D	Haunch Moment Variation	Mid-Span Moment (Ft. Kips) M_C	Mid-Span Moment Variation	Ratio $\frac{M_C}{M_D}$
1/8	0	0.5	80	0	10	.125	10.10	− 0.9%	0.10		.01
	0	1.0	80	0	10	.25	10.19	Base	0.19	Base	.02
	0	2.0	80	0	10	.50	10.36	+ 1.7%	0.36		.03
1/4	0	0.5	80	0	20	.25	40.77	− 1.6%	0.77		.02
	0	1.0	80	0	20	.50	41.43	Base	1.43	Base	.03
	0	2.0	80	0	20	1.00	42.50	+ 2.6%	2.50		.06
1/2	0	0.5	80	0	40	.50	165.71	− 2.5%	5.71		.03
	0	1.0	80	0	40	1.00	170.00	Base	10.00	Base	.06
	0	2.0	80	0	40	2.00	176.00	+ 3.5%	16.00		.09
1	0	0.5	80	0	80	1.00	680.00	− 3.4%	40.00		.06
	0	1.0	80	0	80	2.00	704.00	Base	64.00	Base	.09
	0	2.0	80	0	80	4.00	731.42	+ 3.9%	91.42		.13

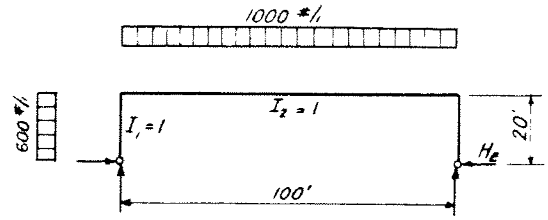
UNIFORM HORIZONTAL WIND LOAD

RIDGE FRAMES



$\frac{h}{l}$	$Q = \frac{f}{h}$	$\frac{I_2}{I_1}$	l Feet	f Feet	h Feet	$K = \frac{I_2}{I_1}\frac{h}{m}$	Haunch Moment (Ft. Kips) M_D	Haunch Moment Variation	Mid-Span Moment (Ft. Kips) M_C	Mid-Span Moment Variation	Ratio $\frac{M_C}{M_D}$
1/8	2.0	0.5	80	20	10	.112	37.06	− 0.4%	21.17	− 1.8%	.57
	2.0	1.0	80	20	10	.224	37.19	Base	21.56	Base	.58
	2.0	2.0	80	20	10	.448	37.44	+ 0.7%	22.32	+ 3.7%	.60
1/4	1.0	0.5	80	20	20	.224	96.78	− 1.0%	33.55	− 6.6%	.35
	1.0	1.0	80	20	20	.448	97.78	Base	35.55	Base	.36
	1.0	2.0	80	20	20	.896	99.60	+ 1.9%	39.21	+ 10.5%	.39
1/2	0.5	0.5	80	20	40	.448	282.08	− 2.2%	63.13	− 12.8%	.22
	0.5	1.0	80	20	40	.896	288.27	Base	72.40	Base	.25
	0.5	2.0	80	20	40	1.792	298.09	+ 3.4%	87.14	+ 21.8%	.29
1	0.25	0.5	80	20	80	.896	920.85	− 3.4%	151.06	− 20.8%	.16
	0.25	1.0	80	20	80	1.792	952.66	Base	190.83	Base	.20
	0.25	2.0	80	20	80	3.584	993.18	+ 4.3%	241.48	+ 26.5%	.24

EXAMPLE No. 1



$$\frac{f}{h} = Q = 0 \qquad K_1 = \frac{1}{1} \text{ x } \frac{20}{50} = 0.4 \ \left(\text{or } K_2 = \frac{1}{1} \text{ x } \frac{20}{100} = 0.2\right)$$

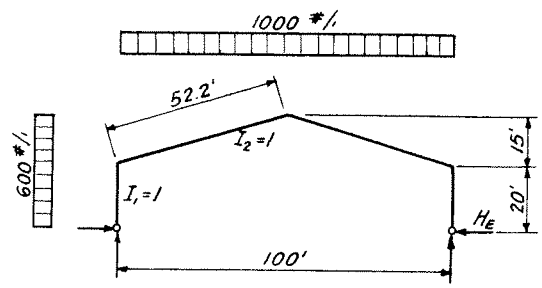
From charts

$C_1 = .0735$ $\qquad H_B \leftarrow \dfrac{.0735 \text{ x } 1000 \text{ x } 100^2}{20} = 36.8$

C_5 (or C_6) $= .26$ $\qquad H_B = .26 \text{ x } 600 \text{ x } 20 = \underline{3.1}$

Total H_B = 39.9 *kips*

EXAMPLE No. 2



$$\frac{f}{h} = Q = \frac{15}{20} = .75 \qquad K_1 = \frac{1}{1} \text{ x } \frac{20}{52.2} = 0.38$$

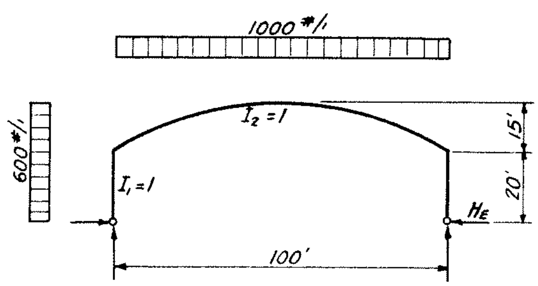
From charts

$C_1 = .0590$ $\qquad H_B = \dfrac{.059 \text{ x } 1000 \text{ x } 100^2}{20} = 29.5$

$C_6 = .525$ $\qquad H_B = .525 \text{ x } 600 \text{ x } 20 = \underline{6.3}$

Total H_B = 35.8 *kips*

EXAMPLE No. 3



$$Q = .75 \qquad K_2 = \frac{1}{1} \text{ x } \frac{20}{100} = 0.2$$

From charts

$C_1 = .0546$ $\qquad H_B = \dfrac{.0546 \text{ x } 1000 \text{ x } 100^2}{20} = 27.3$

$C_6 = .485$ $\qquad H_B = .485 \text{ x } 600 \text{ x } 20 = \underline{5.8}$

Total H_B = 33.1 *kips*

Since, in any given problem, the height, roof rise if any, and span, are established at the outset, the only remaining variable affecting the distribution of moments in the frame is the ratio I_2/I_1. The exact value of this variable can only be determined when the sizes of the members in the frame have been established. How much error may be involved in the distribution of moments if I_2/I_1 is taken as unity in assuming the value of K can be seen from a study of Tables I and II. Covering a wide range of height-to-span and ridge-rise-to-column-height ratios, the moment distribution for the base condition, $I_2/I_1 = 1.0$, is compared in these tables, with the conditions where I_2/I_1 is respectively half and twice as great. Also given is the relationship of midspan moment to haunch moment. Since it is sometimes easier to visualize the effect of variables when they are given numerical values within the range encountered in design practice, a constant 80-ft. span, and constant numerical values for vertical and horizontal loading, are assumed in the table. The percentage variations and the ratio M_C/M_D will, of course, remain the same when h/l, Q and K have the values shown, regardless of the actual span length and load intensity.

It will be noted that, percentagewise, the "error" resulting from a preliminary assumption that $I_2/I_1 = 1.0$ will, in many cases, be so small that it can be neglected. In the case of ridge-type buildings, where the ratio of M_C to M_D is small, considerations other than the bending moment at the crown will usually result in the selection of a rafter section more nearly the size of the column section than this relationship might otherwise indicate. The distribution of moments due to wind loading is affected but very little by the relationship of I_2/I_1, within the range generally encountered.

For those cases where the wrong assumption as to the final value of I_2/I_1 would have the most pronounced effect on moment distribution, notably in frames having the greater height-to-span ratios, a little study of the tabulated values for the ratio M_C/M_D should permit the designer to select an assumed value of I_2/I_1, sufficiently close to the final value that no correction will be required later.

Except in the case of small frames the knee section is usually enlarged by the use of a straight or curved haunch. The increase in moment of inertia thus provided at the knee causes an increase in the horizontal reactions induced by vertical loading. This increase will vary from about 2 to 8% for a haunched knee proportioned in accordance with the recommendations outlined in the next section. Although many engineers neglect the effect of a haunch entirely, it is on the side of conservative design, in cases where knees are haunched, to increase the horizontal thrust by 5% in computing the moment at the knee resulting from vertical loading.

The horizontal reactions may be approximated, in the manner above outlined, for variable depth girders and columns. In frames of this sort the increased moments of inertia near the knee, as compared with those at the center of the girder and the base of the columns, will usually result in horizontal reactions 5% to 20% greater than provided by the charts. It is therefore recommended that such frames be proportioned on the basis of horizontal reactions 10 to 15% greater than indicated by the charts, and that the design be checked by the more laborious "exact" procedure outlined on page 12.

As in most texts dealing with rigid frames, a parabolic form has been used for the roof girders in deriving the formulas for the horizontal reactions on pg. 19, and in the charts on pp. 16 and 17. The resulting formulas are much simpler to use than any that could be derived for a corresponding circular arc.

For relatively flat arches, say up to a rise-to-span ratio of 0.2, which covers the range of most problems, it is doubtful whether the eye can detect the difference between a roof constructed to a parabola and one laid out on a circular arc. Furthermore there is little difference in stress distribution, as between the two forms, up to this limit of rise; sections proportioned on the basis of one type of curve may be transferred to the other without a recalculation of stresses.

For steeper rises, however, there are more noticeable differences in stress distribution as between the two types of curves. In the extreme case—where the rise is equal to one-half the span and the circular arc is tangent to the columns—the horizontal reaction for the parabolic form will be more than 15% larger than for a circular arc of the same rise. Here the moment at the crown of the circular arc may be several times that of the moment at the crown of the corresponding parabolic form, which as the length of the columns approaches zero as a limit, disappears entirely under a symmetrically disposed system of vertical loading.

For a semi-circular arc, therefore—indeed for any single-centered circular arc whose middle ordinate exceeds 0.2 of its chord length—stress analysis should be by the "exact" procedure. However, a compound, circular curve, closely approximating a parabola, may be substituted here with little if any effect upon the stress analysis based on the true parabola.

DESIGN OF MEMBERS

Except in the area of a haunched knee, the proportioning of a rigid frame involves only those design procedures normally employed in the design of any beam, built-up girder or column subject to bending and direct stress.

The stress distribution within the knee section has been the subject of considerable research, both in this country and in Europe. Data obtained from large scale model tests conducted at the National Bureau of Standards,[1, 2, 3] and at Lehigh University,[4] together with a development and summary of theory by Dr. Friedrich Bleich,[5] constitute valuable sources of information in regard to this problem.

These data indicate that, if certain precautions are observed regarding its shape, the knee section may be safely designed in accordance with conventional practices with little if any sacrifice in economy. It is therefore recommended that the knee section be proportioned and designed on the following basis:

Rule 1. That the critical design sections be taken

(a) At the inside face of column and bottom of girder for a straight knee,

(b) At the points of tangency for a circular haunched knee,

(c) At the extremities and common intersection point for haunches made up of tapered, or trapezoidal, segments.

Rule 2. That the allowable bending stress at these critical sections be limited to 20,000 p.s.i., modified where necessary according to the formula

$$F_b = \frac{12{,}000{,}000}{ld/bt};$$

the allowable direct compressive stress be limited to $F_a = 17{,}000 - .485\ l^2/r^2$; and that the maximum combined stress be limited by the provisions of Sect. 12 (a) of the A.I.S.C. Specification.

Rule 3. That the maximum combined stress be determined by the conventional formula $f = P/A + Mc/I$.

Rule 4. That positive lateral support be provided for the compression (inner) flange in the region of the knee. (See discussion on page 4.)

Rule 5. That the average shear stress on the web, computed at the critical design sections referred to in Rule No. 1, be limited to 13,000 p.s.i.*

Rule 6. That in the case of a curved haunch, the ratio R/d be not less than that determined by the curve in Fig. 4.**

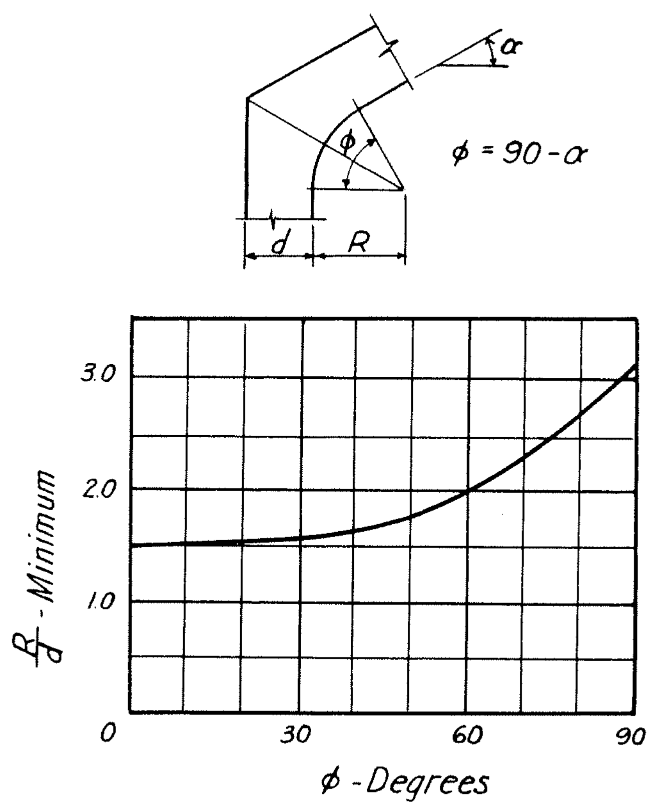


Figure 4

Rule 7. That, in the case of a curved haunch, the relationship b^2/Rt be not more than 2, where b is the width, t is the thickness

[1] "Strength of a Riveted Steel Rigid Frame Having Straight Flanges", by Ambrose H. Stang, Martin Greenspan & William Osgood. Research Paper No. 1130, National Bureau of Standards, Vol. 21, 1938, p. 269.

[2] "Strength of a Riveted Steel Rigid Frame Having a Curved Inner Flange", by Ambrose H. Stang, Martin Greenspan & William Osgood, Research Paper No. 1161, loc. cit. p. 853.

[3] "Strength of a Welded Steel Rigid Frame" by Ambrose H. Stang and Martin Greenspan. Research Paper No. 1224, loc. cit., Vol. 23, 1939, p. 145.

[4] "An Investigation of Steel Rigid Frames" by Inge Lyse and W. E. Black. Paper No. 2130, American Society of Civil Engineers Transactions, Vol. 107, 1942.

[5] "Design of Rigid Frame Knees" by Dr. Friedrich Bleich, American Institute of Steel Construction, July 1943.

* Note: Shear stress intensity is seldom critical when webs of usual thickness are used.

** Note: Occasionally architectural considerations may dictate the use of a curved inner flange of radius smaller than indicated under Rule No. 6. In such cases the critical design section may, with little loss in economy, be taken as if no curved haunch were to be employed (See Rule No. 1 (a)). The thickness of the curved flange used however should satisfy the requirements of Rule No. 7.

and R is the radius of curvature of the inner flange, all in inches.***

Depth of Wide Flange Shape	R
12″	5′0″
14″	5′0″
16″	5′0″ (except for heaviest series)
18″	5′0″ (except for heaviest series)
21″	5′0″ (except for heaviest series)
24″	7′6″
27″	8′6″
30″	8′6″
33″	9′0″
36″	9′0″

Rule 8. That stiffeners be provided at the mid-point, and at or near the extremities, of a curved knee; in line with the lower flange of the girder and the inner flange of the column, in the case of a straight knee.

*** Note: If it is desired to maintain the smallest radius of curvature consistent with Rule No. 6 it may sometimes be found necessary to increase somewhat the thickness, t, of the curved inner flange, over that of the flanges of the prismatic sections. The following table of radii will satisfy the condition $b^2/Rt < 2$, with curved flanges no thicker than those of the wide flange shapes required for the columns and girders.

THEORY OF ANALYSIS—TWO HINGED FRAMES

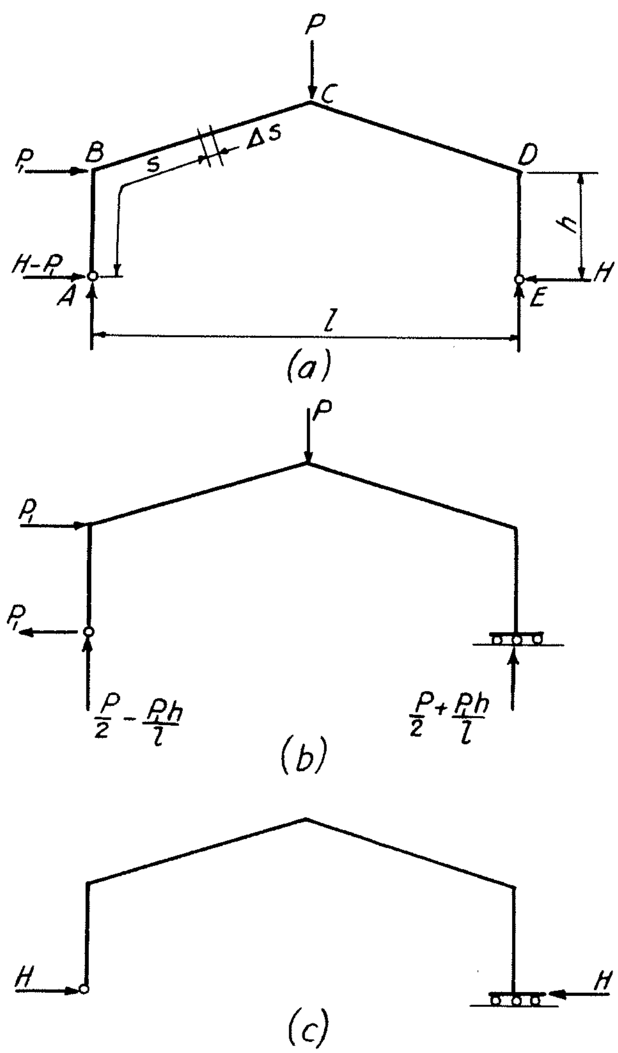


Figure 5

The given frame (Fig. 5a) is replaced by the two statically determinate frames Fig. 5b and 5c. The algebraic sum of the stresses at any point in these two imaginary frames equals the true stress at the corresponding point in the given frame.

Let M_s = Moment at any point due to the loads and reactions of Fig. 5b.

m = Moment at any point due to a unit horizontal load at point E.

Positive moments will be those which cause tension on the inside face of the frame.

Under the action of the loads and reactions of Fig. 5b the leg of the frame at point E will move horizontally to the right a distance Δ.

The value of Δ may be expressed by the common deflection formula

$$\Delta = \sum \frac{M_s\, m\, \Delta s}{EI}$$

In order to achieve a total deflection of zero at point E (the required condition in Fig. 5a) a force H must be applied at point E (Fig. 5c) which will cause a horizontal deflection Δ′ equal and opposite in direction to Δ. If m is the moment at any point due to a unit horizontal load at point E, then Hm is the moment due to the horizontal load H. The deflection due to H may then be expressed as

$$\Delta' = \sum \frac{(Hm)\, m\, \Delta s}{EI} = H \sum \frac{m^2\, \Delta s}{EI} = -\Delta$$

or,

$$H \sum \frac{m^2\, \Delta s}{EI} = -\sum \frac{M_s\, m\, \Delta s}{EI}$$

Noting that m at any point is equal to the ordinate y of that point taken from the base of the frame,

$$H = -\frac{\sum \frac{M_s\, y\, \Delta s}{EI}}{\sum \frac{y^2\, \Delta s}{EI}}$$

Moments which cause tension on the inside of the frame are considered positive. The direction of the reactions is evident by inspection and no further sign convention is required.

Precise analyses (usually required only in the case of large frames composed of members having non-uniform moments of inertia) may be made using the above expression for H. The frame is divided into a number of increments of length Δs (measured along the neutral axis). Each increment, Δs, is multiplied by y (measured vertically from the base line to the centroid of the increment), times the value of M_s at this centroid, and divided by the average moment of inertia of the increment. The sum of these calculations for each increment, divided by the summation of corresponding calculations for $\frac{y^2 \Delta s}{I}$ will give an accurate value for H provided the increments are made sufficiently small. Examples are shown on pages 22, 25 and 28.

MINOR CONSIDERATIONS

As noted on page 5, temperature, direct stress and tie deformation nearly always have but a very minor effect on the stresses in a rigid frame. In most cases they may be neglected entirely and in no case is an exact analysis of their effect justified. The approximate horizontal deflection that would occur, due to each of these factors, if one column were free to move horizontally, would be, in inches

Δ_t, deflection due to temperature $= etl$

Δ_{ar}, deflection due to direct stress $= Hl/AE$

Δ_{at}, deflection due to tie elongation $= Hl/A_tE$

where,

e = coefficient of expansion (.0000067)

t = temperature range (°F)

H = Maximum horizontal reaction under service loads

A = Average cross-sectional area of girder (Square inches)

A_t = Cross-sectional area of tie (Square inches) and l is given in inches.

To derive the correction, H_1, to be made, due to the combined effect of these factors on the computed value of H under service loads, it is only necessary to divide the deflections outlined above, by the deflection due to a unit horizontal load at point E.

Thus

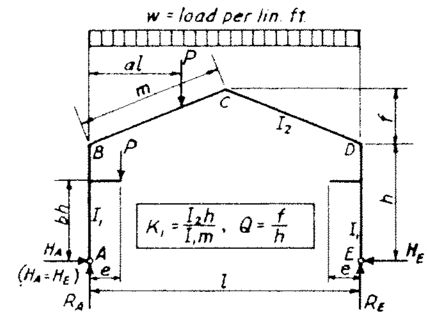
$$H_1 = \frac{\Delta_t + \Delta_{ar} + \Delta_{at}}{\sum \frac{y^2 \Delta s}{EI}}$$

Algebraic signs for each of these secondary considerations is self evident. A tendency of the legs of the frame to move inward, or a tendency of the supports to move outward, will decrease the nominal reaction H. Thus the effect of the direct stress, a decrease in temperature and tie elongation is to reduce the value of H derived by the charts on pages 14 to 17 or by the formulas on pages 18 and 19.

RIDGE & RECTANGULAR FRAMES

(Rectangular when $Q = 0$)

MOMENT COEFFICIENTS-HINGED COLUMN BASES



$H_E = C_1 wl^2/h$ for uniform vertical load on full span

($H_E = C_1 wl^2/2h$ when half-span only is loaded)

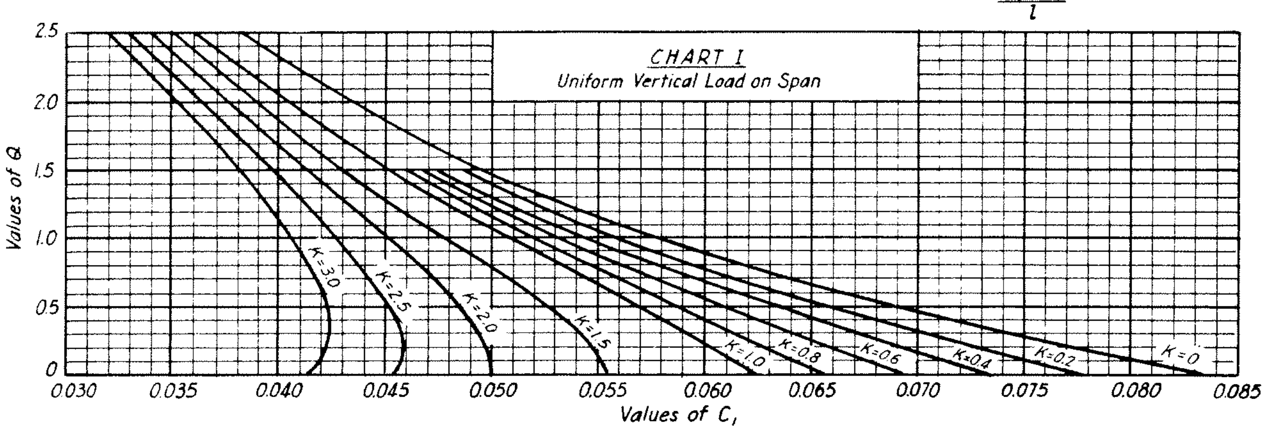
$H_E = C_2 C_1 Pl/h$ for concentrated load P on span, distant al from either end.

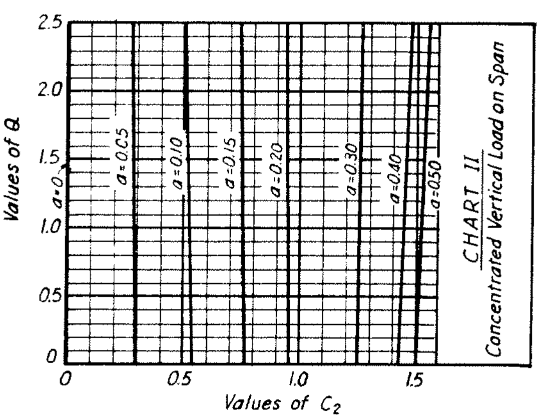
($H_E = 2C_2 C_1 Pl/h$ for equal concentrated loads P, distant al from both ends)

$H_E = C_3 Pe/h$ for concentrated load P on either bracket of length e

($H_E = 2C_3 Pe/h$ for equal concentrated loads P on both brackets)

For uniform load full span	$R_A = \frac{wl}{2}$	$R_E = \frac{wl}{2}$
For uniform load left half span	$R_A = \frac{3wl}{8}$	$R_E = \frac{wl}{8}$
For concentrated roof load P	$R_A = P(1-a)$	$R_E = Pa$
For concentrated bracket load P	$R_A = \frac{P(l-e)}{l}$	$R_E = \frac{Pe}{l}$





Illustrative Problem: (Chart II)

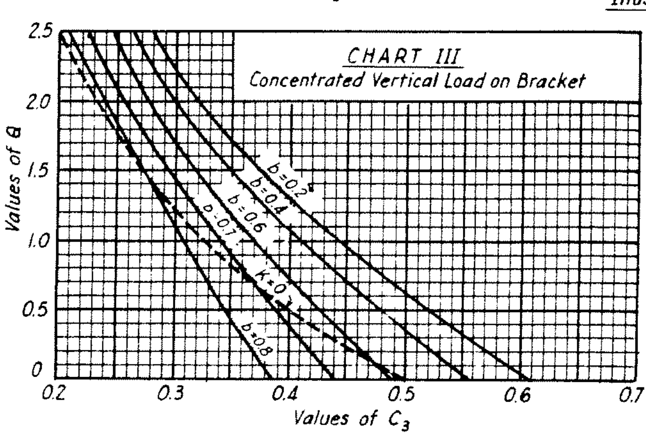
Given: $P = 20^K$; $l = 80'$; $al = 20'(a = 0.25)$; $h = 18$; $f = 10'$; $m = 41.23'$

From Chart I (using $K_1 = 0.44$ and $Q = 0.56$) $C_1 = 0.062$

From Chart II (using $a = 0.25$) $C_2 = 1.12$

Then $H_E = H_A = 0.062 \times 1.12 \times 20 \times \frac{80}{18} = 6.17^K$

Note: When a second 20^K load is symmetrically placed on other side of ℄ $H_E = H_A = 12.34^K$



Illustrative Problem: (Chart III)

Given: $P = 20^K$; $l = 80'$; $e = 2'$; $h = 18'$; $bh = 14'(b = 0.78)$ $h = 10'$; $m = 41.23'$

Entering Chart III (using $b = 0.78$ and $Q = 0.56$)

$C_3 = 0.355$, when $K_1 = 3.0$

$C_3 = 0.385$, when $K_1 = 0$

$C_3 = 0.38$, when $K_1 = 0.44$ (by interpolation)

Then $H_E = H_A = 0.38 \times 20 \times \frac{2}{18} = 0.84^K$

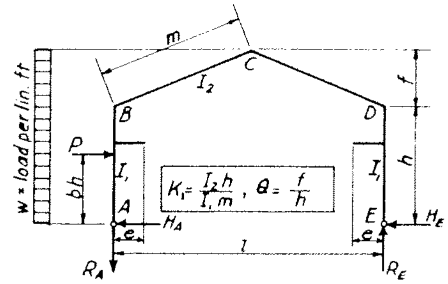
Note: When both brackets have 20^K loads $H_E = H_A = 1.68^K$

Note: Solid lines (Chart III) plotted for $K = 3.0$ (varying values of b)
Dotted line (Chart III) plotted for $K = 0.0$ (all values of b)

RIDGE & RECTANGULAR FRAMES

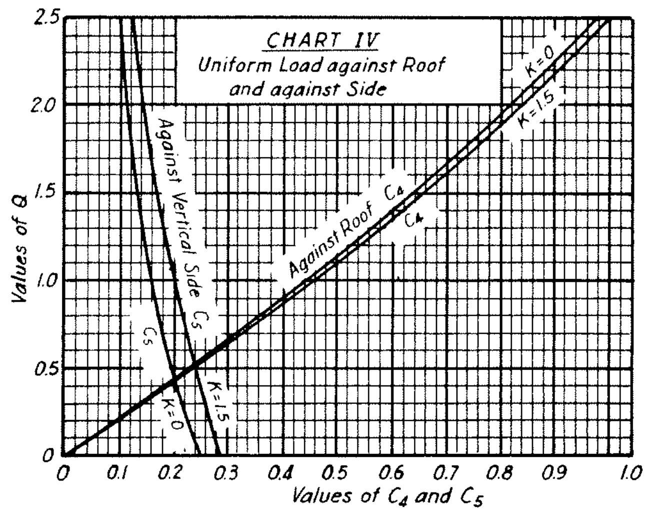
(Rectangular when $Q = 0$)

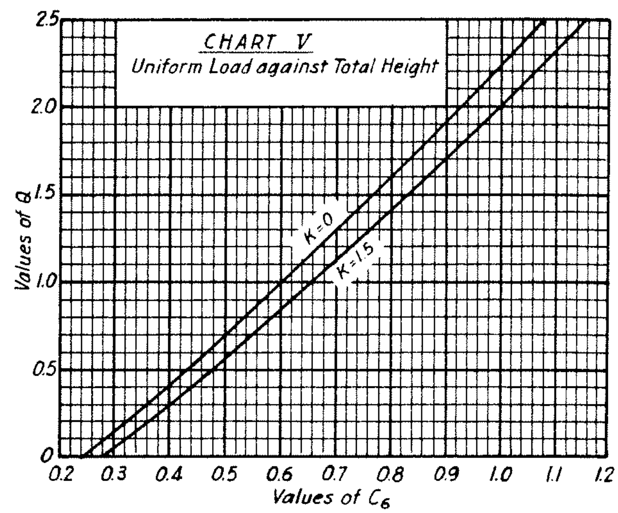
MOMENT COEFFICIENTS-HINGED COLUMN BASES

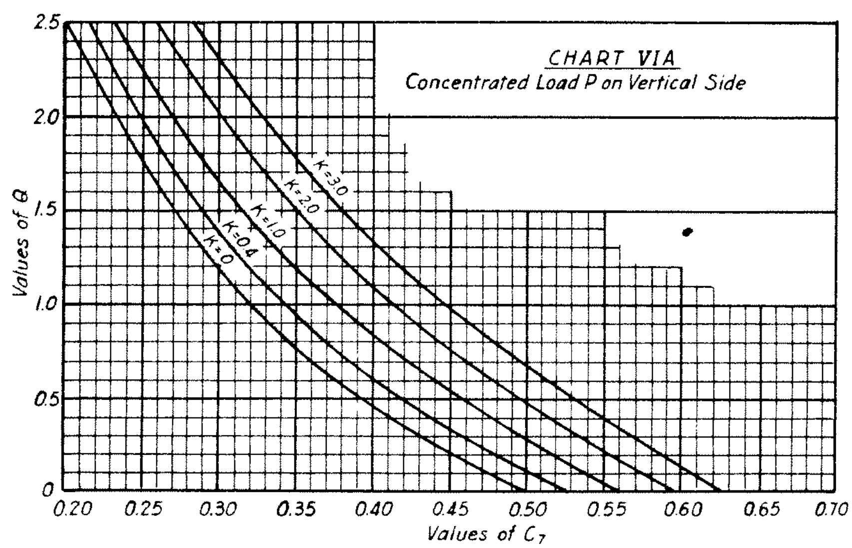


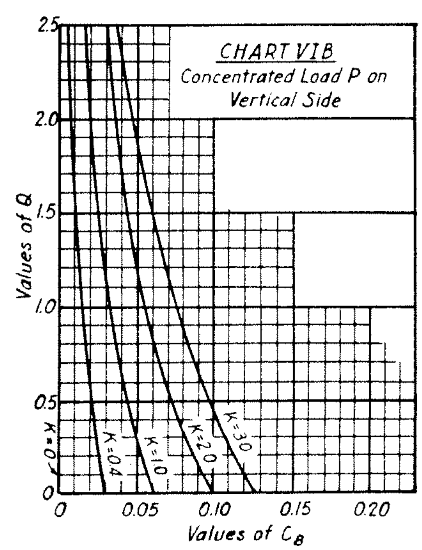
$H_E = C_4 wh$, for uniform load against roof
$H_E = C_5 wh$, for uniform load against vertical side
$H_E = C_6 wh$, for uniform load against total height.
$H_E = Pb(C_7 - b^2 C_8)$, for concentrated load P on one vertical side ($b \leq 1.0$)
($H_E = P_1$ for concentrated load P on both vertical sides at same elevation.

For concentrated horizontal load P	$H_A = P - H_E$	$R_A = R_E = \frac{Pbh}{l}$
For uniform load against roof	$H_A = wf - H_E$	$R_A = R_E = \frac{wf(2h+f)}{2l}$
For uniform load against vertical side	$H_A = wh - H_E$	$R_A = R_E = \frac{wh^2}{2l}$
For uniform load against total height	$H_A = w(h+f) - H_E$	$R_A = R_E = \frac{w(h+f)^2}{2l}$





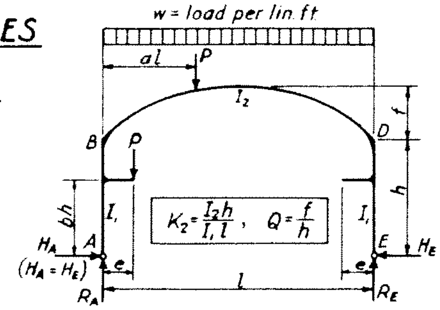




PARABOLIC & RECTANGULAR FRAMES

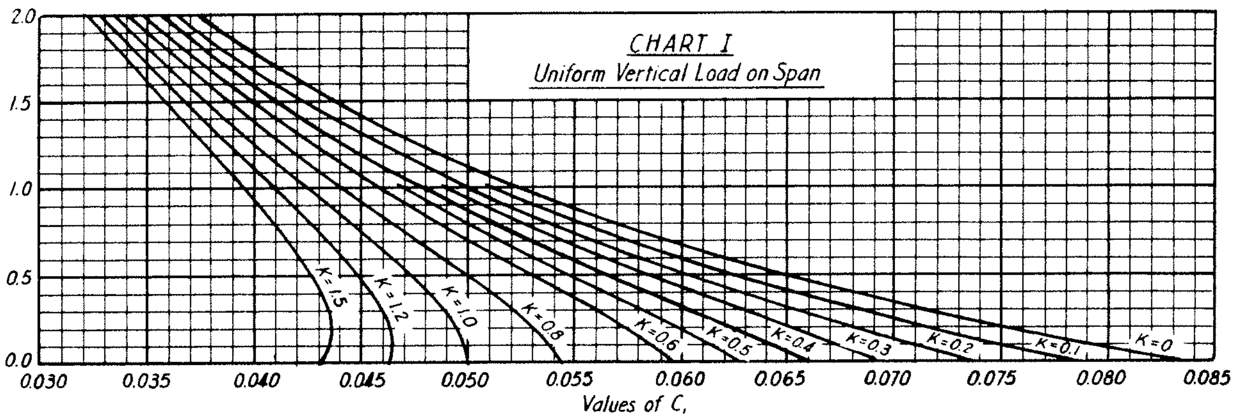
(Rectangular when $Q=0$)

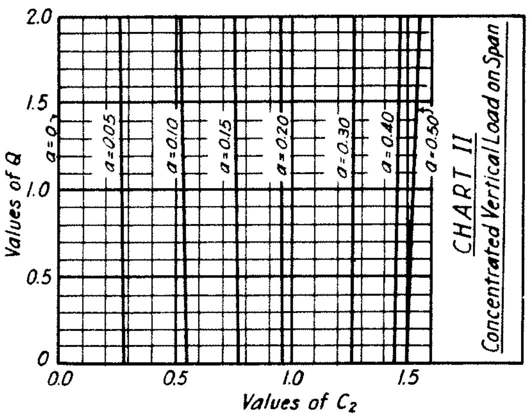
MOMENT COEFFICIENTS - HINGED COLUMN BASES



$H_E = C_1 w l^2/h$ for uniform vertical load on full span
($H_E = C_1 w l/2h$ when half-span only is loaded)
$H_E = C_2 C_1 P l/h$ for concentrated load P on span, distant al from either end.
($H_E = 2C_2 C_1 P l/h$ for equal concentrated loads P, distant al from both ends)
$H_E = C_3 Pe/h$ for concentrated load P on either bracket of length e.
($H_E = 2C_3 Pe/h$ for equal concentrated loads P on both brackets)

For uniform load full span	$R_A = \frac{wl}{2}$	$R_E = \frac{wl}{2}$
For uniform load left half span	$R_A = \frac{3wl}{8}$	$R_E = \frac{wl}{8}$
For concentrated roof load P	$R_A = P(1-a)$	$R_E = Pa$
For concentrated bracket load P	$R_A = \frac{P(l-e)}{l}$	$R_E = \frac{Pe}{l}$





Illustrative Problem: (Chart II)

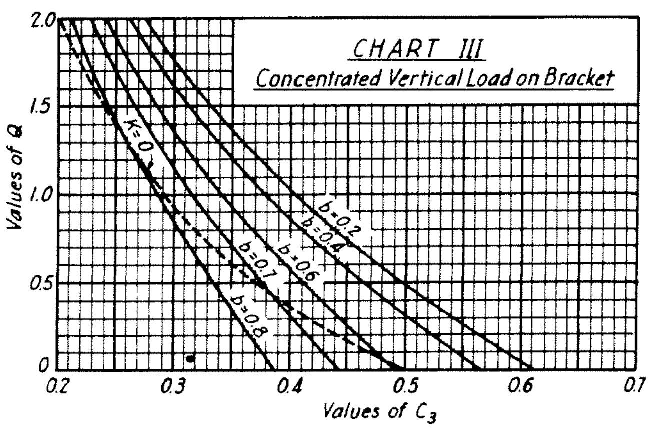
Given: $P = 20^k$; $l = 80'$; $al = 20'(a = 0.25)$; $h = 18'$; $f = 10'$

From Chart I (using $K_2 = 0.23$ and $Q = 0.56$) $C_1 = 0.0586$

From Chart II (using $a = 0.25$) $C_2 = 1.12$

Then $H_E = H_A = 0.0586 \times 1.12 \times 20 \times \frac{80}{18} = 5.82^k$

Note: When a second 20^k load is symmetrically placed on other side of ℄, $H_E = H_A = 11.64^k$



Illustrative Problem: (Chart III)

Given: $P = 20^k$; $l = 80'$; $e = 2'$; $h = 18'$; $bh = 14'(b = 0.78)$; $h = 10'$

Entering Chart III (using $b = 0.78$ and $Q = 0.56$)

$C_3 = 0.34$, when $K_2 = 1.5$

$C_3 = 0.36$, when $K_2 = 0$

$C_3 = 0.35$, when $K_2 = 0.23$ (by interpolation)

Then $H_E = H_A = 0.35 \times 20 \times \frac{2}{18} = 0.78^k$

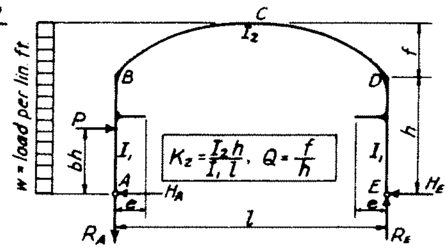
Note: When both brackets have 20^k loads, $H_E = H_A = 1.56^k$

Note: Solid lines (Chart III) plotted for $K_2 = 1.50$ (varying values of b)
Dotted line (Chart III) plotted for $K_2 = 0$ (all values of b)

PARABOLIC & RECTANGULAR FRAMES

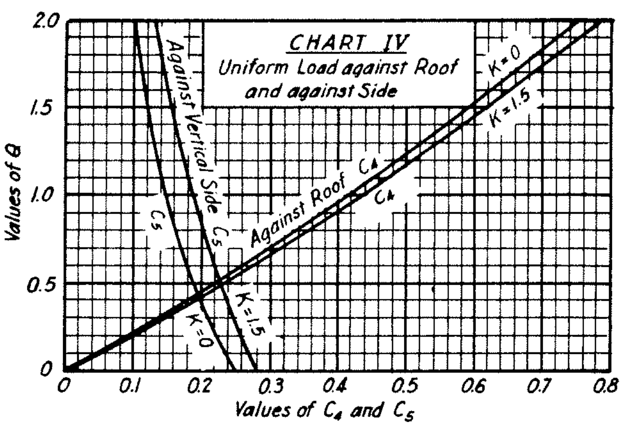
(Rectangular when $Q = 0$)

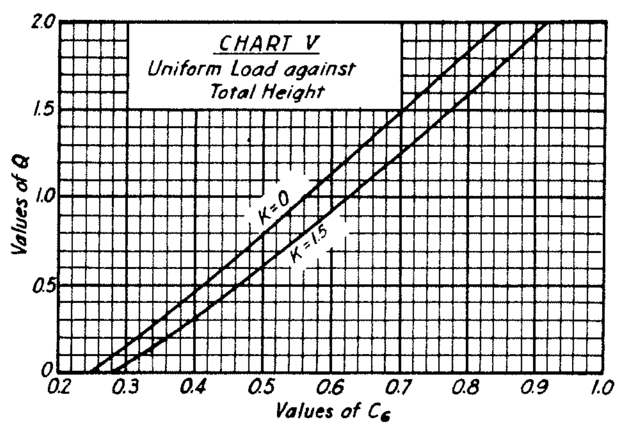
MOMENT COEFFICIENTS - HINGED COLUMN BASES

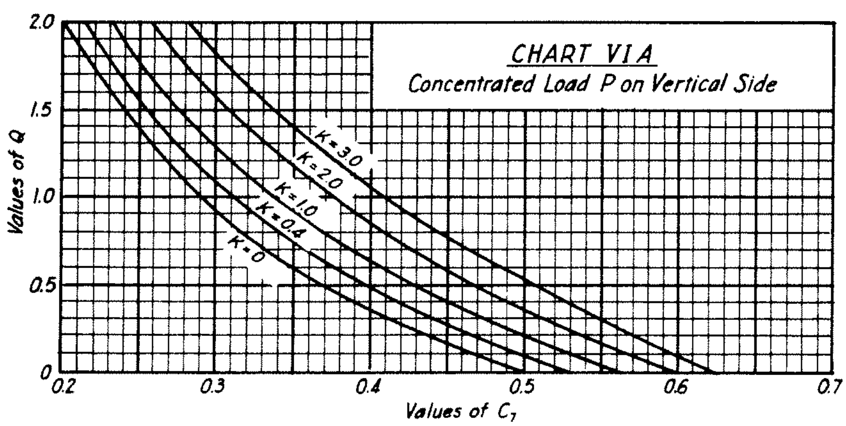


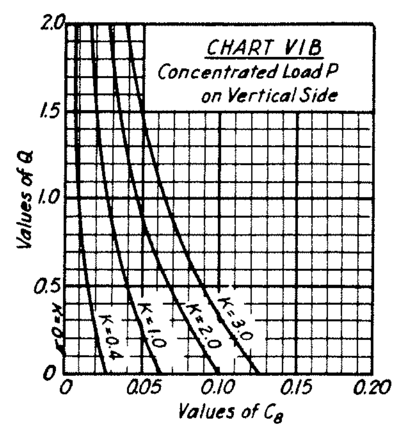
$H_E = C_4 wh$ for uniform load against roof
$H_E = C_5 wh$ for uniform load against vertical side
$H_E = C_6 wh$ for uniform load against total height
$H_E = Pb(C_7 - b^2 C_8)$ for concentrated load P on one vertical side ($b \leq 1.0$)
($H_E = P$, for concentrated load P on both vertical sides at same elevation)

For concentrated horizontal load P	$H_A = P - H_E$	$R_A = R_E = \frac{Pbh}{l}$
For uniform load against roof	$H_A = wf - H_E$	$R_A = R_E = \frac{wf(2h+f)}{2l}$
For uniform load against vertical side	$H_A = wh - H_E$	$R_A = R_E = \frac{wh^2}{2l}$
For uniform load against total height	$H_A = w(h+f) - H_E$	$R_A = R_E = \frac{w(h+f)^2}{2l}$









Illustrative Problem: (Charts VIA & VIB)

Given: $P = 20^k$; $l = 80'$; $bh = 12'$ ($b = 0.67$); $h = 18'$

Entering Chart VI A (using $K = 0.23$ and $Q = 0.56$) $C_7 = 0.375$

Entering Chart VI B (using $K = 0.23$ and $Q = 0.56$) $C_8 = 0.010$

Then $H_E = 20 \times 0.67(0.375 - 0.67^2 \times 0.010) = 4.93^k$ and

$H_A = 20 - 4.93 = 15.07^k$

RIDGE AND RECTANGULAR FRAMES

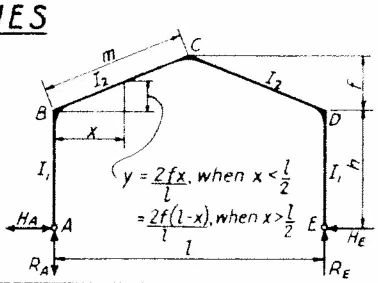
(RECTANGULAR WHEN $Q = 0$)

General Formulas:

$K = \frac{I_2 h}{I_1 m}$ $Q = \frac{f}{h}$

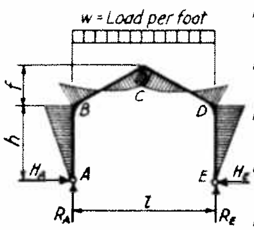
$N = 4(K + 3 + 3Q + Q^2)$

* In formulas for M_x, "x" is always measured from point B.



Plus sign (+) denotes moments which cause tension on the inside of the frame when the vertical loads act downward and the horizontal loads, applied to the left side of the frame, act toward the right. The direction of the reactions, and the signs of all terms in the moment formulas, are shown correctly for this condition. When the direction of the loads (but not their position) is reversed the direction of the reactions may, and the signs for all moments will, be reversed.

CASE I - UNIFORMLY DISTRIBUTED VERTICAL LOAD ENTIRE SPAN



$R_A = R_E = R = \frac{wl}{2}$

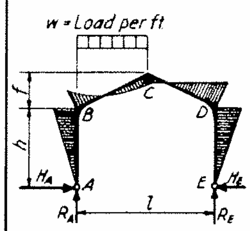
$H_A = H_E = H = \frac{wl^2}{8hN}(8 + 5Q)$

$M_B = M_D = -Hh$

$M_C = \frac{Rl}{4} - H(h+f)$

$M_x = \frac{wx(l-x)}{2} - H(h+y)$ *

CASE IA - UNIFORMLY DISTRIBUTED VERTICAL LOAD HALF SPAN (LEFT)



$R_A = \frac{3wl}{8}$ $R_E = \frac{wl}{8}$

$H_A = H_E = H = \frac{wl^2}{16hN}(8 + 5Q)$

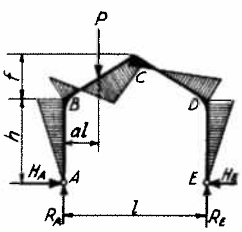
$M_B = M_D = -Hh$

$M_C = \frac{R_E l}{2} - H(h+f)$

$M_x = R_A x - \frac{wx^2}{2} - H(h+y)$, when $x < \frac{l}{2}$ *

$= R_E(l-x) - H(h+y)$, when $x > \frac{l}{2}$ *

CASE II - ONE CONCENTRATED ROOF LOAD AT ANY POSITION



$R_A = P - Pa$ $R_E = Pa$

$H_A = H_E = H = \frac{Pla}{hN}(6 - 6a + 3Q - 4Qa^2)$

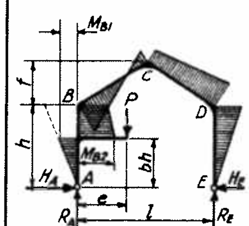
$M_B = M_D = -Hh$

$M_C = \frac{R_E l}{2} - H(h+f)$

$M_x = R_A x - H(h+y)$, when $x < al$ *

$= R_E(l-x) - H(h+y)$, when $x > al$ *

CASE III - BRACKET LOAD ON ONE COLUMN (LEFT COLUMN)



$R_A = P - \frac{Pe}{l}$ $R_E = \frac{Pe}{l}$

$H_A = H_E = H = \frac{3Pe}{hN}(K - b^2K + 2 + Q)$

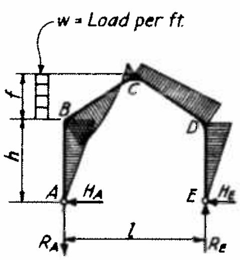
$M_B = Pe - Hh$ $M_D = -Hh$

$M_C = \frac{Pe}{2} - H(h+f)$

$M_{B1} = -Hbh$ $M_{B2} = Pe - Hbh$

$M_x = R_E(l-x) - H(h+y)$ *

CASE IVA - UNIFORM HORIZONTAL LOAD INCLINED (ROOF) PORTION ONLY



$R_A = R_E = R = \frac{wf(2h+f)}{2l}$

$H_A = wf - H_E$

$H_E = \frac{wf}{4N}(8K + 24 + 20Q + 5Q^2)$

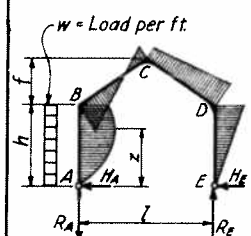
$M_B = H_A h$ $M_D = -H_E h$

$M_C = \frac{Rl}{2} - H_E(h+f)$

$M_x = H_A(h+y) - Rx - \frac{wy^2}{2}$, when $x < \frac{l}{2}$ *

$= R(l-x) - H_E(h+y)$, when $x > \frac{l}{2}$ *

CASE IVB - UNIFORM HORIZONTAL LOAD VERTICAL (COLUMN) PORTION ONLY



$R_A = R_E = R = \frac{wh^2}{2l}$

$H_A = wh - H_E$ $H_E = \frac{wh}{4N}(5K + 12 + 6Q)$

$M_B = H_A h - \frac{wh^2}{2}$

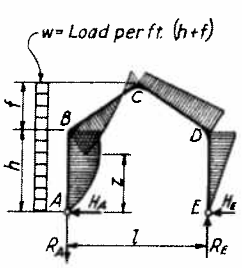
$M_D = -H_E h$

$M_C = \frac{Rl}{2} - H_E(h+f)$

$M_x = R(l-x) - H_E(h+y)$ *

$z = \frac{H_A}{w}$

CASE V - UNIFORM HORIZONTAL LOAD ON BOTH INCLINED AND VERTICAL SURFACES



$R_A = R_E = R = \frac{w(h+f)^2}{2l}$

$H_A = w(h+f) - H_E$

$H_E = \frac{wh}{4N}(5K + 12 + 8KQ + 30Q + 20Q^2 + 5Q^3)$

$M_B = H_A h - \frac{wh^2}{2}$ $M_D = -H_E h$

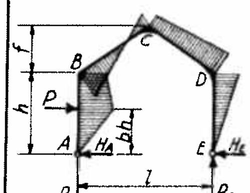
$M_C = \frac{Rl}{2} - H_E(h+f)$

$M_x = H_A(h+y) - Rx - \frac{w(h+y)^2}{2}$, when $x < \frac{l}{2}$ *

$= R(l-x) - H_E(h+y)$, when $x > \frac{l}{2}$ *

$z = \frac{H_A}{w}$

CASE VI - ONE HORIZONTAL CONCENTRATED LOAD AT ANY POSITION ON COLUMN ($b \leq 1.0$)



$R_A = R_E = R = \frac{Pbh}{l}$

$H_A = P - H_E$

$H_E = \frac{Pb}{N}(3K - b^2K + 6 + 3Q)$

$M_B = H_A h - Ph(1-b)$ $M_D = -H_E h$

$M_C = \frac{Rl}{2} - H_E(h+f)$

$M_x = R(l-x) - H_E(h+y)$ *

PARABOLIC AND RECTANGULAR FRAMES

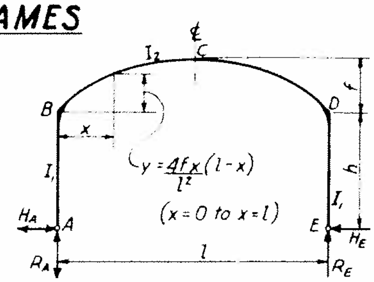
(RECTANGULAR WHEN Q = 0)

General Formulas:

$K = \frac{I_2 h}{I_1 l}$ $Q = \frac{f}{h}$

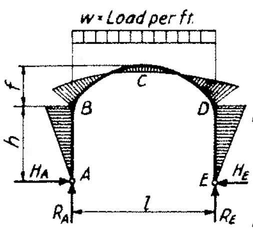
$N = 2(10K + 15 + 20Q + 8Q^2)$

* In formulas for M_x, "x" is always measured from point B.



Plus sign (+) denotes moments which cause tension on the inside of the frame when the vertical loads act downward and the horizontal loads applied to the left side of the frame, act toward the right. The direction of the reactions and the signs of all terms in the moment formulas, are shown correctly for this condition. When the direction of the loads (but not their position) is reversed the direction of the reactions may, and the signs for all moments will, be reversed.

CASE I – UNIFORMLY DISTRIBUTED VERTICAL LOAD ENTIRE SPAN



$R_A = R_E = R = \frac{wl}{2}$

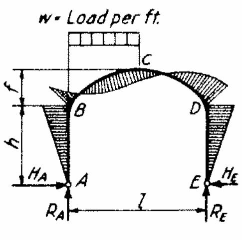
$H_A = H_E = H = \frac{wl^2}{2hN}(5 + 4Q)$

$M_B = M_D = -Hh$

$M_C = \frac{Rl}{4} - H(h + f)$

$M_x = \frac{wx(l - x)}{2} - H(h + y)$ *

CASE IA – UNIFORMLY DISTRIBUTED VERTICAL LOAD HALF SPAN (LEFT)



$R_A = \frac{3wl}{8}$ $R_E = \frac{wl}{8}$

$H_A = H_E = H = \frac{wl^2}{4hN}(5 + 4Q)$

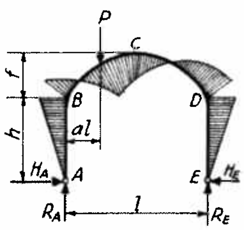
$M_B = M_D = -Hh$

$M_C = \frac{R_E l}{2} - H(h + f)$

$M_x = R_A x - \frac{wx^2}{2} - H(h + y)$, when $x < \frac{l}{2}$ *

$= R_E(l - x) - H(h + y)$, when $x > \frac{l}{2}$ *

CASE II – ONE CONCENTRATED ROOF LOAD AT ANY POSITION



$R_A = P - Pa$ $R_E = Pa$

$H_A = H_E = H = \frac{5Pla}{hN}(3 - 3a + 2Q - 4Qa^2 + 2Qa^3)$

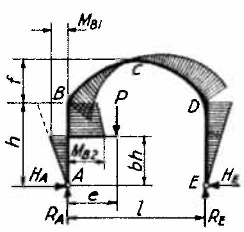
$M_B = M_D = -Hh$

$M_C = \frac{R_E l}{2} - H(h + f)$

$M_x = R_A x - H(h + y)$, when $x < al$ *

$= R_E(l - x) - H(h + y)$, when $x > al$ *

CASE III – BRACKET LOAD ON ONE COLUMN (LEFT COLUMN)



$R_A = P - \frac{Pe}{l}$ $R_E = \frac{Pe}{l}$

$H_A = H_E = H = \frac{5Pe}{hN}(3K - 3b^2K + 3 + 2Q)$

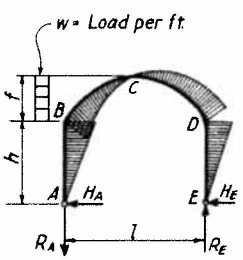
$M_B = Pe - Hh$ $M_D = -Hh$

$M_C = \frac{Pe}{2} - H(h + f)$

$M_{B1} = -Hbh$ $M_{B2} = Pe - Hbh$

$M_x = R_E(l - x) - H(h + y)$ *

CASE IVA – UNIFORM HORIZONTAL LOAD INCLINED (ROOF) PORTION ONLY



$R_A = R_E = R = \frac{wf(2h + f)}{2l}$

$H_A = wf - H_E$

$H_E = \frac{wf}{7N}(70K + 105 + 112Q + 32Q^2)$

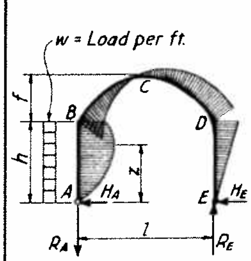
$M_B = H_A h$ $M_D = -H_E h$

$M_C = \frac{Rl}{2} - H_E(h + f)$

$M_x = H_A(h + y) - Rx - \frac{wy^2}{2}$, when $x < \frac{l}{2}$ *

$= R(l - x) - H_E(h + y)$, when $x > \frac{l}{2}$ *

CASE IVB – UNIFORM HORIZONTAL LOAD VERTICAL (COLUMN) PORTION ONLY



$R_A = R_E = R = \frac{wh^2}{2l}$

$H_A = wh - H_E$

$H_E = \frac{5wh}{4N}(5K + 6 + 4Q)$

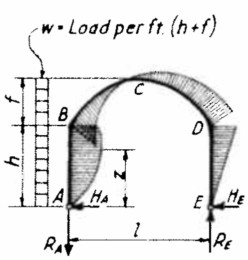
$M_B = H_A h - \frac{wh^2}{2}$ $M_D = -H_E h$

$M_C = \frac{Rl}{2} - H_E(h + f)$

$M_x = R(l - x) - H_E(h + y)$ *

$z = \frac{H_A}{w}$

CASE V – UNIFORM HORIZONTAL LOAD ON BOTH INCLINED AND VERTICAL SURFACES



$R_A = R_E = R = \frac{w(h + f)^2}{2l}$

$H_A = w(h + f) - H_E$

$H_E = \frac{wh}{28N}(175K + 210 + 280KQ + 560Q + 448Q^2 + 128Q^3)$

$M_B = H_A h - \frac{wh^2}{2}$ $M_D = -H_E h$

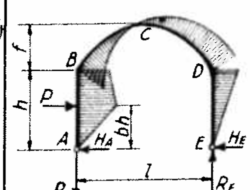
$M_C = \frac{Rl}{2} - H_E(h + f)$

$M_x = H_A(h + y) - Rx - \frac{w(h + y)^2}{2}$, when $x < \frac{l}{2}$ *

$= R(l - x) - H_E(h + y)$, when $x > \frac{l}{2}$ *

$z = \frac{H_A}{w}$

CASE VI – ONE HORIZONTAL CONCENTRATED LOAD AT ANY POSITION ON COLUMN ($b \leq 1.0$)



$R_A = R_E = R = \frac{Pbh}{l}$

$H_A = P - H_E$

$H_E = \frac{5Pb}{N}(3K - b^2K + 3 + 2Q)$

$M_B = H_A h - Ph(1 - b)$ $M_D = -H_E h$

$M_C = \frac{Rl}{2} - H_E(h + f)$

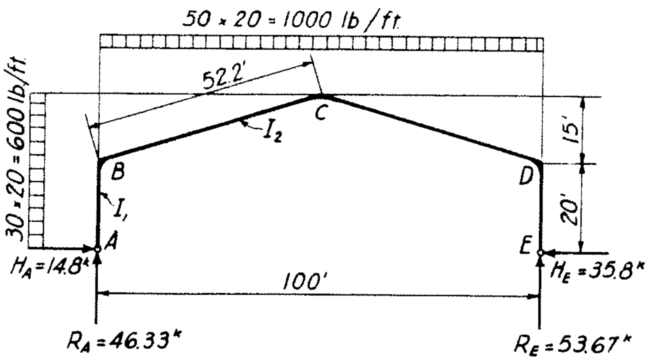
$M_x = R(l - x) - H_E(h + y)$ *

DESIGN PROBLEM #1

Given Data:

See sketch below for dimensions
Spacing of frames 20′-0″ o.c.
Type of sections—Rolled WF's
No increase of section at knee
Vertical load 50 lbs. per sq. ft. total
Wind load 30 lbs. per sq. ft. horiz. projection

Reactions:



$$Q = \frac{15}{20} = 0.75, \text{ Assume } \frac{I_2}{I_1} = 1.0,$$

$$K = \frac{1}{1} \text{ x } \frac{20}{52.2} = 0.38$$

From Chart I, Page 14 $C_1 = 0.059$

$$H_B = 0.059 \text{ x } 1.0 \text{ x } \frac{100^2}{20} = 29.5$$

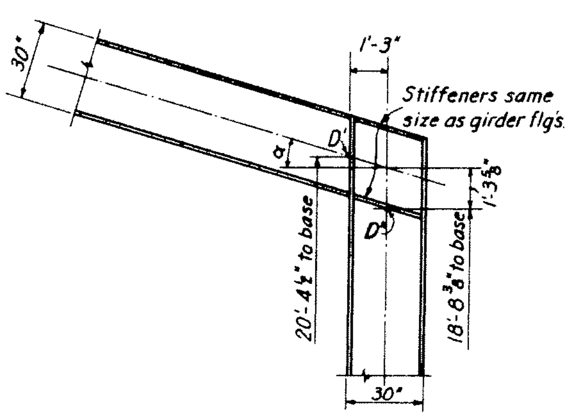
From Chart V, Page 15 $C_6 = 0.525$

$$H_B = 0.525 \text{ x } 0.6 \text{ x } 20 = 6.3$$

$$\text{Total } H_B = 35.8^k$$

Sketch of Knee:

Assume 30 WF Col. and Girder



Girder:

Moment at D′:

With wind:	Without wind:
35.8 x 20.38 = −730	29.5 x 20.38 = −601
$1.0 \text{ x } \frac{1.25^2}{2} = -\ 1$	$1.0 \text{ x } \frac{1.25^2}{2} = -\ 1$
53.67 x 1.25 = + 67	50 x 1.25 = + 63
−664′k	−539′k

Without wind is critical condition.

Axial force at D′:

$$P = 29.5 \cos \alpha + 50 \sin \alpha = 42.6^k$$

Try 30 WF 124

Bracing assumed 15′-0″ o.c.

$$\frac{l}{r} = \frac{15 \text{ x } 12}{2.16} = 83.3, \quad F_a = 13.64 \ ksi$$

$$\frac{ld}{bt} = \frac{15 \text{ x } 12 \text{ x } 30.16}{10.52 \text{ x } 0.93} = 555, \quad F_b = 20.00 \ ksi$$

$$\frac{42.6}{36.45 \text{ x } 13.64} + \frac{539 \text{ x } 12}{354.6 \text{ x } 20} = .086 + .912 = .998$$

o.k.

Moment at Crown:

$$\frac{1.0 \text{ x } 100^2}{8} = +1250$$

$$29.5 \text{ x } 35 = -1032$$

$$+\ 218^{\prime k} \quad \text{o.k.}$$

Column:

Moment at D″:

29.5 x 18.70 = 552′k

Axial force at D″ = 50^k

Try 30 WF 124

Col. assumed braced at mid height.

$$\frac{l}{r} = \frac{10 \text{ x } 12}{2.16} = 55.6, \quad F_a = 15.50 \ ksi$$

$$\frac{ld}{bt} = \frac{10 \text{ x } 12 \text{ x } 30.16}{10.52 \text{ x } 0.93} = 370, \quad F_b = 20.00 \ ksi$$

$$\frac{50}{36.45 \text{ x } 15.50} + \frac{552 \text{ x } 12}{354.6 \text{ x } 20} = .088 + .934 = 1.022$$

A 2.2% overstress would generally be permitted.

Web Shear:

Shear at D′:

$$50 \text{ x } \cos \alpha - 29.5 \sin \alpha = 39.4^k$$

$$\text{Unit shear} = \frac{39.4}{30.16 \text{ x } 0.585} = 2.24 \ ksi$$

Calculated weight of frame (exclusive of base details) = 18,540 lbs.

DESIGN PROBLEM #2

Given Data:

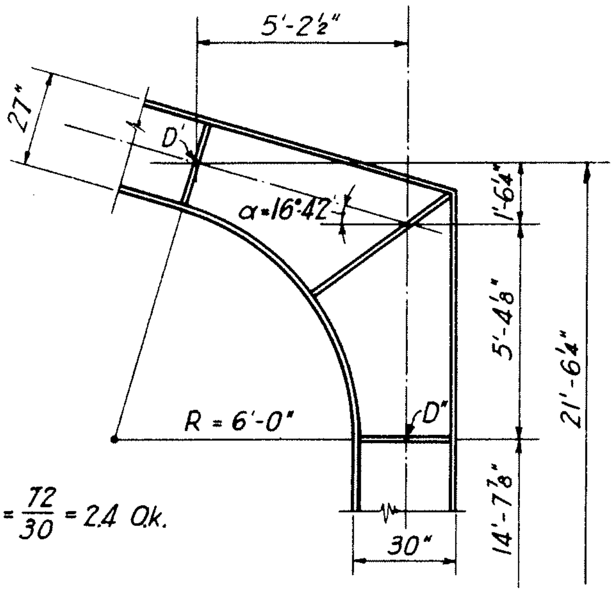
Same as problem #1 except inner flange at knee to be curved on a 6′-0″ radius.

Reactions:

See design problem #1.
Design not controlled by wind.
Increase H_E by 5% (See recommendation Page 9)
$H_E = 1.05 \times 29.5 = 31.0$ *kips*

Sketch of Knee:

Assume 30 WF Col. & 27 WF Girder



Girder:

Moment at D′:

$21.52 \times 31.0 = -667$

$1.0 \times \frac{5.21^2}{2} = -14$

$5.21 \times 50 = +261$

$-420'^k$

Axial force at D′:

$31.0 \cos\alpha + (50-5.21)\sin\alpha = 42.6^k$

Try 27 WF *114*

Bracing assumed 15′-0″ o.c.

$\frac{l}{r} = \frac{15 \times 12}{2.11} = 85.3, \quad F_a = 13.47\ ksi$

$\frac{ld}{bt} = \frac{15 \times 12 \times 27.28}{10.07 \times 0.932} = 523, \quad F_b = 20.00\ ksi$

$\frac{42.6}{33.53 \times 13.47} + \frac{420 \times 12}{299.2 \times 20} = .094 + .842 = .936$ o.k.

Column:

Moment at D″:

$31.0 \times 14.66 = 454'^k$

Axial force $= 50^k$

Try 30 WF *108*

Column assumed braced at mid height.

$\frac{l}{r} = \frac{10 \times 12}{2.06} = 57.8, \quad F_a = 15.35\ ksi$

$\frac{ld}{bt} < 600, \quad F_b = 20.00\ ksi$

$\frac{50}{31.77 \times 15.35} + \frac{454 \times 12}{299.2 \times 20} = .103 + .910$
$= 1.013$
(overstress = 1.3%)

Haunch:

Unit shear stress in web of haunch is low. ⅜″ plate ample for stress. Provide curved flange plates with area at least equal to that of smaller beam flange, say 10½″ x ⅞″ plate.

$\frac{b^2}{Rt} = 1.75$ o.k. (See Rule 7)

Provide radial stiffeners at knee centerline and at points of tangency.

Correction for variation of I's:

The ratio $\frac{I_2}{I_1}$ originally assumed as 1.0 is actually 0.91 on the basis of the sections selected. The Charts indicate that the value of K revised accordingly will increase the horizontal reaction by less than 1%, therefore no change in the design is required.

Calculated weight of frame (exclusive of base details) = 16,780 lbs.

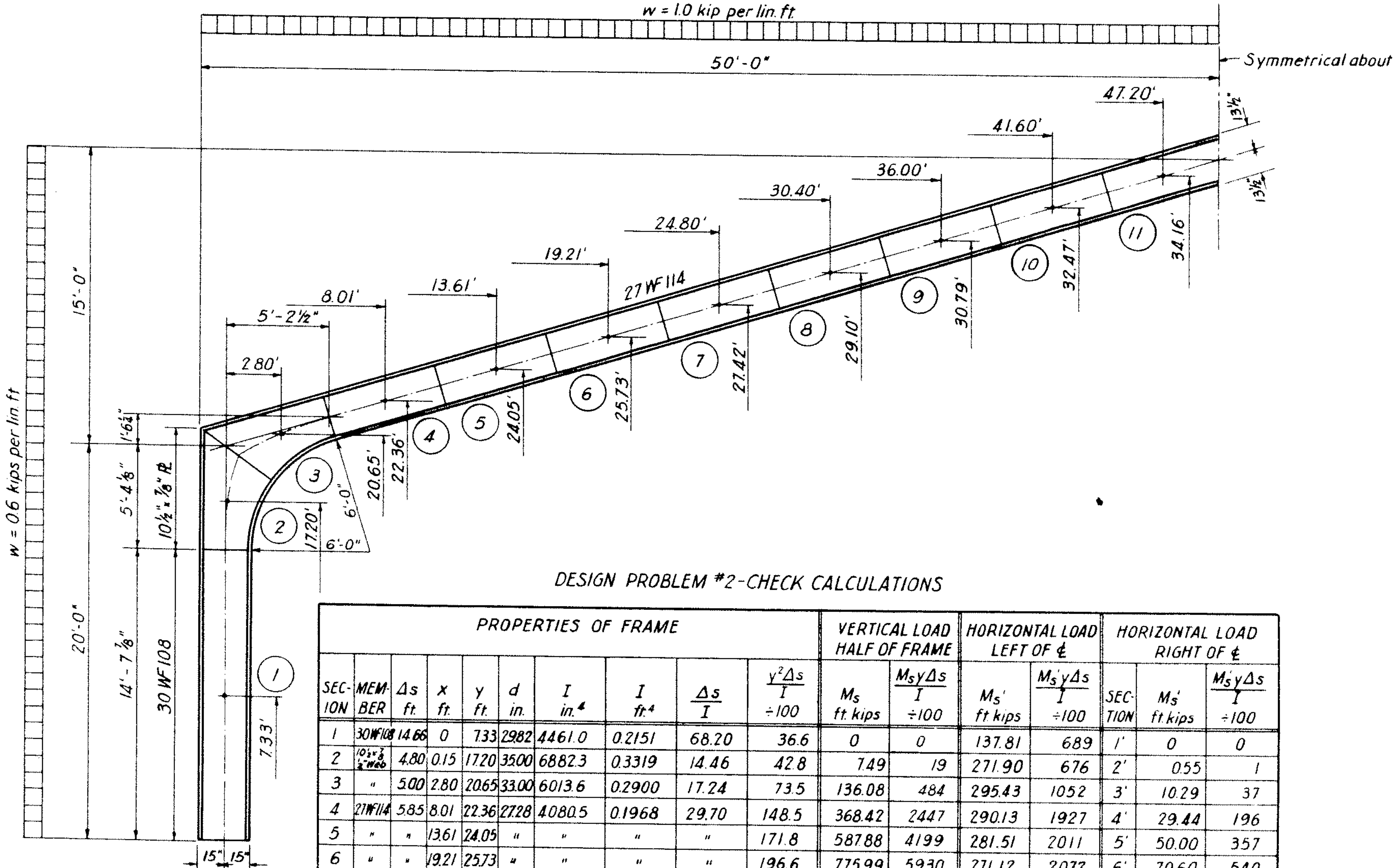
Check by "exact" method:

While it was found that the critical design condition did not involve wind loading, the horizontal reaction, H_E, for both vertical and horizontal loading will be computed by the "exact" procedure for purposes of comparison.

Since the statical moment produced by the vertical loading is symmetrical about the centerline of the frame, the summations of $\frac{M_s\, y\, \Delta s}{I}$ and $\frac{y^2\, \Delta s}{I}$ for half of the frame may be used in computing H_E.

For vertical loading:

$$H_B = \frac{64,707.0}{2,085.9} = 31.02^k$$



DESIGN PROBLEM #2-CHECK CALCULATIONS

PROPERTIES OF FRAME										VERTICAL LOAD HALF OF FRAME		HORIZONTAL LOAD LEFT OF ℄		HORIZONTAL LOAD RIGHT OF ℄		
SEC-TION	MEM-BER	Δs ft.	x ft.	y ft.	d in.	I in.^{4}	I ft.^{4}	$\frac{\Delta s}{I}$	$\frac{y^2\Delta s}{I}$ ÷100	M_s ft. kips	$\frac{M_s y \Delta s}{I}$ ÷100	M_s' ft kips	$\frac{M_s' y \Delta s}{I}$ ÷100	SEC-TION	M_s' ft. kips	$\frac{M_s' y \Delta s}{I}$ ÷100
1	30WF108	14.66	0	7.33	29.82	4461.0	0.2151	68.20	36.6	0	0	137.81	689	1'	0	0
2	10½"x⅞" ½" Web	4.80	0.15	17.20	35.00	6882.3	0.3319	14.46	42.8	7.49	19	271.90	676	2'	0.55	1
3	"	5.00	2.80	20.65	33.00	6013.6	0.2900	17.24	73.5	136.08	484	295.43	1052	3'	10.29	37
4	27WF114	5.85	8.01	22.36	27.28	4080.5	0.1968	29.70	148.5	368.42	2447	290.13	1927	4'	29.44	196
5	"	"	13.61	24.05	"	"	"	"	171.8	587.88	4199	281.51	2011	5'	50.00	357
6	"	"	19.21	25.73	"	"	"	"	196.6	775.99	5930	271.12	2072	6'	70.60	540
7	"	"	24.80	27.42	"	"	"	"	223.3	932.48	7594	259.12	2110	7'	91.14	742
8	"	"	30.40	29.10	"	"	"	"	251.5	1057.92	9143	245.34	2120	8'	111.72	966
9	"	"	36.00	30.79	"	"	"	"	281.6	1152.00	10535	229.88	2102	9'	132.30	1210
10	"	"	41.60	32.47	"	"	"	"	313.1	1214.72	11714	212.70	2051	10'	152.88	1474
11	"	"	47.20	34.16	"	"	"	"	346.6	1246.08	12642	193.83	1967	11'	173.46	1760
									2085.9		64707		18777			7283

18777

For entire frame = 26060

DESIGN PROBLEM #2 (Con'd)

Comparing this value with the 31.0^k value used in the design indicates agreement within 0.1% of error.

In computing the statical moments produced by the horizontal loading the entire horizontal reaction is placed at the column base on the windward side, since the base on the lee side is treated as if free to deflect horizontally. The resulting statical moments are not symmetrical about the centerline of the frame and it is necessary to calculate $\sum \frac{M_s\, y\, \Delta s}{I}$ and $\sum \frac{y^2\, \Delta s}{I}$ for the full frame.

In the table of calculations Sect. 1′, 2′, etc., to the right of the centerline, correspond to Sect. 1, 2, etc., on the left side.

For horizontal loading:

$$H_E = \frac{26{,}060}{2 \text{ x } 2085.9} = 6.25^k$$

which is in complete agreement with the value derived from the charts, within the limits of accuracy required.

Since the expression $\sum \frac{y^2\, \Delta s}{E\, I}$ is an index of the horizontal displacement of point E, with respect to point A, produced by a unit horizontal force at E, it may be used to study the influence of factors not generally included in ordinary design assumptions.

Expressed in inches, the displacement due to a unit force of 1 kip may be written as

$$\Delta = \sum \frac{1000 \text{ x } 12^2 y^2 \text{ x } 12\Delta s}{E \text{ x } 12^4\, I}$$

$$= \sum \frac{83.33\, y^2\, \Delta s}{E\, I}$$

where y, Δs and I are given in feet, as in the table of calculations.

For the frame as designed

$$\Delta = \frac{83.33 \text{ x } 2 \text{ x } 2085.9 \text{ x } 100}{29{,}000{,}000} = 1.20 \text{ in.}$$

Effect of foundation movement:

The roof girders were proportioned for a moment of 420′^{k} at D′. Equating M_C to this moment

$$M_C = 420 = \frac{1.0 \text{ x } 100^2}{8} - 35H_E$$

where H_E = 23.71^k, instead of the 31.02^k reaction produced by the design vertical loading—a decrease of 7.31^k. Then, since 1.20 in. is the equivalent of a 1 kip change in horizontal reaction, it follows that an 8¾ in. displacement (outward) would be required to overstress the 24 WF 114 roof girders at the ridge. Hence, for any ordinary foundation conditions, a tie rod between column bases would be unnecessary as far as stress in the frame is concerned.

Effect of temperature change and frame distortion due to axial forces:

A drop of 60° F in the temperature of the frame would tend to reduce the span length by

$$\Delta_t = .0000067 \text{ x } 60° \text{ x } 100' \text{ x } 12 = .482''$$

The shortening effect of axial stress in the roof girders having an area of 35.01 sq. in. $\left(\frac{33.53}{Cos\ \alpha}\right)$ would be

$$\Delta_{ar} = \frac{31{,}020^{\#} \text{ x } 100' \text{ x } 12}{35.01 \text{ x } 29 \text{ x } 10^6} = .042''$$

The stretching effect of axial stress in a 1¾″ ϕ tie rod would be

$$\Delta_{at} = \frac{31{,}020^{\#} \text{ x } 100' \text{ x } 12}{2.41 \text{ x } 29 \text{ x } 10^6} = .533''$$

The maximum combined effect of these factors would be a decrease in the horizontal reaction of

$$\frac{.482 + .042 + .533}{1.20} = 0.88^k$$

Such a reduction would increase the ridge moment from 164′^{k} to 195′^{k} (18.9%) which is still considerably less than that for which the roof girders were designed.

Only when the frame is fully loaded could the increase in horizontal reaction (produced by a rise in temperature) have any significance at the haunch. But, under full loading the releaving effect here, of frame distortion due to axial stress, more than offsets the effect of the temperature rise.

It will be seen, therefore, that at no point in the frame as designed is a consideration of these factors of any importance.

DESIGN PROBLEM #3

Given Data:

Same as for Example #1. Haunch to be strengthened with straight variable sections as per sketch.

$$\frac{h}{l} = \frac{1}{5} \text{ and } \frac{f}{h} = 0.75, \frac{M_C}{M_D} = \text{about } .25$$

(See Table I)

A girder proportioned for $0.25\,M_D$ would be lighter than desirable on a 100 ft. span. Nevertheless by extending the haunched section to a point where the bending moment is more nearly equal to the mid span moment than at the haunch, an appreciable saving in material may be effected.

Reactions:

$$\text{Assume } K = \frac{1}{2} \times \frac{20}{52.2} = 0.19$$

From Chart I, Page 14 $\quad C_1 = 0.061$

$H_B = 0.061 \times 1.0 \times \frac{100^2}{20} =$	30.5
Increase 5% (See recommendation Page 9)	1.5
	32.0^k

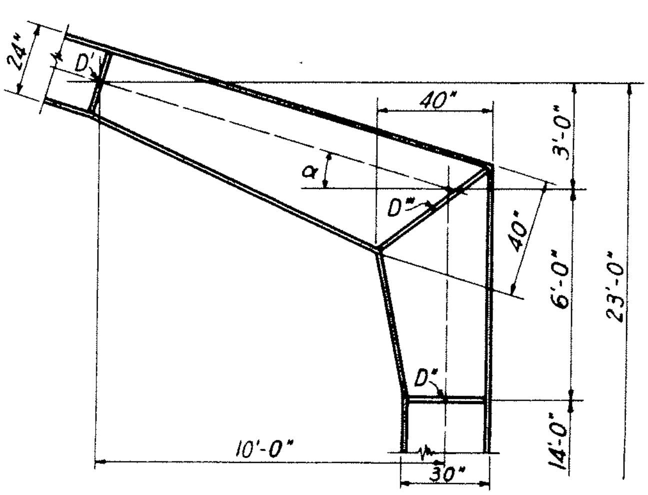
From Chart V, Page 15 $\quad C_6 = 0.52$

$H_B = 0.52 \times 0.6 \times 20 =$	6.2
Total H_B =	38.2^k

Vertical reactions same as for Problem #1

Sketch of Knee:

Assume 30 WF Col. and 24 WF Girder



Girder:

Moment at D'

With wind	Without wind
$38.2 \times 23.0 = -879$	$32.0 \times 23.0 = -736$
$1.0 \times \frac{10^2}{2} = -50$	$1.0 \times \frac{10^2}{2} = -50$
$53.67 \times 10 = +537$	$50.0 \times 10 = +500$
$-392'^k$	$-286'^k$

With wind is critical condition.

Axial force at D':

$38.2 \cos\alpha + (53.7-10) \sin\alpha = 49.1^k$

Try 24 WF 94

Bracing assumed 15'-0" o.c.

$$\frac{l}{r} = \frac{15 \times 12}{1.92} = 94, \quad F_a = 12.72 \times 1.33 = 16.92 \textit{ ksi}$$

$$\frac{ld}{bt} = 15 \times 12 \times 3.07 = 553,$$

$$F_b = 20.0 \times 1.33 = 26.67 \textit{ ksi}$$

$$\frac{49.1}{27.63 \times 16.92} + \frac{392 \times 12}{220.9 \times 26.67} = .105 + .798$$
$$= .903 \quad \text{o.k.}$$

Column:

Without wind critical condition.

Moment at D'

$32.0 \times 14 = 448'^k$

Axial force = 50^k

Try 30 WF 108

Assume braced at midheight.

$$\frac{l}{r} = \frac{10 \times 12}{2.06} = 58.3 \qquad F_a = 15.35 \textit{ ksi}$$

$$\frac{ld}{bt} < 600 \qquad F_b = 20.00 \textit{ ksi}$$

$$\frac{448 \times 12}{299.2 \times 20} + \frac{50}{31.77 \times 15.35} = .898 + .103 = 1.001$$

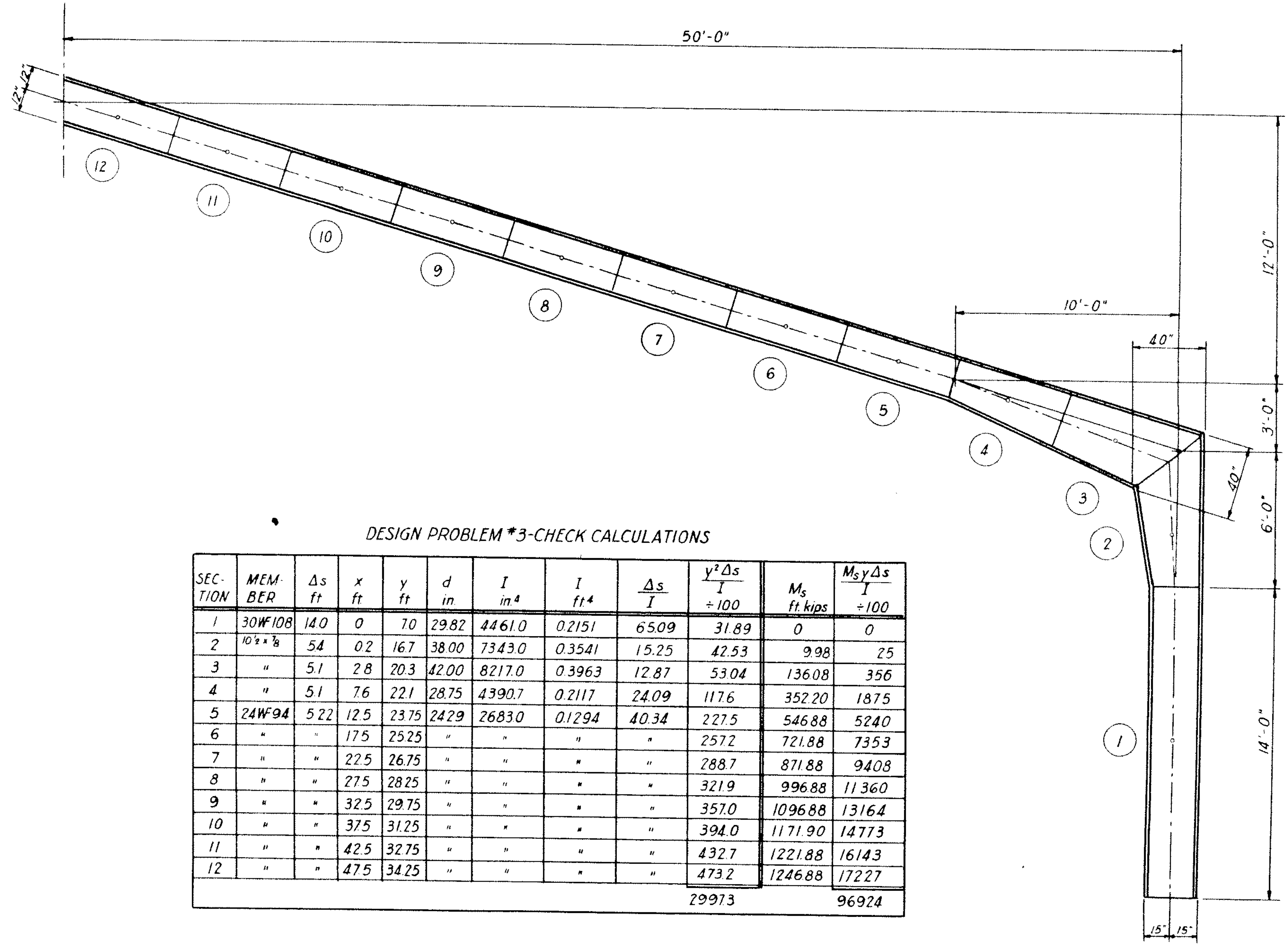
(0.1% overstress o.k.)

Correction for variation of $\frac{I_2}{I_1}$

For the rolled shapes

$$\frac{I_2}{I_1} = \frac{2683}{4461} = 0.60$$

Since 0.5 was assumed for this ratio the calculated horizontal reactions are in error on the "safe side" (by less than 1.0%).



DESIGN PROBLEM #3-CHECK CALCULATIONS

SEC-TION	MEM-BER	Δs ft	x ft	y ft	d in	I in.^{4}	I ft.^{4}	$\frac{\Delta s}{I}$	$\frac{y^2 \Delta s}{I}$ ÷100	M_s ft.kips	$\frac{M_s y \Delta s}{I}$ ÷100
1	30WF108	14.0	0	7.0	29.82	4461.0	0.2151	65.09	31.89	0	0
2	$10\frac{1}{2} \times \frac{7}{8}$	5.4	0.2	16.7	38.00	7343.0	0.3541	15.25	42.53	9.98	25
3	"	5.1	2.8	20.3	42.00	8217.0	0.3963	12.87	53.04	136.08	356
4	"	5.1	7.6	22.1	28.75	4390.7	0.2117	24.09	117.6	352.20	1875
5	24WF94	5.22	12.5	23.75	24.29	2683.0	0.1294	40.34	227.5	546.88	5240
6	"	"	17.5	25.25	"	"	"	"	257.2	721.88	7353
7	"	"	22.5	26.75	"	"	"	"	288.7	871.88	9408
8	"	"	27.5	28.25	"	"	"	"	321.9	996.88	11 360
9	"	"	32.5	29.75	"	"	"	"	357.0	1096.88	13164
10	"	"	37.5	31.25	"	"	"	"	394.0	1171.90	14773
11	"	"	42.5	32.75	"	"	"	"	432.7	1221.88	16143
12	"	"	47.5	34.25	"	"	"	"	473.2	1246.88	17227
									2997.3		96924

$$H_E = \frac{96924}{2997.3} = 32.34^K$$

DESIGN PROBLEM #3 (Con'd)

Design of Haunch:

Moment at D‴ < 32.0 x 20 < 640′ᵏ. Use ½″ web plates and 10½″ x ⅞″ flange plates. (Flange plates may be tapered to 9″ to match 24 WF if desired.)

For $d = 40''$ $I = 9363$, $S = 468$

$$F_b < \frac{640 \text{ x } 12}{468} < 16.4 \; ksi$$

Calculated weight of frame (exclusive of base details) = 15,290#

Check by "exact" method:

Calculated by means of increments $H_B = 32.34^k$, instead of 32.0^k as used in the design—an error (on the "unsafe" side at the haunch) of about 1%. Correcting the design computation at D″ accordingly gives

$$\frac{453 \text{ x } 12}{299.2 \text{ x } 20} + \frac{50}{31.77 \text{ x } 15.35} = .908 + .103 = 1.011$$

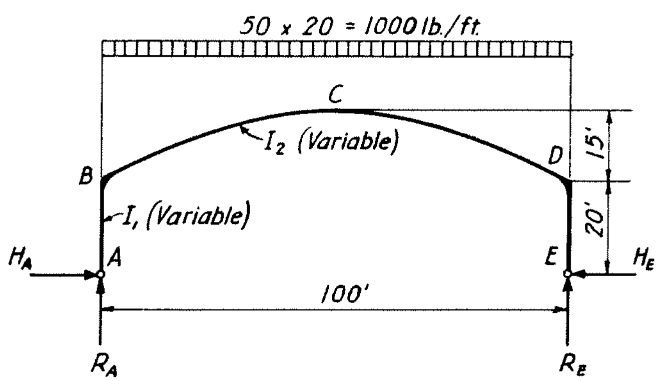
DESIGN PROBLEM #4

Given Data:

See sketch below for dimensions.
Spacing of frames 20′-0″
Type of sections—3 plates welded to provide variable depth I's.
Inner flange curved to 7′-0″ radius at haunches.
Vertical load 50 lbs. per sq. ft. total.

Reactions:

In the following calculations curve BCD will be assumed as a parabola with axis through C. However, since the ratio of rise to span is relatively small (0.15), the sections chosen could be used on the arc of a circle passing through points B, C and D, without appreciable error in the calculations. (See page 9)



$Q = \frac{15}{20} = 0.75$, Assume $\frac{I_2}{I_1} = 1.0$,

$K = \frac{1}{1} \times \frac{20}{100} = 0.2$

From Chart I, Page 16 $C_1 = 0.0548$

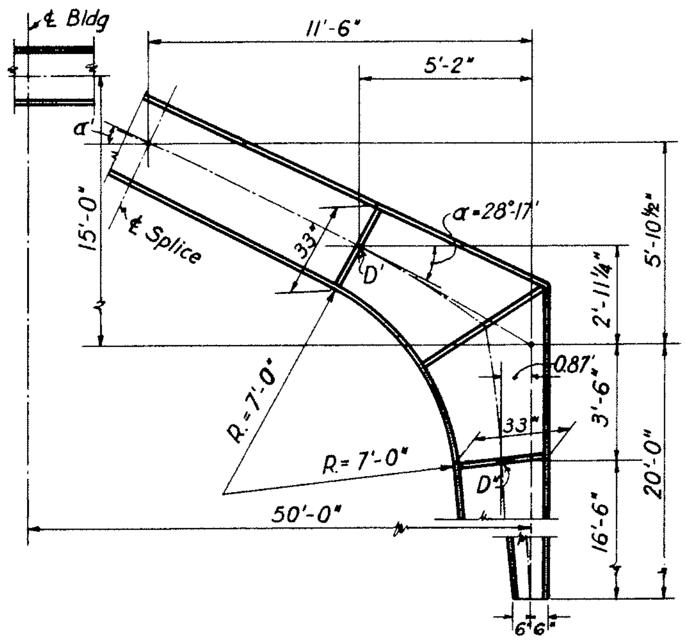
$$H_B = 0.0548 \text{ x } \frac{100^2}{20} = 27.4$$

Add 15% (See Page 9) $= 4.1$

$$H_B = 31.5^k$$

Sketch of Knee:

Assume 33″ depth at points of tangency to 7′-0″ inner flange radius and uniformly varying depths tapered to 20″ at the crown and 12″ at the column base.



⅜″ Web Plate

Flg. Plates	d	A	S_x	r_y
10 x 1	33	31.63	367	2.29
10 x 1	12	23.75		
10 x ¾	31.40	26.21	277	2.16
10 x ¾	20	21.94	159	2.39

DESIGN PROBLEM #4 (Con'd)

Girder:

Moment at D′

$31.5 \times 22.94 = -723$

$1.0 \times \frac{5.2^2}{2} = -\ 14$

$50.0 \times 5.2 = +260$

$-477'^{k}$

Axial force at D′:

$(50.0-5.2)\ \sin\alpha + 31.5\ \cos\alpha = 49.0^{k}$

Bracing assumed 15′-0″ o.c.

Try 10″ x 1″ Flg. Pls, d = 33″

$\frac{ld}{bt} = \frac{15 \times 12 \times 33}{10 \times 1.0} = 594$ o.k. (See Rule 8)

$F_b = 20.00\ ksi$

$\frac{l}{r} = \frac{15 \times 12}{2.29} = 78.6, \qquad F_a = 14.00\ ksi$

$$\frac{477 \times 12}{367 \times 20} + \frac{49.0}{31.63 \times 14.00}$$

$= .780 + .111 = .891$ o.k.

Moment at splice: (Try 11′-0″ from D)

$31.5 \times 25.87 = -815$

$1.0 \times \frac{11}{2} \quad -\ 61$

$50.0 \times 11 = +550$

$-326'^{k}$

Axial force at splice:

$(50.0-11.0)\ \sin\alpha' + 31.5\ \cos\alpha' = 45.1^{k}$

where $\tan\alpha' = \frac{4 \times 15}{l} - \frac{8 \times 11 \times 15}{l^2} = 0.468$

Try 10″ x ¾″ Flg. Pls., d = 31.40″

$\frac{ld}{bt} = \frac{15 \times 12 \times 31.40}{10 \times 0.75} = 754$

$F_b = 15.92\ ksi$

$\frac{l}{r} = \frac{15 \times 12}{2.16} = 83.3, \qquad F_a = 13.64\ ksi$

$$\frac{326 \times 12}{277 \times 15.92} + \frac{45.1}{26.21 \times 13.64} = 1.013$$

Locate splice 11′-6″ from D instead of 11′-0″.

$$\frac{306 \times 12}{277 \times 15.92} + \frac{45.1}{26.21 \times 13.63} = 0.959 \quad \text{o.k.}$$

Moment at Crown:

In computing the moments in the vicinity of the haunch the horizontal reaction, obtained by the charts, was increased 15% to compensate for the varying stiffness of the tapered members, which is greatest in this region. If the percentage correction used is too large the sections chosen for the haunch will be unnecessarily strong. Such is not the case at the crown where the use of too great a correction factor in the given problem would tend to underestimate the moment for which this section should be designed. To provide for this contingency, therefore, the correction factor here will be halved.

$M_C = 1.0 \times \frac{100^2}{8} = +1250$

$27.4 \times 1.075 \times 35 = -1031$

$+\ 219'^{k}$

Axial force at crown:

$27.4 \times 1.075 = 29.5^{k}$

Try 10″ x ¾″ Flg. Pls., d = 20″

$\frac{ld}{bt} = < 600 \qquad F_b = 20.00\ ksi$

$\frac{l}{r} = \frac{15 \times 12}{2.39} = 75.3 \qquad F_a = 14.25\ ksi$

$$\frac{219 \times 12}{159 \times 20} + \frac{29.5}{21.94 \times 14.25} = 0.920 \quad \text{o.k.}$$

Column:

Moment at D″:

$31.5 \times 16.5 = -520$

$50.0 \times .87 = +\ 44$

$-476'^{k}$

Axial force at D″ = 50^{k}

Column assumed braced at mid-height.

Try 10″ x 1″ Flg. Pls., d = 33″

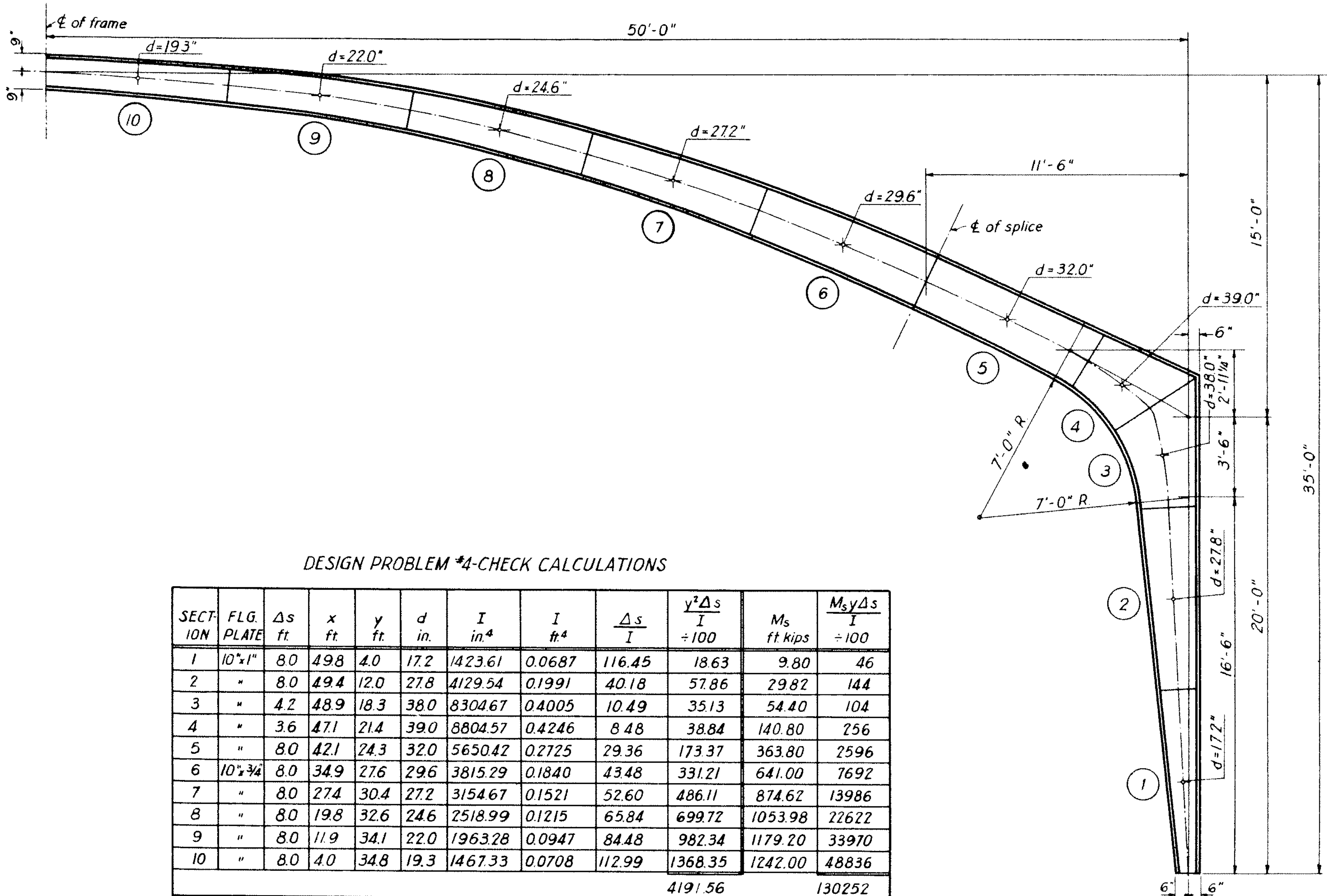
$\frac{ld}{bt} < 600 \qquad F_b = 20.00\ ksi$

$\frac{l}{r} = \frac{10 \times 12}{2.29} = 52.4 \qquad F_a = 15.67\ ksi$

$$\frac{476 \times 12}{367 \times 20} + \frac{50}{31.63 \times 15.67} = 0.879 \quad \text{o.k.}$$

Direct stress at column base:

$$\frac{50.0}{23.75} = 2.11\ ksi$$



DESIGN PROBLEM #4-CHECK CALCULATIONS

SECTION	FLG. PLATE	Δs ft.	x ft.	y ft.	d in.	I in.⁴	I ft.⁴	$\frac{\Delta s}{I}$	$\frac{y^2 \Delta s}{I} \div 100$	M_s ft.kips	$\frac{M_s y \Delta s}{I} \div 100$
1	10"x1"	8.0	49.8	4.0	17.2	1423.61	0.0687	116.45	18.63	9.80	46
2	"	8.0	49.4	12.0	27.8	4129.54	0.1991	40.18	57.86	29.82	144
3	"	4.2	48.9	18.3	38.0	8304.67	0.4005	10.49	35.13	54.40	104
4	"	3.6	47.1	21.4	39.0	8804.57	0.4246	8.48	38.84	140.80	256
5	"	8.0	42.1	24.3	32.0	5650.42	0.2725	29.36	173.37	363.80	2596
6	10"x¾"	8.0	34.9	27.6	29.6	3815.29	0.1840	43.48	331.21	641.00	7692
7	"	8.0	27.4	30.4	27.2	3154.67	0.1521	52.60	486.11	874.62	13986
8	"	8.0	19.8	32.6	24.6	2518.99	0.1215	65.84	699.72	1053.98	22622
9	"	8.0	11.9	34.1	22.0	1963.28	0.0947	84.48	982.34	1179.20	33970
10	"	8.0	4.0	34.8	19.3	1467.33	0.0708	112.99	1368.35	1242.00	48836
									4191.56		130252

$$H_E = \frac{130252}{4191.56} = 31.08^{K}$$

DESIGN PROBLEM #4 (Con'd)

Haunch:

$$Tan\ \alpha \text{ (at D)} = \frac{4f}{l} = 0.600 \qquad \alpha = 31^\circ$$

$$\phi = \frac{90^\circ - 31^\circ}{2} = 29^\circ - 30'$$

$$\frac{R}{d} = \frac{84}{33} = 2.55 > 1.6 \quad \text{o.k.}$$

(See Fig. 4, p. 10)

$$\frac{b^2}{Rt} = \frac{10 \text{ x } 10}{84 \text{ x } 1.0} = 1.19 < 2.0 \quad \text{o.k.}$$

Calculated weight of frame (exclusive of base details) = 14,020 lbs.

Check Analysis by "exact" method:

In the foregoing calculations the horizontal thrust, derived from a use of the charts, was arbitrarily increased by 15% in computing the moments at the haunch. To "play safe" the horizontal thrust was increased but half of this amount in computing the moment at the crown, since, contrary to the condition at the haunch, the use of too large a correction factor for H will underestimate the moment at the crown. In setting up the following analysis the 15% increase will be assumed to be correct, and the girder section at the crown will be designed for the smaller moment.

On this basis

$$M_C = \frac{1.0 \text{ x } 100^2}{8} - 31.5 \text{ x } 35 = +148'^k$$

Try 10" x ¾" Flange Plates, d = 18"

$A = 21.19$ sq. in. $\quad S_x = 140$ in.^{3} $\quad r_y = 2.43$ in.

$$\frac{l}{r} = \frac{15 \text{ x } 12}{2.43} = 74.1, \quad F_a = 14.34 \ ksi$$

$$\frac{ld}{bt} < 600, \qquad F_b = 20.00 \ ksi$$

$$\frac{31.5}{21.19 \text{ x } 14.34} + \frac{148 \text{ x } 12}{140 \text{ x } 20} = 0.738 \quad \text{o.k.}$$

The depth of the roof girder then, will be uniformly varied, from 18" at the crown to 33" at the point of tangency of the 7'-0" radius curve of the inner flange at the haunch. All other features as to the composition of the frame will remain as before.

From the calculations given in the accompanying table it will be seen that the value derived for H, 31.08 kips, is 1% less than the value used in the foregoing computations. (The correct adjustment for H would have been 13.4% instead of 15%.) The calculated moments at the haunch may be reduced by 1%, a refinement which would not justify a redesign of the sections in that portion of the frame.

Using the "exact" value for H, the moment at the crown would be

$$\frac{1.0 \text{ x } 100^2}{8} - 31.08 \text{ x } 35 = +162'^k,$$

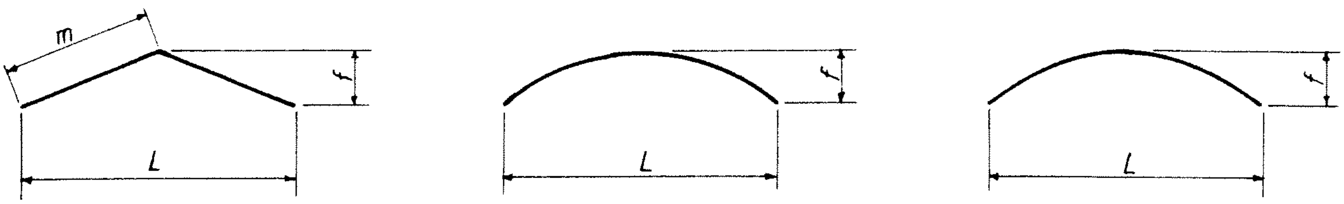
an increase of 9.5% over the value assumed at the beginning of this analysis, but a decrease of 26% from the value assumed in the original design.

Testing the stresses in the revised 18" crown section for the correct moment

$$\frac{31.08}{21.19 \text{ x } 14.34} + \frac{162 \text{ x } 12}{140 \text{ x } 20} = 0.794 \quad \text{o.k.}$$

LENGTH OF ROOF PROFILE, L_1

$L_1 = CL$



Type A	**Type B**	**Type C**
$m = L_1/2$	Circular Arc	Parabolic Arc
$c = \sqrt{4r^2 + 1}$	$c = \dfrac{.01745 \, sin^{-1} x}{x}$ Where $x = \dfrac{4r}{4r^2 + 1}$	$c = \dfrac{1}{2}\left[Y + \dfrac{.5756}{r} \, log_{10} \, (Y + 4r) \right]$ Where $Y = \sqrt{16r^2 + 1}$