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Bracing for Flexural Buckling in Cold-Formed Steel Framed Walls

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Abstract
The objective of this paper is to explore the flexural bracing requirements in cold-formed steel stud walls using an all-steel design philosophy, i.e., bracing that employs mechanical bridging alone, without sheathing. Bracing strength and stiffness demands in cold-formed steel framed walls must be adequate to ensure safety, but not overly conservative so that the requirements cannot be practically met. The current cold-formed steel design specification, AISI-S100-12, requires the brace for a single compression member to have stiffness equal to twice the ideal brace stiffness, but related proposals for braces in multiple stud walls including brace force accumulation and minimum brace stiffness have not yet been adopted. Elastic critical load and second order elastic analyses are conducted herein to determine an adequate level of stiffness for a single braced compression member, and relationships between strength and stiffness for braced multiple studs to that of a single stud. Statistics of measured member imperfections are incorporated to provide an equivalent imperfection for multiple stud walls. Design by second order analysis is utilized to determine how alternating the direction of studs affects strength and stiffness requirements. For a single braced compression member, the impact of allowing a minimum of 4/3 of the ideal brace stiffness, instead of twice, is explored as an alternative to current requirements. New design expressions for brace stiffness and strength, incorporating the notion of a minimum brace stiffness, and the equivalent imperfection, are provided. The new expressions provide the designer with greater flexibility in developing solutions that meet the necessary stiffness and strength.

1. INTRODUCTION

The current cold-formed steel design specification, AISI-S100-12, requires the following brace strength and stiffness for an individual compression member:

$$F_{br,1} = 0.01P_r \tag{1}$$

$$\beta_{br,1} = \frac{2\left[4-(2/n)\right]}{L_b}\left(\frac{P_r}{\phi}\right) \tag{2}$$

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where, $F_{br,1}$ is the required brace strength, $\beta_{br,1}$ is the required brace stiffness, P_r is the required axial compression on the member being braced, ϕ is the resistance factor, L_b is distance between braces, and n is the number of braces along the length of the stud.

The development of the preceding bracing provisions is explained in Sputo and Turner (2006) and in Sputo and Beery (2008). They essentially follow the pioneering work of Winter (1960), and parallel AISC-360-10 and the contributions of Yura, Helwig, (see e.g. Yura and Helwig 2009) and others as summarized in Ziemian (2010). The background of these bracing provisions is discussed in the following section to provide the fundamental expressions from which minor modifications are recommended herein.

2. BACKGROUND

2.1 Illustration of basic bracing behavior

Ideal brace stiffness, β_i, for a single mid-height brace is defined as the brace stiffness such that the elastic critical buckling load (P_{cr}) with the bracing equals pure second mode buckling, i.e.:

$$P_{cr} = \frac{\pi^2 EI}{\left(L/2\right)^2} \tag{3}$$

where E is the material modulus, I the moment of inertia, and L the column length. Increasing the brace stiffness beyond β_i will not increase the elastic buckling load of the column. This is numerically illustrated through an elastic critical buckling load analysis in MASTAN2 (McGuire et al. 2000) as shown in Fig. 1(a). In the example the pin-ended column is modeled as a 362S162-68 [50 ksi] stud (AISI S200-12) with L = 96 in., and E = 29,500 ksi. The brace is modeled as a simple truss element with stiffness:

$$\beta = \frac{EA_{br}}{L_{br}} \tag{4}$$

where L_{br} = 24 in. and E = 29,500 ksi. To find β_i, the brace area, A_{br}, is incremented until the buckling load equals P_{cr} of Eq. (3).

To illustrate how the bracing force evolves a second order (geometric nonlinear) elastic analysis of the column with an initial first mode imperfection of peak magnitude $\Delta_0 = L/1000$ is conducted at varying levels of β as illustrated in Fig. 1(b). Brace forces for an ideal brace $\beta = \beta_i$ are infinite – i.e., the member cannot develop the desired second mode behavior when starting from an initially imperfect geometry. However, developing the second mode P_{cr} is possible for higher brace stiffness, and the brace force required decreases as the brace becomes stiffer.

Practically, the AISI-S100-12 provisions (following current practice), utilize twice the ideal brace stiffness: $2\beta_i$. This brace stiffness insures that the brace forces are minimized and provides a significant tolerance for provided brace stiffness as brace forces demonstrate little change around $2\beta_i$. However, lower brace stiffness, e.g. $1.33\beta_i$ also provides reduced brace forces, and in cases where it is practically infeasible to provide $2\beta_i$ may provide equally adequate bracing.

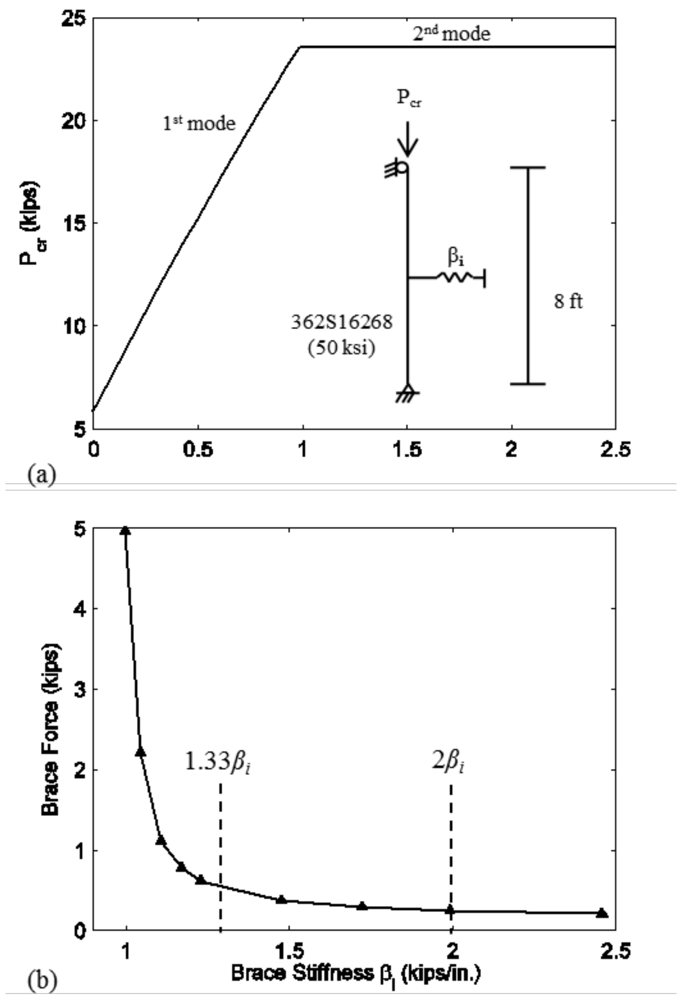


Figure 1: Single stud braced at mid-height (a) buckling load, and (b) brace force, vs. brace stiffness

2.2 Bar spring model

Stiffness and strength requirements for a mid-height brace of a single stud can be derived from a classic bar spring model. Consider the model of Fig. 2(a) with axial load *P*, stud height *L*, brace stiffness β, and lateral deflection of the stud at mid-height Δ. The ideal brace stiffness, β_i, is found by summing the moment about the deformed geometry of the bottom half of the free-body diagram (Fig. 2(b)):

$$P\Delta = \beta \frac{\Delta}{2}\frac{L}{2} \tag{5}$$

and setting the axial compressive load equal to the column buckling load, $P=P_{cr}$, then:

$$\beta_i = \frac{4P_{cr}}{L} \tag{6}$$

To determine the brace force we first note that the force in the brace is simply:

$$F_{br,1} = \beta\Delta \tag{7}$$

Considering the model with initial imperfection Δ_0 (Fig. 2(c)) and again summing the moment in the deformed geometry:

$$P(\Delta_0 + \Delta) = \beta\frac{\Delta}{2}\frac{L}{2} \tag{8}$$

Rearranging to solve for mid-height displacement:

$$\Delta = \frac{P\Delta_0}{\frac{\beta L}{4} - P} \tag{9}$$

Applying Eq. (9), when the critical buckling load ($P = P_{cr}$) is applied to the stud with a brace of stiffness $\beta = \beta_i$, the deflection of the stud at mid-height will be large (theoretically $\Delta = \infty$). When P_{cr} is applied to a stud having initial imperfection Δ_0 with a brace at mid-height of $2\beta_i$, the stud will deflect Δ_0. Using Eq. (7) with Δ_0 = L/1000, the resulting brace force is 0.8% of the applied load:

$$F_{br,1} = 2\beta_i\Delta_0 = \frac{8P}{L}\frac{L}{1000} = 0.8\%P \tag{10}$$

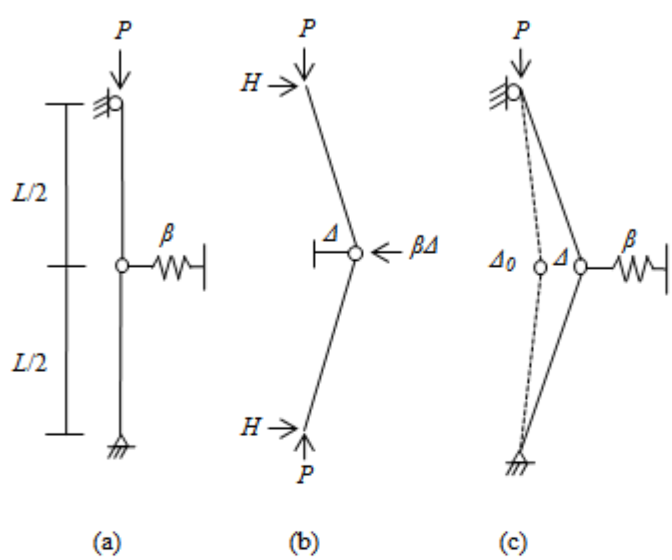


Figure 2: Free body diagrams (a) bar spring model, (b) deformed geometry of an initial straight column, and (c) deformed geometry of a column with initial imperfection

Requiring braces with twice the critical brace stiffness may be conservative. At a brace stiffness of $1.33\beta_i$ the brace forces are double that of a brace with $2\beta_i$ stiffness, but still remain small (refer to Fig. 2(b)). Using Eqs. (9) and (7) with β=$1.33\beta_i$ and P=P_{cr} yields a brace force of 1.6% of the applied load, P:

$$\Delta = \frac{P\Delta_0}{\frac{\beta L}{4} - P} = \frac{P_{cr}\Delta_0}{1.33P_{cr} - P_{cr}} = 3\Delta_0 \tag{11}$$

$$F_{br,1} = 1.33\beta_i 3\Delta_0 = 1.6\%P \tag{12}$$

Decreasing the brace stiffness from $2\beta_i$ to $1.33\beta_i$ doubles the required brace force (from 0.8%P to 1.6%P) but is still manageable. Thus, a designer potentially has some greater flexibility in terms of required stiffness if the new required strength can be accommodated.

2.3 Flexible column model

The bar spring model is an idealization where the column is infinitely stiff. Realistically, the column flexibility plays a role in the solution as well. To illustrate these effects a numerical study with MASTAN2 and the same 326S162-68 stud column is performed. First a column model with only two elements is created to approximate the bar spring model. As the brace stiffness increased the brace force asymptotes to the value predicted by the bar spring model.

Next, the column is modeled with 100 elements to accurately reflect the column bending and the initial half sine wave imperfection. Analyses are completed for various brace stiffness as summarized in Table 1. Even with large brace stiffness, the brace force asymptotically reaches a 33.6% greater brace force than predicted with the two element bar spring model. This is a result of the flexibility of the column; the column deflects 33.6% greater than predicted by the bar spring model, and hence through Eq. (7) the brace force must also increase. A 600S200-118 column was also modeled to study the effects of column stiffness on brace force. It was determined that the 33.6% difference in brace force is independent of column stiffness.

Table 1: Comparison of brace force for various brace stiffness and columns

		362S162-68		600S200-118	
β/β_i	F_{br} (%P) Eq.'s 7,9	F_{br} (%P) MASTAN2	% difference	F_{br} (%P) MASTAN2	% difference
100 element stud					
1.3	1.733	2.336	34.77	–	–
2	0.800	1.072	33.95	–	–
10	0.444	0.594	33.67	0.596	34.01
100	0.404	0.540	33.61	0.541	33.85
1000	0.400	0.535	33.55	–	–
2 element stud					
1.3	1.733	1.397	19.432		
2	0.800	0.728	8.972		
10	0.444	0.440	1.063		
100	0.404	0.405	0.207		

Specifying either $2\beta_i$ or 1.33 β_i yields a true brace force that is approximately 34% greater than the brace force calculated with the bar spring model, Eq. (13), i.e.

$$F_{br,1,true} = 1.34 F_{br,1} \tag{13}$$

Considering Eq. (13) one finds that F_{br} is ~1%P for a 2β_i brace, and ~2%P for a 1.33β_i brace.

3. MULTIPLE STUD WALL WITH MID-HEIGHT BRACING

3.1 Previous Research

Sputo and Beery (2008) conducted a study to determine strength and stiffness requirements for multiple stud walls with identical same imperfections. Using a similar procedure for determining β_i for single studs, brace cross sectional areas (all braces having the same area) were incremented until the wall reached 2nd mode buckling for an elastic critical load analysis. As $2\beta_i$ is required for design, the analysis was re-run with $2\beta_i$ braces and the resulting end brace force recorded.

Sputo and Berry (2008) determined that there is a direct linear relationship between brace force and number of studs:

$$F_{br,n} = \left[n_s\right] F_{br,1} \tag{14}$$

and the required brace stiffness is related to the stiffness of a single stud as:

$$\beta_{br,n} = \beta_{br,1}\left[0.4n_s^2 + 0.5n_s\right] \text{ for } n_s > 1 \tag{15}$$

where n_s is the number of studs in walls braced on one end, $F_{br,n}$ is the maximum brace force, and $\beta_{br,n}$ is the required brace stiffness. In this case $F_{br,1}$ refers to the required brace force of a single braced column as determined through the same structural model.

The above equations are an empirical fit to the results obtained through the elastic critical load analysis. Interestingly, the results do not reflect a general second order elastic analysis. Further, the equations do not shed any light on the mechanics of the brace stiffness and strength requirements for multiple braced studs, thus further exploration is conducted.

3.2 Derivation

Consider a multiple stud wall braced at one end. Brace forces accumulate through the wall system as they approach the end support, theoretically linearly per Fig. 3(a). Each brace behaves as a spring, thus the equivalent stiffness of the system can be calculated as the stiffness resulting from a springs in series:

$$\frac{F}{\beta_{eq}} = \frac{F}{\beta} + \frac{2F}{\beta} + \frac{3F}{\beta} + \frac{4F}{\beta} + \frac{5F}{\beta} \tag{16}$$

Note, that practically the bracing stiffness is a constant β as a single member is used for the bracing (bridging). A more efficient solution would be to use stiffer bracing as the forces accumulate, and this could be accommodated in Eq. 16. Nonetheless, solving Eq. (16) for β_{eq}:

$$\beta_{eq} = \frac{\beta}{(1+2+3+4+5)} = \beta\left(\sum_{i=1}^{n_s} i\right)^{-1} \tag{17}$$

The equivalent stiffness of the wall system is reduced due to the brace force accumulation. Therefore, the required stiffness of braces in a multiple stud wall increases and is related to the required stiffness of a single braced stud, as given in Eq. (18),

$$\beta_{br,n} = \beta_{br,1}\left(\sum_{i=1}^{n_s} i\right) \tag{18}$$

3.3 Comparison with Sputo and Beery proposed expressions

The theoretical expression of Eq. (18) is compared with MASTAN2 analysis (multiple 362S162-68 studs with equal $L/1000$ imperfections) and the empirically derived expression of Sputo and Beery, Eq. (15), in Fig. 4(a). The theoretical expression overly predicts the required brace stiffness, and Eq. (15) better tracks the simulation results. Eq. (15) was empirically fit to MASTAN2 analysis, so the agreement is not a surprise; however, here full second order elastic analysis was conducted to determine the brace stiffness as opposed to elastic critical load analysis used in Sputo and Beery.

The primary error in the theoretical model develops from the assumed force accumulation. Only for infinitely stiff bracing do the forces develop per Fig. 3(a). For practical brace and column stiffness some of this load is transferred into the column supports, and in fact for the studied case the force accumulation is as provided in Fig. 3(b). The error in assuming linear brace force accumulation is modest, as show in Fig. 4(b). Further, assuming linear brace force accumulation is simple and provides a mechanical match between assumed brace forces, Eq. (14), and stiffness, Eq. 18, but does lead to a more conservative formulation.

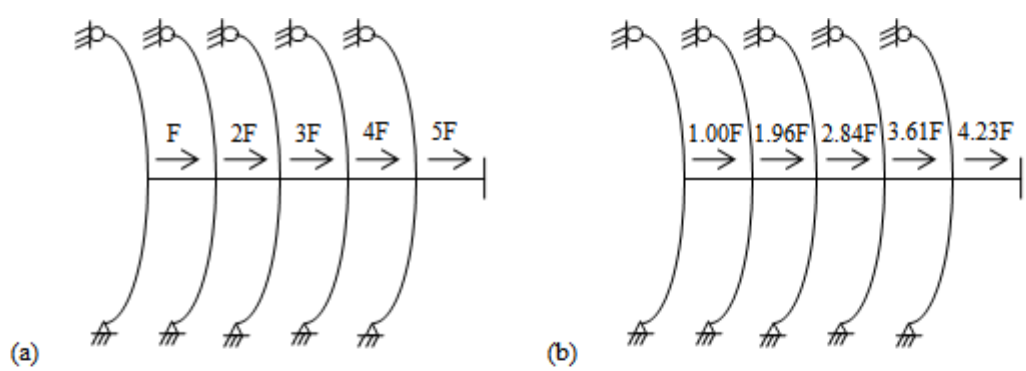


Figure 3: Brace force distribution for a 5 stud wall (a) theoretical with $\infty\beta_i$ studs, and (b) from numerical model with $2\beta_i$ studs, ($F = F_{br,1}$)

3.5 "No" accumulation for +/- L/1000 alternating imperfections

It is important to recognize that the direct accumulation of the bracing force is due to the assumption that all of the studs have maximum imperfections and all imperfections are in the same direction. If one considers the case of "flipped" studs, where the stud orientation is flipped from stud to stud, resulting in + and - imperfections along the stud wall the bracing forces no longer accumulate, as illustrated in Fig. 4(b). The accumulation is only a function of the imperfection, not the brace stiffness, as confirmed by completing analysis at $1.33\beta_i$ as well as $2\beta_i$; neither of which lead to accumulation when "flipped" studs are employed. Note, in addition that this conclusion impacts the required brace stiffness, as the brace force in Eq. (16) now is always just F, so the required bracing stiffness only increases linearly in this case.

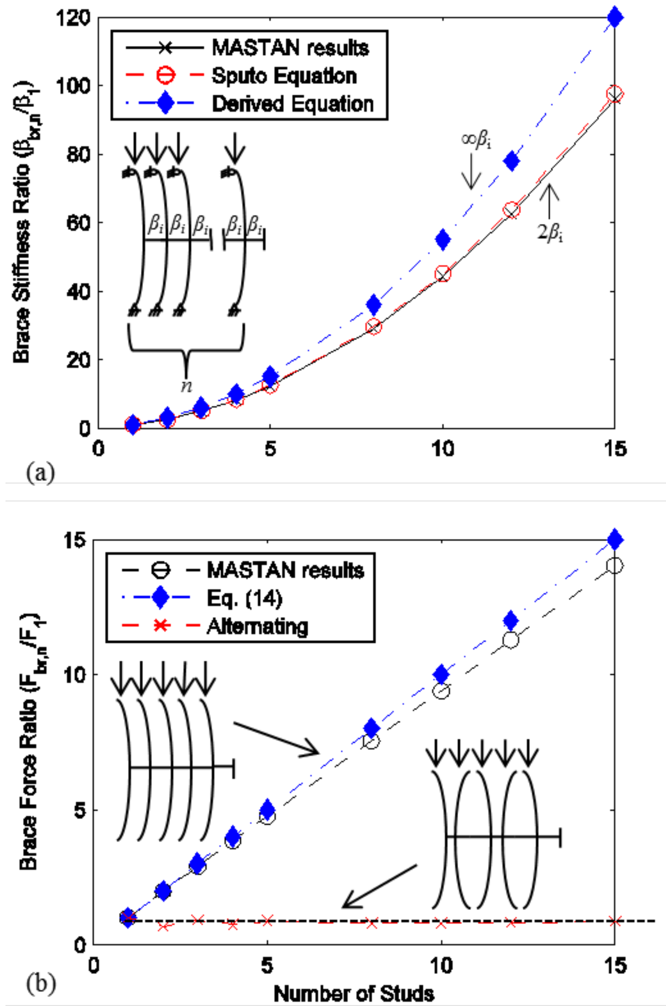


Figure 4: (a) brace stiffness ratio, and (b) brace force ratio, vs. number of wall studs

3.6 Expected accumulation for random imperfections

The use of $L/1000$ for imperfection magnitude is related to the history of maximum measured imperfections in structural steel columns (Ziemian 2010) and has over time been utilized as a manufacturing tolerance in steel studs as well. As detailed in Zeinoddini (2011) expected imperfections in cold-formed steel studs are less than $L/1000$. Based on available data the probability that a random imperfection in a single stud is less than $L/1000$ is 95%.

Now consider a wall with n_s studs. The brace force accumulates based on the actual imperfections. Most of the imperfections (95% of the time!) are considerably less than $L/1000$. For n studs, we seek an imperfection level that has the same confidence level as L/1000 for a single stud. That is, what average level of imperfection can one have 95% confidence won't be exceeded? This was addressed in Zeinoddini (2011) and is found to be:

$$\Delta_0 = 1.69\frac{L/3054}{\sqrt{n_s}} + L/2242 \tag{19}$$

At n=1 (one stud) the equivalent imperfection is $L/1000$ as n increases the result asymptotically approaches the average of the imperfection measurements: $L/2242$.

For a 5 stud wall the equivalent imperfection is $L/1442$. Using Eqs. (7), (9), and (14), the maximum brace force for a 5 stud wall with equivalent imperfection can be compared. With $2\beta_i$ braces and equivalent imperfection of $L/1442$, $F_{br,1}$ = 0.55%P and $F_{br,5}$ = 2.8%P compared with $F_{br,5}$ = 4.0%P for $\Delta_0 = L/1000$. For $1.33\beta_i$ braces and $\Delta_0 = L/1000$, $F_{br,1}$ = 1.1%P and $F_{br,5}$ = 5.5%P, which is 31% less than studs with $\Delta_0 = L/1000$, where $F_{br,5}$ = 8.0%P.

4. RECCOMENDATIONS FOR DESIGN

4.1 Required stiffness and strength for single stud

It is recommended that bracing design utilize slightly more freedom than current approaches. Therefore, it is proposed that the brace stiffness be allowed to be as little as $1.33\beta_i$:

$$\beta_{br,1} \geq 1.33\beta_i = 1.33\left(\frac{4P_r}{L}\right) \tag{20}$$

Further, the actual brace stiffness should be used to determine the brace force (thus higher stiffness is rewarded, but not required). Utilizing Eq.'s (6), (7), and (9) for the simple case of a single mid-height brace this results in:

$$F_{br,1} = 1.34\beta_{br,1}\Delta = 1.34\beta_{br,1}\frac{\Delta_0}{\left(\beta_{br,1}/\beta_i\right)-1} = 1.34\beta_{br,1}\frac{L/1000}{\left(\beta_{br,1}/\beta_i\right)-1} \tag{21}$$

Note, Eq. (21) utilizes the empirically determined "true" brace force by increasing the bar-spring model solution by 1.34. If $\beta_{br,1} = 2\beta_i$ Eq. (21) results in $F_{br,1} = 0.01072P_r$, essentially the same bracing force as current design of Eq. (1). However, in the preceding formulation lower brace stiffness is allowed.

4.2 Required stiffness and strength for wall with multiple studs (anchored on one end only)
Extending the philosophy of bracing for one stud to multiple studs the required bracing stiffness is recommended to be greater than 1.33 time the ideal stiffness, taking into account the increasing forces in the bracing per Eq. (18). The required brace stiffness is

$$\beta_{br,n} \geq 1.33\beta_i \left(\sum_{i=1}^{n_s} i \right) = 1.33\left(\frac{4P_r}{L} \right)\left(\sum_{i=1}^{n_s} i \right) \tag{22}$$

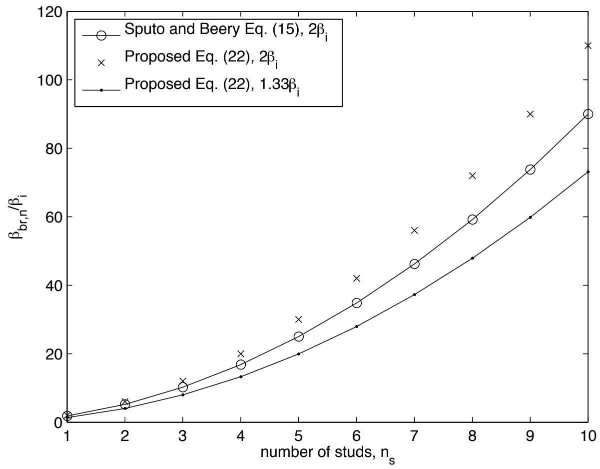
This required stiffness increases quickly. Use of the lower $1.33\beta_i$ provides some relief, but as noted previously, part of the difficulty is the use of the same brace for bracing all studs, which becomes progressively less effective as the brace forces accumulate. To determine the brace force it is first worthwhile to note the equivalent brace stiffness from the multiple stud system, from Eq. (17):

$$\beta_{eq} = \beta_{br,n} \left(\sum_{i=1}^{n_s} i \right)^{-1} \tag{23}$$

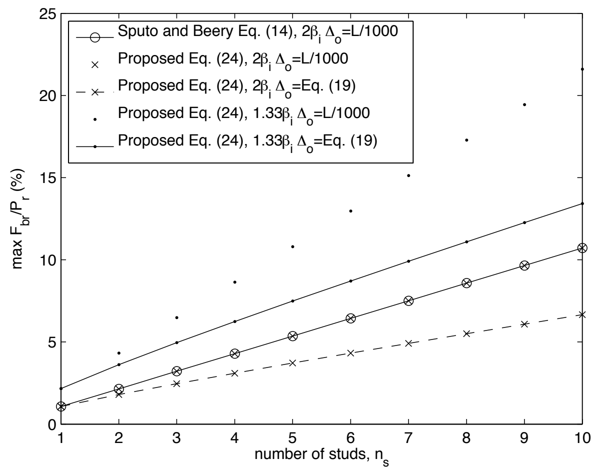
Now, assuming a linear accumulation of the brace force, but taking advantage of the equivalent imperfection size as the number of studs increase (Eq. 19), then the maximum brace force developed at the anchor of the braces is:

$$F_{br,n} = n_s 1.34\beta_{eq}\Delta = 1.34\beta_{eq} \frac{\Delta_0}{\left(\beta_{eq}/\beta_i\right)-1} = 1.34\beta_{eq} \frac{1}{\left(\beta_{eq}/\beta_i\right)-1}\left[\frac{1.69L}{3054\sqrt{n_s}} + \frac{L}{2242} \right] \tag{24}$$

4.3 Discussion
The recommended brace stiffness (Eq. 22) and brace strength (Eq. 24) may be compared to the earlier proposals of Sputo and Beery (2008). The comparison of expressions for brace stiffness are provided in Fig. 5(a) and indicate that use of a lower required brace stiffness may be beneficial in some cases, but the accumulation results in significant challenges in field implementation. The comparisons of required brace forces, as provided in Fig. 5(b), is slightly more complicated as the developed forces are a function of the provided stiffness and the imperfection magnitude. As illustrated in Fig. 5(b), if the minimum brace stiffness ($1.33\beta_i$) is used and it is assumed that the imperfections are all at maximum values (L/1000) the maximum brace force for 10 studs is greater than 20%P_r! If the notion of a statistically equivalent imperfection is employed, i.e., Eq. (19), then the maximum brace force for 10 studs reduces to 13%P_r. If a higher brace stiffness is provided ($2\beta_i$) and Eq. (19) is again used for the initial imperfection, as in the proposed Eq. (24), then the maximum brace force for 10 studs is reduced to 7%P_r. The results indicate that controlling brace forces is realistic and possible (stiffness is a greater challenge). In addition, the results indicate the large range of possible design solutions available to the engineer trying to provide adequate brace stiffness and strength.



(a)



(b)

Figure 5: Comparison of expressions for (a) brace stiffnessa, and (b) brace force, vs. number of wall studs

An important addition in bracing design per AISI-S100 (2012) is the allowance for direct second order elastic analysis, as completed in this paper, to replace Eq.'s (1) and (2). This method can be utilized to great advantage for non-standard cases, such as in the study conducted herein with alternating imperfections. In addition, non-standard bracing configurations, with changing brace numbers, brace stiffness, struts that remove partial brace forces, etc. all can be included directly in the analysis and use to determine brace forces in design.

This paper addresses only stud flexural buckling. Bracing strength and stiffness requirements for discrete bridging in flexural-torsional buckling are not currently available. Direct second order elastic analysis using a beam element that properly accounts for flexural-torsional buckling is one possible design option. In this case Green et al.'s (2006) work provides some estimation of the torsional bracing stiffness available from standard stud bridging details, and Zeinoddini (2011) provides expected imperfection magnitudes in camber and twist including analogs to Eq. (19) for these imperfections. Direct development of the bracing stiffness and strength in combined buckling modes (like flexural-torsional buckling) can be challenging, see Vieira and Schafer (2013) for a solution to this problem utilizing sheathing bracing that could be extended to discrete bracing.

5. CONCLUSIONS

Bracing strength and stiffness requirements for stability bracing against flexural buckling of a single column or a multiple column wall with a mid-height brace are investigated. Classical bar-spring solutions are compared with direct second order analysis using numerical (beam finite elements) solutions and empirically shown to differ by 34%. The origins of the brace stiffness requirements to be twice the ideal brace stiffness are revisited, and the impact of utilizing lower brace stiffness explored. For multiple stud walls, the accumulation of required bracing stiffness is demonstrated through a simple model of braces in series and assuming linear accumulation of brace force. Initial imperfections have a significant impact on brace forces. For example, it is theoretically shown that for the case of alternating imperfections, brace forces do not accumulate. For multiple stud walls an equivalent imperfection (with the same probability of exceedance in multiple stud walls as $L/1000$ in a single stud) is recommended to replace the use of maximum $L/1000$ imperfections. These findings are drawn together into a recommended method that provides required brace stiffness and strength requirements. The method allows the designer to use as little as 4/3 the ideal brace stiffness, and incorporates the beneficial equivalent imperfection when determined brace forces. The resulting method provides the design greater flexibility in developing an adequate bracing system and complements new provisions for analysis-based calculation of bracing strength and stiffness requirements.

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