Building a Better Grid

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Proposed improvements to 2D steel I-girder bridge analysis.

TWO-DIMENSIONAL ANALYSIS METHODS are a popular choice when it comes to steel bridge design.

They are relatively simple and quick to perform, and there are several commercial software packages that facilitate their use. But many current 2D analysis packages use simplifying assumptions that can significantly reduce the accuracy of their results. Recent research has identified and quantified these limitations and has also led to some proposed improvements that can dramatically increase the accuracy of 2D models.

As part of the recently completed National Cooperative Highway Research Program (NCHRP) Research Project 12-79, 60 different I-girder bridges were analyzed using "1D" (i.e., various approximate analysis methods), 2D and 3D analysis methods, and the results were evaluated to determine the accuracy of the 1D and 2D methods compared to the benchmark 3D analyses. Depending on the geometric complexity of the bridge, the research, published in NCHRP Report 725 (available for free at www.trb.org/nchrp), showed that 2D analysis methods could produce noticeably erroneous results in several response categories when analyzing steel I-girder bridges.

Two basic issues were identified as the primary causes of the inaccuracies:

- Omitting warping stiffness, or more precisely the stiffness due to restraint of warping, when modeling the torsional properties of I-girders
- Incomplete modeling of the stiffness of truss-type cross-frames

Fortunately, these issues can be addressed by simple improvements to the modeling of I-girders and truss-type cross-frames in 2D analysis methods. Incorporating these modeling improvements increases the accuracy of 2D analysis methods dramatically, allowing the extension of the use of these popular and useful analysis methods to a wider range of bridges.

2D Tutorial

To understand these issues, a basic understanding of 2D analysis methods is helpful. In simple terms, in a 2D analysis, the entire bridge is modeled using a two-dimensional array of nodes and line elements. Both the girders and cross-frames are modeled in one horizontal plane using line elements. In a traditional 2D grid analysis, the bridge deck is also effectively modeled in strips as part of the line elements used to model the girders and cross-frames (see Figure 1). There is also a variant 2D analysis method commonly called the plate-and-eccentric beam method, in which the girders and cross-frames are still modeled using line elements, but the deck is modeled using plate or shell elements, offset from the line elements used to model the girders and cross-frames (see Figure 2). Many of the limitations of traditional 2D analysis methods, including those discussed in this article, are associated with the modeling of I-girders and truss-type cross-frames using single line elements.

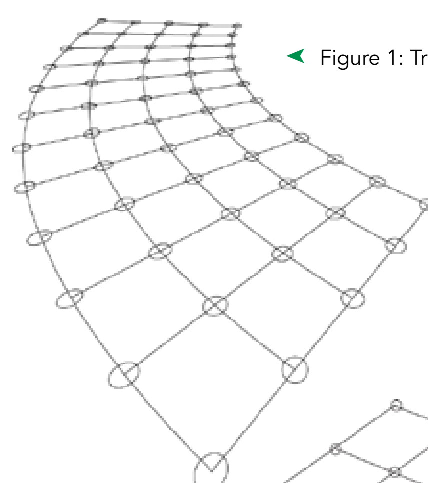


Figure 1: Traditional 2D grid model.

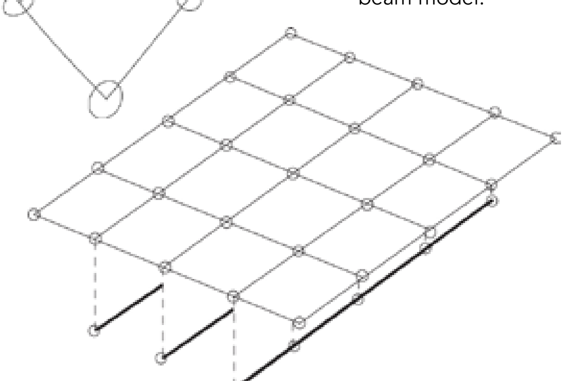


Figure 2: 2D plate-and-eccentric-beam model.

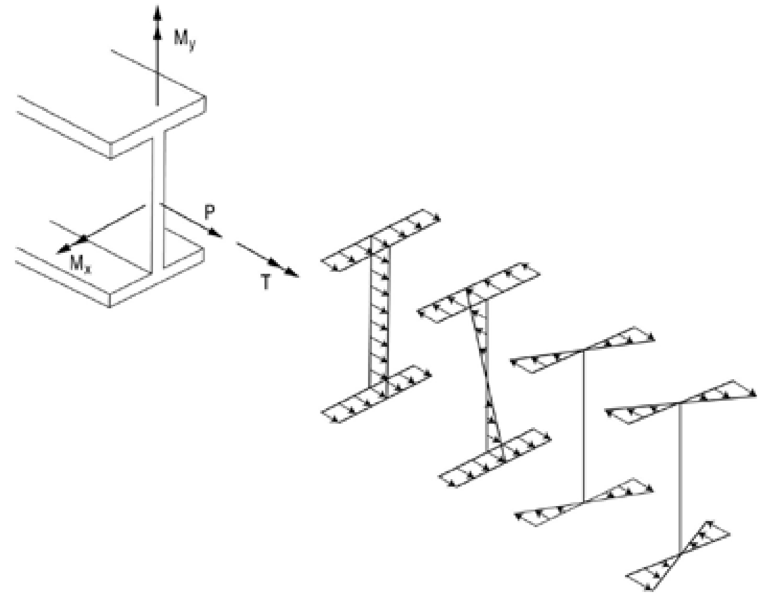
Modeling Torsional Stiffness

Torsion is carried in I-girders by two mechanisms. The first is St. Venant (pure) torsional shear and the second is warping. St. Venant torsional shear is a shear flow around the perimeter of the cross section, while warping involves a cross-bending of the flanges (i.e., bending of the flanges in opposite directions; see Figure 3) in response to torsion.

The St. Venant torsional stiffness of I-girders is relatively low, due largely to their open cross-section geometry. The St. Venant torsional shear flow around the perimeter of the cross section can only develop force couples across the thickness of any given segment of the cross section. Without a significant force couple distance between these shear flows, the ability of I-girders to resist torque via St. Venant torsional response is limited.

Since I-girders have low St. Venant torsional stiffness, they resist torsion primarily by warping. When an I-girder is twisted, longitudinal stresses develop in the girder as the flanges undergo the corresponding cross-bending actions. In fact, the warping of the flanges is a major source of *flange lateral bending* in I-girders; other sources including actions such as lateral wind loads. The separate bending of the flanges in opposite directions is also associated with corresponding shear stresses acting in opposite directions in each of the flanges. These stresses, multiplied by the distance between the flange centroids, produce a couple that is the warping contribution to the girder internal torque.

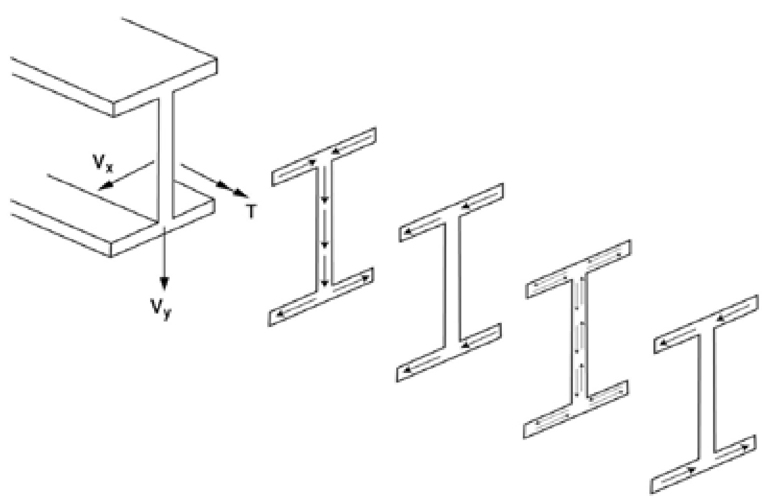
The total state of normal stress in an I-girder is a combination of any axial stress, major-axis bending stress, bending stresses from girder weak-axis moments and warping normal stress (Figure 3). The total state of shear stress in an I-girder is a combination of vertical shear stress, horizontal shear stress, some small amount of St. Venant torsional shear stresses and warping shear stress (Figure 4).



$$\text{Total Normal Stress} = \sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y} + \text{Warping Normal Stress}$$

Figure 3: Primary normal stresses that can occur in an I-girder. Cross-bending of the flanges is illustrated here.

Figure 4: Primary shear stresses that can occur in an I-girder.



$$\text{Total Shear Stress} = \tau = \frac{V_y Q_x}{I_x t} + \frac{V_x Q_y}{I_y t} + \text{St. Venant Torsion} + \text{Warping Torsion}$$

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I-girders in bridges are often subject to torsion. In a curved girder bridge, torsion occurs under vertical loading as a result of the curved alignment of the girders. Torsion can also occur in straight I-girders when the bridge has skewed supports. Since I-girders carry torsion primarily by means of warping (or restraint of warping), omitting the warping stiffness when analyzing an I-girder bridge means that a key stiffness parameter is omitted in the analysis.

The significance of omitting the warping stiffness varies based on the geometric complexity of the bridge. In bridges with significant curvature, significant skew or both, the girders are subject to significant torsional loading, and omitting the warping stiffness can significantly reduce the accuracy of the analysis in several important response categories. The relationship between geometric complexity and the potential inaccuracy of 2D analysis methods was quantified in the NCHRP 12-79 research and presented in a simple scorecard format which will be presented later in this article.

As an example, consider the bridge shown in Figure 5, a three-span curved steel I-girder bridge with skewed intermediate supports. Figure 6 shows a comparison of predicted vertical displacements for one of the girders in this bridge. It can be seen that the traditional 2D analysis (orange dashed line) predicts dramatically different deflections compared to the corresponding 3D analyses of this same bridge (black solid line). The reason is that the traditional 2D analysis omits the warping stiffness in modeling the torsional response of the girders.

To correct this error, the warping stiffness should be considered when modeling I-girders in 2D analysis methods, and there are a number of ways to accomplish this. One approach proposed by the NCHRP 12-79 research team is by means of the development of an equivalent torsional constant, J_{eq}, which includes an estimate of the warping stiffness of the girders. A full derivation is presented in NCHRP Report 725; the resulting equations for J_{eq} are presented below. For the case of a portion of an I-girder between two cross-frames within a continuous portion of a span, the equation for J_{eq} is:

$$J_{eq(fx-fx)} = J\left[1-\frac{\sinh(pL_b)}{pL_b}+\frac{[\cosh(pL_b)-1]^2}{pL_b\sinh(pL_b)}\right]^{-1}$$

where L_b is the length between the cross-frame locations, $p=\sqrt{\frac{GJ}{EC_w}}$, GJ is the physical St. Venant torsional rigidity of the girder cross section, and EC_w is the warping rigidity of the girder cross section.

For the case of a portion of an I-girder between two cross-frames at the end of a span, the equation for J_{eq} is:

$$J_{eq(s-fx)} = J\left[1-\frac{\sinh(pL_b)}{pL_b\cosh(pL_b)}\right]^{-1}$$

By using J_{eq}, the accuracy of a 2D analysis can be dramatically improved, as we'll discuss later.

Modeling Truss-Type Cross-Frame Stiffness

Another limitation of many current 2D analysis packages is tied to how truss-type cross-frames are modeled. A truss-type cross-frame is an open-web structure featuring a bottom chord, diagonals and possibly a top chord. Many current 2D analysis packages make significant simplifying assumptions when modeling the structural response of these truss structures as a single line element.

One method commonly used to determine the "equivalent" stiffness of a cross-frame modeled as a line element is by calculating an equivalent flexural stiffness. In this approach, the truss-type cross-frame is modeled separately, and a unit force couple is applied to one end. Deflections in the direction of loading are calculated and used to determine an equivalent end rotation. The equivalent end rotation and unit force cou-

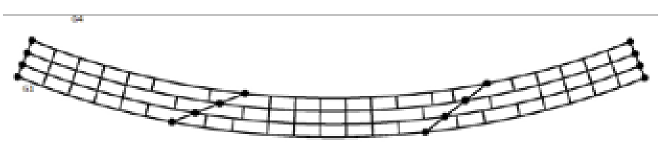
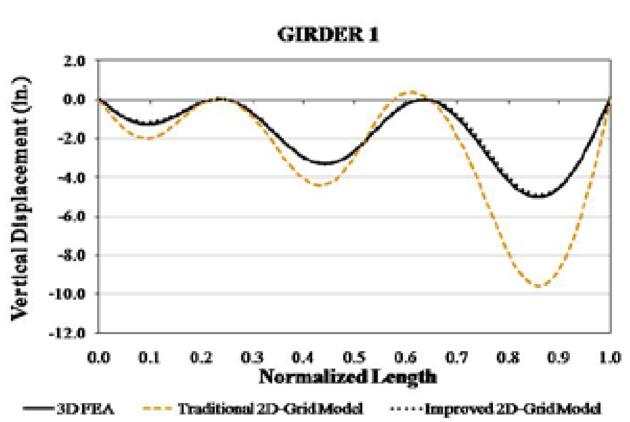
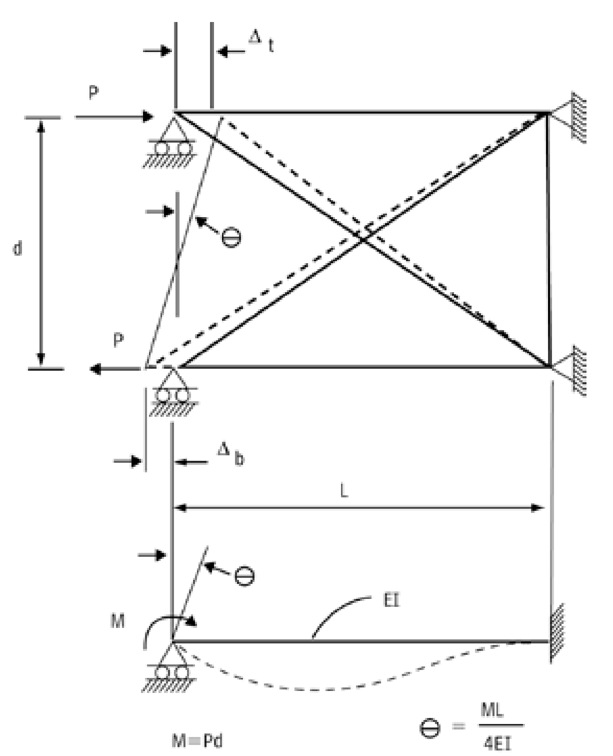


Figure 5: Example three-span curved girder bridge with skewed interior supports.

Figure 6: Comparison of vertical displacement predictions from a 3D analysis, a traditional 2D grid analysis and an improved 2D grid analysis for a girder in the bridge shown in Figure 6.

Figure 7: Current "flexure stiffness method" for approximating the stiffness of a truss-type cross-frame.





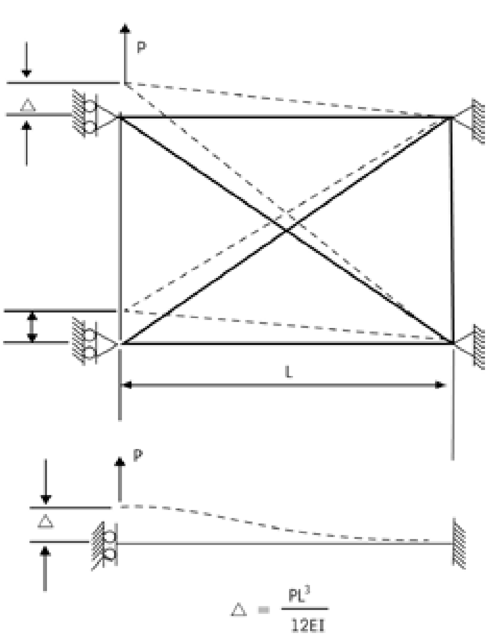
ple are then analyzed as a propped cantilever with Euler-Bernoulli beam theory (i.e., no consideration of shear deformations) to back-calculate the associated equivalent moment of inertia of the cross frame. This moment of inertia is then used as the primary stiffness property of the Euler-Bernoulli line element used in the grid analysis to model the cross frame stiffness (Figure 7, previous page).

A second method is to calculate equivalent shear stiffness. In this approach, the truss-type diaphragm is modeled separately, and a unit vertical force is applied to one end. Vertical deflections are calculated and are used as a transverse deflection to back-calculate the associated shear racking stiffness of the cross frame using an Euler-Bernouli beam element. This is then used as the primary stiffness property of the line element used in the grid analysis to model the cross frame stiffness (Figure 8).

Neither method represents the true stiffness of a truss-type cross-frame, and many key stiffness parameters are omitted from consideration. Also, neither approach considers that there are significant equivalent beam flexure and beam shear deformations in both of the above figures.

For a straight bridge with little or no skew, the effect of these simplified approximations of cross-frame stiffness are negligible since the cross-frames play a relatively insignificant role in the distribution

Figure 8: Current "shear stiffness method" for approximating the stiffness of a truss-type cross-frame.



of load through the structural framing of the bridges. But in bridges with significant curvature and/or skew, the girders and cross-frames function together as a system, and the cross-frames play a significant role in the distribution of load through the structural framing. For these types of bridges, incorrect representation of cross-frame stiffnesses can result in incorrect calculation of the loads in the girders and cross-frames.

NCHRP Report 725 provides a complete discussion of this issue and recommends two alternative approaches that can be implemented to provide improved estimates of the stiffness of truss-type cross-frames in 2D models:

1) An improved approximation using shear-deformable (Timoshenko) beam element representation of the cross-frame
2) An "exact" beam element representation of the truss-type cross-frame based on virtual work concepts, and implemented via user-defined beam elements

The shear-deformable (Timoshenko) beam approach simply involves the calculation of an equivalent moment of inertia, I_{eq}, as well as an equivalent shear area A_{seq} for a shear-deformable (Timoshenko) beam element representation of the cross-frame. In this approach, the equivalent moment of inertia is determined first, based on pure flexural deformation of the cross-frame (with zero shear). The cross-frame is supported as a cantilever at one end and is subjected to a force couple applied at the corner joints at the other end, producing constant bending moment. The associated horizontal displacements are determined at the free end of the cantilever, and the corresponding end rotation is equated to the value from the beam pure flexure solution $M/(EI_{eq}/L)$.

In the second step of the improved calculation, using an equivalent Timoshenko beam element rather than an Euler-Bernoulli element, the cross-frame is still supported as a cantilever but is subjected to a unit transverse shear at its tip. Using the equivalent moment of inertia, I_{eq}, determined from the beam pure flexure solution, and the calculated deflection under a unit shear load, the equivalent shear area is found by solving this equation:

$$\Delta = VL^3 / 3EI_{eq} + VL / GA_{seq}.$$

Impact of Proposed Improvements

By incorporating the above two proposed improvements, the accuracy of 2D analysis methods can be significantly improved, to the point where their results correlate much better with 3D analysis results for a wide range of bridge geometries. Table 1 shows "scores" for 1D analysis methods, traditional 2D analysis methods and the improved 2D analysis methods. These letter grade scores represent the error indices for the various methods, categories of structural response and bridge geometries. A letter grade of "A" indicates the 1D or 2D method exhibits up to 6% error when compared to a 3D analysis. A "B" indicates 7% to 12% error, a "C" indicates 12% to 20% error, a "D" indicates 20% to 30% error and an "F" indicates greater than 30% error when compared to a 3D analysis.

It can be seen from the table (opposite page) that traditional 2D analysis methods produce reasonably accurate results for a fairly limited range of cases, but the improved 2D methods produce reasonably accurate results for a wide range of cases.

As a specific example of the improvements of the accuracy of 2D methods, consider the three-span curved girder bridge with skewed interior supports previously shown in Figure 5. As mentioned previously, Figure 6 shows the predictions of vertical displacements for a girder in this bridge. The traditional 2D grid analysis (orange dashed line) shows poor correlation with the 3D analysis results (black solid line), but the improved 2D grid analysis (black dashed line) shows very good correlation.

These proposed improvements to 2D analysis methods will be presented in further detail in the upcoming 2nd Edition of the AASHTO/NSBA Steel Bridge Collaboration Guideline G13.1, *Guidelines for Steel Girder Bridge Analysis* and are presented in full detail in NCHRP Report 725.

The hope of the NCHRP 12-79 research team is that these proposed improvements to 2D analysis methods will be incorporated into many of the commercial 2D steel bridge design software packages in the near future. MSC

Table 1: Recommended Levels of Analysis, I-Girder Bridges, Non-Composite Dead Load Analysis Models

Response	**Geometry**	**Worst-Case Scores**			**Mode of Scores**		
		Traditional 2D-Grid	ID-Line Girder	Improved 2D-Grid[g]	Traditional 2D-Grid	ID-Line Girder	Improved 2D-Grid[g]
Major-Axis Bending Stresses	C ($I_c \leq 1$)	B	B	A	A	B	A
	C ($I_c > 1$)	D	C	A	B	C	A
	S ($I_s < 0.30$)	B	B	A	A	A	A
	S ($0.30 \leq I_s < 0.65$)	B	C	A	B	B	A
	S ($I_s > 0.65$)	D	D	A	C	C	A
	C&S ($I_c > 0.5$&$I_s > 0.1$)	D	F	A	B	C	A
Vertical Displacements	C ($I_c < 1$)	B	C	A	A	B	A
	C ($I_c > 1$)	F	D	A	F	C	A
	S ($I_s < 0.30$)	B	A	A	A	A	A
	S ($0.30 \leq I_s < 0.65$)	B	B	A	A	B	A
	S ($I_s \geq 0.65$)	D	D	A	C	C	A
	C&S ($I_c > 0.5$&$I_s > 0.1$)	F	F	A	F	C	A
Cross-Frame Forces	C ($I_c \leq 1$)	C	C	B	B	B	A
	C ($I_c > 1$)	F	D	B	C	C	A
	S ($I_s < 0.30$)	NA[a]	NA[a]	B	NA[a]	NA[a]	A
	S ($0.30 \leq I_s < 0.65$)	F[b]	NA[c]	B	F[b]	NA[c]	A
	S ($I_s \geq 0.65$)	F[b]	NA[c]	B	F[b]	NA[c]	A
	C&S ($I_c > 0.5$&$I_s > 0.1$)	F[b]	NA[c]	B	F[b]	NA[c]	A
Flange Lateral Bending Stresses	C ($I_c \leq 1$)	C	C	C	B	B	B
	C ($I_c > 1$)	F	D	C	C	C	B
	S ($I_s < 0.30$)	NA[d]	NA[d]	NA[d]	NA[d]	NA[d]	NA[d]
	S ($0.30 \leq I_s < 0.65$)	F[b]	NA[e]	C	F[b]	NA[e]	B
	S ($I_s > 0.65$)	F[b]	NA[e]	C	F[b]	NA[e]	B
	C&S ($I_c > 0.5$&$I_s > 0.1$)	F[b]	NA[e]	C	F[b]	NA[e]	B
Girder Layover at Bearings	C ($I_c < 1$)	NA[f]	NA[f]	NA[f]	NA[f]	NA[f]	NA[f]
	C ($I_c > 1$)	NA[f]	NA[f]	NA[f]	NA[f]	NA[f]	NA[f]
	S ($I_s < 0.30$)	B	A	A	A	A	A
	S ($0.30 \leq I_s < 0.65$)	B	B	A	A	B	A
	S ($I_s \geq 0.65$)	D	D	B	C	C	A
	C&S ($I_c > 0.5$&$I_s > 0.1$)	F	F	B	F	C	A

Notes:

[a] Magnitudes should be negligible for bridges that are properly designed and detailed. The cross-frame design is likely to be controlled by considerations other than gravity-load forces.

[b] Results are highly inaccurate due to modeling deficiencies addressed in Ch. 6 of the NCHRP 12-79 Task 8 report. The improved 2D-grid method discussed in this Ch. 6 provides an accurate estimate of these forces.

[c] Line-girder analysis provides no estimate of cross-frame forces associated with skew.

[d] The flange lateral bending stresses tend to be small. AASHTO Article C6.10.1 may be used as a conservative estimate of the flange lateral bending stresses due to skew.

[e] Line-girder analysis provides no estimate of girder flange lateral bending stresses associated with skew.

[f] Magnitudes should be negligible for bridges that are properly designed & detailed.

[g] The improved 2D-grid method requires the use of an equivalent St. Venant torsion constant, which estimates the influence of the girder warping response on the torsional stiffness, as well as a Timoshenko beam cross-frame model that accounts for both the shear and bending flexibility of the cross-frames. See Articles 3.11 and 3.12 of the NCHRP 725 Report for detailed discussions of these improvements. In addition, the improved 2D-grid method is limited to the analysis of systems with at least two girders connected by enough cross-frames such that I_c is less than or equal to 20.