The question and answers for this month's Steel Quiz were contributed by Hamza Sekkak, a PhD student at the Illinois Institute of Technology. Thank you, Hamza!

Refer to Figure 1. Given that weld size, $a$, is $5 / 16 \mathrm{in}$. and $F_{E X X}$ is 70 ksi , solve for weld available strength, $\phi R_{n}$, using:

1 Table 8-4
2 Instantaneous center of rotation method
3 Elastic method
4 Plastic method


1 Using Table 8-4. $k=0$, because the force applied is out-ofplane with regard to the cross-sectional plane of the plate. From Table 8-4:

$$
\begin{aligned}
C & =1.84 \\
\phi R_{n} & =\phi C C_{1} D I=(0.75)(1.84)(1.0)(5)(9)=62.1 \mathrm{kips}
\end{aligned}
$$

2 Using the instantaneous center of rotation method. Break half of the weld length into equal segments (see Figure 2).

Figure 2


Select a trial location for the instantaneous center of rotation, ro. Compute coordinates of the centroids of the segments and their angles. Compute the deformations $\Delta_{m i}$ and $\Delta_{u i}$ using the following equations:

$$
\begin{aligned}
\Delta_{m i} & =0.209\left(\theta_{i}+2\right)^{-0.32} w \\
\Delta_{u i} & =1.087\left(\theta_{i}+6\right)^{-0.65} w \leq 0.17 w
\end{aligned}
$$

where $\theta_{i}$ is the segment angle in degrees and $w$ is the weld size in in.

Compute $\Delta_{i}$ as follows:

$$
\Delta_{i}=r_{i} \frac{\Delta_{u c r}}{r_{c r i}}=r_{i}(0.0046)
$$

Compute $R_{n}$, the resistance of each segment:

$$
R_{i}=0.60 F_{E X X} t_{e} l_{w}\left(1.0+0.50 \sin ^{1.5} \theta_{i}\right)\left[\frac{\Delta_{i}}{\Delta_{m i}}\left(1.9-0.9 \frac{\Delta_{i}}{\Delta_{m i}}\right)\right]^{0.3}
$$

| Vertical <br> Segments | Length <br> $I_{w}$ (in.) | $X$ (in.) | $Y$ (in.) | $r_{i}$ (in.) | $R_{i}$ <br> $(k i p)$ | $\left(R_{i}\right)_{x}$ <br> $(k i p)$ | $\left(R_{i}\right)_{y}$ <br> $(k i p)$ | $R_{i} r_{i}$ <br> $(k i p-i n)$. |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.5 | 0.82 | 4.25 | 4.329 | 6.87 | 6.75 | 1.31 | 29.75 |
| 2 | 0.5 | 0.82 | 3.75 | 3.839 | 6.89 | 6.72 | 1.48 | 26.44 |
| 3 | 0.5 | 0.82 | 3.25 | 3.353 | 6.82 | 6.61 | 1.68 | 22.86 |
| 4 | 0.5 | 0.82 | 2.75 | 2.871 | 6.66 | 6.38 | 1.91 | 19.13 |
| 5 | 0.5 | 0.82 | 2.25 | 2.396 | 6.41 | 6.02 | 2.20 | 15.35 |
| 6 | 0.5 | 0.82 | 1.75 | 1.934 | 6.02 | 5.44 | 2.56 | 11.64 |
| 7 | 0.5 | 0.82 | 1.25 | 1.497 | 5.44 | 4.54 | 2.99 | 8.14 |
| 8 | 0.5 | 0.82 | 0.75 | 1.114 | 4.56 | 3.07 | 3.38 | 5.09 |
| 9 | 0.5 | 0.82 | 0.25 | 0.861 | 3.34 | 0.97 | 3.20 | 2.88 |
|  |  |  |  |  |  |  |  |  |

Check rotational and force equilibrium until both values become the same ( $r_{0}=0.824 \mathrm{in}$.).

Rotational equilibrium:

$$
R_{n}=\frac{2\left(\sum R_{i} r_{i}\right)}{e+r_{0}}=\frac{2(141.3)}{6.824}=41.4 \mathrm{kips}
$$

Force equilibrium:

$$
R_{n}=2 \sum\left(R_{i}\right)_{y}=41.4 \mathrm{kips}
$$

Finally:

$$
\phi R_{n}=(0.75)(2 \text { weld lines }) \times 41.4 \text { kips }=62.1 \text { kips }
$$

## 3 Using the elastic method.

The moment of inertia of the weld is:

$$
I_{x}=2\left(\frac{9^{3}}{12}\right)=121.50 \mathrm{in}^{3}
$$

Solve for the welding strength from the following:

$$
\begin{aligned}
& \sqrt{\left(\frac{R_{n}}{2 l}\right)^{2}+\left(\frac{R_{n} e c}{I_{p}}\right)^{2}}=\left(0.707 \times \frac{5}{16} \mathrm{in} .\right) 0.6(70 \mathrm{ksi}) \\
& R_{n}=40.5 \mathrm{kips} \rightarrow \phi R_{n}=30.4 \mathrm{kips}
\end{aligned}
$$

## 4 Using the plastic method.

For one weld:

$$
\begin{aligned}
& f_{w}=\sqrt{f_{v}^{2}+\left(f_{a}+f_{b}\right)^{2}} \\
& f_{v}=\frac{R_{n}}{l}, f_{a}=0, f_{b}=\frac{4 M}{l^{2}}=\frac{4 R_{n} e}{l^{2}} \\
& f_{w}=\left(\frac{5}{16} \text { in. }\right)(0.707)(0.6)(70 \mathrm{ksi})
\end{aligned}
$$

For two welds:

$$
R_{n}=58.7 \mathrm{kips} \rightarrow \phi R_{n}=44.0 \mathrm{kips}
$$

Notes: The Steel Quiz submitted by Hamza Sekkak did not account for the directional strength increase when applying the plastic method. Section J2.4.(b) of the AISC Specification for Structural Steel Buildings (ANSI/AISC 360) states: "For fillet welds, the available strength is permitted to be determined accounting for a directional strength increase of (1.0 $+0.50 \sin ^{1.5} \theta$ ) if strain compatibility of the various weld elements is considered." Though not explicitly addressed in the use of the plastic method, strain compatibility will likely not be a problem for the condition shown. This is a matter of engineering judgment. If the directional strength increase is to be included, it can be done as follows:

$$
\begin{aligned}
& \theta=\operatorname{atan}\left(\frac{f_{b}}{f_{v}}\right)=\operatorname{atan}\left(\frac{4 e}{l}\right)=\operatorname{atan}\left(\frac{4(6)}{9}\right)=69.4^{\circ} \\
& {\left[1.0+0.50 \sin ^{1.5}\left(69.4^{\circ}\right)\right]=1.4}
\end{aligned}
$$

Accounting for the directional strength increase $\phi R_{n}=63.9$ kips, this is within $3 \%$ of the strength predicted by the instantaneous center of rotation method, which explicitly considers strain compatibility of the various weld elements.
-Larry Muir, PE, AISC Director of Technical Assistance

## STEEL SOLUTIONS CENTER

If you are interested in submitting one question or an entire quiz, contact AISC's Steel Solutions Center at 866.ASK.AISC or solutions@aisc.org.

