Steel Bridge Design Handbook

CHAPTER 8

Structural Analysis

February 2022

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Foreword

The Steel Bridge Design Handbook covers a full range of topics and design examples to provide bridge engineers with the information needed to make knowledgeable decisions regarding the selection, design, fabrication, and construction of steel bridges. The Handbook has a long history, dating back to the 1970s in various forms and publications. The more recent editions of the Handbook were developed and maintained by the Federal Highway Administration (FHWA) Office of Bridges and Structures as FHWA Report No. FHWA-IF-12-052 published in November 2012, and FHWA Report No. FHWA-HIF-16-002 published in December 2015. The previous development and maintenance of the Handbook by the FHWA, their consultants, and their technical reviewers is gratefully appreciated and acknowledged.

This current edition of the Handbook is maintained by the National Steel Bridge Alliance (NSBA), a division of the American Institute of Steel Construction (AISC). This Handbook, published in 2021, has been updated and revised to be consistent with the 9th edition of the AASHTO LRFD Bridge Design Specifications which was released in 2020. The updates and revisions to various chapters and design examples have been performed, as noted, by HDR, M.A. Grubb & Associates, Don White, Ph.D., and NSBA. Furthermore, the updates and revisions have been reviewed independently by Francesco Russo, Ph.D., P.E., Brandon Chavel, Ph.D., P.E., and NSBA.

The Handbook consists of 19 chapters and 6 design examples. The chapters and design examples of the Handbook are published separately for ease of use, and available for free download at the NSBA website, www.aisc.org/nsba.

The users of the Steel Bridge Design Handbook are encouraged to submit ideas and suggestions for enhancements that can be implemented in future editions to the NSBA and AISC at solutions@aisc.org.

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7. Supplementary Notes The previous edition of this Handbook was published as FHWA-HIF-16-002 and was developed to be current with the 7th edition of the AASHTO LRFD Bridge Design Specifications. This edition of the Handbook was updated to be current with the 9th edition of the AASHTO LRFD Bridge Design Specifications, released in 2020.	
8. Abstract An important aspect of the structural analysis process is the selection of the mathematical model and associated analysis method. Few absolute guidelines are available for the selection of an analysis method. The number of permutations resulting from various combinations of complicating physical features and mathematical models is virtually boundless. This decision should be based on an evaluation of the nature and complexity of the structure, a thorough understanding of the expected behavior, and knowledge of the capabilities and limitations of the various analysis options. Therefore, this module provides an overview for the structural analysis of steel girder bridges. Discussions include the applicable loads, descriptions of the various tools and techniques available, and considerations for selecting the appropriate application or technique for a given bridge.	
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Steel Bridge Design Handbook: Structural Analysis

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1.0 INTRODUCTION

An overview of structural analysis of steel girder bridges is provided in this portion of the Steel Bridge Design Handbook. Discussions include the applicable loads, descriptions of the various tools and techniques available, and considerations for selecting the appropriate application or technique for a given bridge.

An important aspect of the structural analysis process is the selection of the mathematical approach and associated analysis method, but few absolute guidelines are available for the selection of an analysis method. The number of permutations resulting from various combinations of complicating physical features and mathematical models is virtually boundless. This decision should be based on an evaluation of the nature and complexity of the structure, a thorough understanding of the expected behavior, and knowledge of the capabilities and limitations of the various analysis options. Selection of a suitable analysis method is key to reasonable and complete analysis results that are calculated to an appropriate level of refinement. It is important to note that all analyses, not matter how complex, can only approximate the behavior of an actual bridge, and it is prudent to use the simplest analysis method that provides reasonable behavior and the necessary results. Creating unnecessarily detailed analyses for bridges is neither required nor productive.

1.1 Analysis Dimensionality

All bridges are three-dimensional systems. However, depending upon its complexity, the actual system can reasonably be reduced to a two-dimensional or one-dimensional model. For example, as outlined in the simplified live load distribution discussion to follow, a three-dimensional bridge can be reduced to a one-dimensional girder with the application of just a few simple modification factors.

This potential for reduction and a clear understanding of dimensionality are important. When and how to make these types of dimensional reductions are important decisions that depend on the objectives of the analysis and are a key focus of this module.

Some bridges may be effectively modeled directly using a 1-D model, e.g., a straight I-girder bridge with little or no skew. A 1-D model considers an individual girder outside the context of the system and models transverse distribution of live load using an empirically based approach.

In moderately more complex bridges, a system can be effectively reduced from three-dimensional (3-D) to a two-dimensional (2-D) grid ("grillage") or plate with eccentric beam ("PEB") model. These models can include the effects of girder eccentricity and cross-frame stiffness in an indirect and/or approximate manner. A two-dimensional model provides a more refined estimate of load distribution through the cross-frames and longitudinal girders considering stiffness of the modeled elements. In some cases (e.g., PEB models) a 2-D modeling approach can also allow for refined distribution of live load, rather than relying on an empirical live load distribution factor approach.

For other structures, a 3-D analysis approach which includes the depth of the system in the model and associated cross-frames, etc., is the most appropriate. Such models often include shell elements or brick elements to model the deck and use shell and/or frame elements for the girders and transverse components, such as cross-frames.

1.2 Categories of Steel Girder Bridges

For discussion of analysis options and tradeoffs, it is useful to group the most common steel girder bridge types into broad categories. These categories are presented below, generally listed in order of increasing behavioral complexity.

- Straight, Non-Skewed, I-Shaped Girder Bridges
- Straight, Non-Skewed, Box-Shaped Girder Bridges
- Straight, Skewed, I-Shaped Girder Bridges
- Straight, Skewed, Box-Shaped Girder Bridges
- Curved I-Shaped Girder Bridges, Skewed or Non-Skewed
- Curved Box-Shaped Girder Bridges, Skewed or Non-Skewed

The choice of analysis method is directly influenced by the nature and complexity of the structure as well as the analysis objectives and the resources available. In general terms, the behavior of straight girders is less complicated than curved girders. Often, straight girders can be analyzed using relatively simple methods. Conversely, curved girders experience torsional effects locally within each girder as well as globally through the entire superstructure, which can significantly alter both the local and global distribution of load through the girders and superstructure. This complex behavior often suggests the need for more rigorous analysis.

Additionally, introducing skew to either a straight or curved girder bridge complicates the behavior and, subsequently, the analysis required to capture it. Compared to a non-skewed bridge, increased differential deflection between girders in a skewed bridge results from the adjacent girders and their supports being at different longitudinal positions. The connection of adjacent girders by cross-frames further affects the distribution of loads in both the girders and the cross-frames. This behavior is influenced by various geometric parameters including girder spacing, length of bridge, radius of curvature, and skew angle. Quantifying these effects often requires a more refined analysis than would be required for a non-skewed bridge.

Box-shaped girders (also commonly referred to as tub girders) are generally considered more complicated structures than I-shaped girders. Box-shaped girders have more pieces and parts than I-shaped girders, and these parts work together as a structural system in more complicated ways than simple I-girders with cross-frames. While simple bridges with box-shaped girders can potentially be modeled using a 1-D approach , this requires a number of simplifying assumptions and several additional calculations at the end of the analysis to determine all of the individual member load effects.

1.3 Complicating Factors

In addition to the various levels of complexity inherently associated with different structure types, other factors can also complicate the analysis of any steel girder bridge. In some cases, these complicating factors can be addressed outside the analysis, perhaps by some simple supplemental hand calculations, but in other cases, these factors may suggest the need for using a more rigorous analysis method. Some of these factors include:

- *Flaring of Bridge Width (Variable Girder Spacing)* – Some of the more simplified analysis approaches cannot effectively address the changes in loads and section properties associated with variable girder spacing.

- *Flaring of Girder Width (Variable Width Box-Shaped Girders)* – Variation of the width of a box-shaped girder to accommodate a flaring bridge width results in changes to many structural parameters. Additionally, the widths of box-shaped tub girders that have variable depth are non-uniform. Some of the more simplified analysis approaches cannot effectively address these changes.

- *Bifurcation (Splitting/Merging) of Bridges and of Girders* – Splitting girder lines at discontinuous bents (i.e., at breaks in superstructure units) may add little, if any, complication to an analysis, but splitting girder lines within a continuous unit and/or away from a support usually involves complicated framing. If a more simplified analysis method is used, the designer must carefully calculate loads in many members, often by hand, and may need to make simplifying assumptions that may lead to cumulative inaccuracies. In these cases, using a more rigorous analysis method could allow for direct modeling of both load and stiffness distribution.

- *Longer Spans* – As span lengths increase, so do the magnitudes of loads and load effects such as moments, shears, and deflections. A more rigorous analysis may be justified in these cases because the greater refinement might lead to economic savings in girder sizes, or, perhaps more importantly, better prediction of deflections, allowing the designer to better address issues.

- *Deeper/Larger Girders* – Deeper girders are often associated with longer span lengths, and the same considerations in choosing an analysis method apply. In addition, some structural response effects become more pronounced in deeper girders. For example, twisting deformations produce greater transverse deflections of cross-frame connection points. Bigger, stiffer girders can also require greater forces to maneuver in the field if they are out of position during erection, making the accurate calculation of deflections more important.

- *Exceeding the Limits of Applicability of the Current Line Girder Live Load Distribution Factors* – The live load distribution factors provided in the *AASHTO LRFD Bridge*

Design Specifications, 9th Edition (referred to herein as the AASHTO LRFD BDS) [1] are not universal, and as with any approximate analysis method, limits to their applicability exist. For example, the AASHTO empirical live load distribution factor approach is not applicable for bridges featuring severe skew, curvature, variable girder pacing, and other complications. Most refined analysis techniques can directly calculate the distribution of live loads to various girders by means of direct structural analysis, thus avoiding the need to use live load distribution factors. Alternatively, as suggested earlier, even relatively simple 1-D (line girder) analysis models can still use live load distribution factors that are based upon rigorous analysis.

- *Type of Diaphragms/Cross-Frames* – Modeling truss-type cross-frames (as opposed to rolled section or built-up plate diaphragms) in more simplified analysis models, such as grid analysis models, involves making simplifying assumptions to approximate the response of a truss structure using a line-element model. Furthermore, the results of the line-element model need to be converted back to individual member forces for design of the cross-frame. More rigorous analysis methods allow direct modeling of truss-type cross-frames.

- *Severe Curvature or Skew* – Skew in a steel girder bridge introduces complexity to the system stiffness and associated load paths; in cases of severe skew, a more rigorous analysis may be warranted even for straight girder bridges in order to more carefully quantify differential deflections, cross-frame forces and associated fatigue stresses and bearing reactions at both service and different construction stages.

- *Variable Web Depth (Haunched Girders)* – More simplified analysis methods either do not allow for non-prismatic girder cross sections or require simplifying assumptions to model the section properties and leave the calculation of stress resultants in nonparallel flanges to the designer. While the approximations introduced in stiffness modeling typically have a negligible effect on the final results of the stiffness analysis used to determine load effects, the more important aspect of modeling non-prismatic girders is the use of the associated section properties in the computation of stresses due to the load effects and the associated resistances. Minor non-prismatic features such as flange transitions with a uniform depth web can typically be ignored for structural analysis. However, such variations must be included in the computations of stresses and resistances.

- *Non-typical Substructures (Integral Substructures, Flexible Substructures such as Single Column Piers or Straddle Bents, or Multiple Fixed Piers)* – Non-typical substructure configurations can noticeably affect the structural response of superstructure elements. For example, if a bent cap provides different vertical stiffness at each bearing, the load distribution among the girders is affected. Likewise, integral substructures present additional stiffness and loading complexity. Thermally induced stresses may be introduced in girders by excessive restraint of expansion/contraction by non-typical substructures. Load effects and resulting forces and stresses can be increased due to the increased stiffness near integral features such as pier caps and abutments.

- *Complicated Bearing Configurations* – In some cases, the orientations of fixed, partially fixed, or free bearings, particularly in curved girder bridges, affect the end restraint provided to girders thereby changing the structural response.

The presence of one or more of these complicating factors should prompt the consideration of increased refinement in one or more of the following:

- Modeling of the stiffness either locally and/or globally,
- Calculation of load effects,
- Calculation of deflections and rotations, and
- Modeling of various load effects (dead load, live load, thermal load, and accounting for the sequence of their application)

In these cases, using a rigorous analysis method may be desirable if it allows direct calculation of important load effects, in lieu of approximating these effects manually outside the main analysis by a less rigorous approach. For example, it may be more desirable to analyze a bridge with variable girder spacing using a more refined analysis technique such as a 2-D or 3-D approach in order to address distribution of live load in a more rigorous manner than could be achieved with a 1-D approach using the AASHTO empirical live load distribution factors with approximate manual adjustments based only on evaluation of girder spacing.

1.4 Summary of Analysis Tools and Techniques

In this module, many analysis methods are discussed and placed in a context of their level of refinement and also their level of complexity. Additional information for analysis methods, including guidance for recommended levels of analysis, can be found in AASHTO/NSBA's *Guidelines for Steel Girder Bridge Analysis* [2] and FHWA's *Manual for Refined Analysis in Bridge Design and Evaluation* [3]. These methods can generally be placed into two broad categories:

Simplified Analysis Methods: Line girder simplification supported by the following mathematical methods:

- Classical beam analysis (for straight girders)
- V-Load Method (for curved I-shaped girders)
- M/R Method (for curved box-shaped girders)

More Rigorous Analysis Methods: Two- or three-dimensional models supported by the following types of numerical models and applications:

- Basic grid "grillage" analysis (two-dimensional analysis)

- Plate with eccentric beam (PEB) analysis (two-dimensional analysis)
- Three-dimensional analysis using a combination of frame, plate, membrane, and/or shell elements

2.0 LOADS

Loading criteria are outlined in detail within NSBA's *Steel Bridge Design Handbook: Loads and Load Combinations* [4]. Application of these loads in an analysis is discussed herein.

2.1 Live Load Modeling

There are two fundamental ways to apply live loads:

- Apply the distributed effects of a full lane to a line girder model
- Apply the 3-D representation of load to a rigorous model.

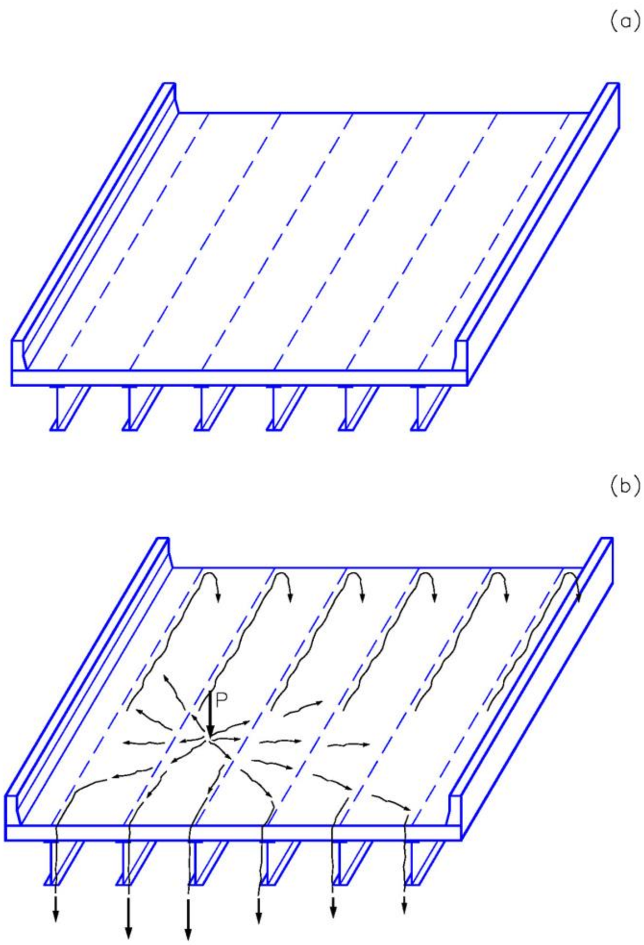


Figure 1 Illustration of live load distribution
(a) Basic girder bridge, (b) Distribution of a concentrated load such as a wheel load through the deck to various girders and then to the girder supports [31].

2.2 Distribution Factor Method—Concepts

factor. For this discussion, bending moment is used for illustration but the concepts are similar for shear, reactions, or deflection. Consider the point load in Figure (b) above. That load, or any set of loads, produces bending moments in an entire span that are resisted in part, and in differing amounts, by the individual beams. If additional loads are added in adjacent lanes, and the lanes themselves are shifted transversely on the bridge deck, the total bending moment produced by all loads changes as does the sharing of load amongst the various beams.

If this analysis is performed using what is referred to as a "refined analysis" in the AASHTO LRFD BDS [1] (i.e., a model of the entire superstructure, considering the relative stiffness of the various structural elements such as the deck, the beams, and the cross-frames or diaphragms) to determine the distribution of loads among the various beams, the sharing of load among beams would be captured directly. However, for simpler bridges such as straight bridges with little or no skew, designers often use a simpler 1-dimensional analysis method termed line girder analysis. In a line girder analysis, the model considers only a single beam without direct consideration of the interaction of the beam with the rest of the structural system. In a line girder analysis, the distribution of dead loads to each beam is computed outside of the analysis using simplified manual methods (such as uniform distribution or tributary width assumptions), and the distribution of live load moments and shears is approximated using what are called live load distribution factors. The AASHTO Bridge Design Specifications provide closed form equations that estimate the fraction of the total force effect (moment or shear) that is to be resisted by the most heavily loaded beam in a cross-section. In most cases, the Specifications provide equations for single lane and multiple lane loading, and when using the equation for the multiple lane loading, it is not known how many lanes control the analysis. For both equations, which beam controls the design, and where the loads are, is not known in this analysis, but importantly for design and analysis, the distribution factor equations were developed to bound the solution.

AASHTO refers to the distribution factors using the letter, g.

The distribution factor, *g*, is defined as

$$g = M_{beam} / M_{lane}$$

Where:

M_{beam} = the force effect (moment in this case) experienced by the subject beam
M_{lane} = the force effect that would be caused in the subject beam by the total effect of a single lane of traffic

In the case of a 1-D line girder analysis, this factor is used to convert the load effects of a fully loaded lane to those in the most heavily loaded interior or exterior beam. Thus, a hypothetical fully loaded lane of load is applied to a line element having the stiffness characteristics of the beam and slab system and these force effects are then multiplied by "g" to obtain the estimated load effect for the girder in the system.

It should be noted that the value of the distribution factor, g, can be greater than 1.0. If a bridge deck is wide enough to carry more than one lane of traffic, it is possible that the structural system (beam size, spacing, connectivity through the deck and cross-frames, etc.) could be configured such that more than one lane's worth of load could be carried by a given beam for a given positioning of the live load.

Furthermore, the value of the distribution factor, g, can be (and usually is) greater than the total number of lanes that can fit on the deck divided by the total number of beams in the cross-section. This is because the distribution factor reflects the maximum loading that a given beam could be subjected to, considering the various possible positioning of the live load. Not all beams are subject to maximum loading at the same time, but each beam could be subject to that maximum moment at a given time.

In the research and specification development work, these factors can be established by analyzing many systems with refined methods to establish the relationship between forces in a most heavily loaded beam and the force effects of a fully loaded lane, or multiple lanes. Hundreds of analyses are performed for bridges and the effects of the relative stiffness of the various components, geometry effects, and load configuration are studied. The results of these analyses are then used to develop empirically based formulas that contain the relevant system parameters as variables. The development of the AASHTO LRFD BDS [1] distribution factors followed this approach, which is documented in NCHRP Project 12-26 [5].

These formulas can then be used by designers to estimate the maximum live load force effects without performing the refined analysis. Some compromise may be made in accuracy, but this method generally gives good results for routine bridges. The critical aspect of using distribution factors is understanding their limitations and the restrictions on their use relative to the bridge under consideration. Often, the same considerations that suggest using a more rigorous modeling approach (e.g., 3-D vs. 2-D modeling) suggest using a more rigorous approach to calculating live load effects.

2.3 Distribution Factor Method—Example

In the first case, wheel loads (a single wheel load is illustrated in Figure 1) are applied to a 1-D model and the actions are determined. These actions are subsequently scaled to account for the transverse load distribution, i.e., the amount of load applied to a girder line. This scale factor is called a distribution factor. Section 4.6.2.2 of the AASHTO LRFD BDS [1] provides equations to determine these factors that are a function of the bridge parameters including girder spacing, S, span length, L, girder stiffness, K_g, and deck thickness, t_s. The live load distribution factor for a composite girder bridge with two or more (multiple) design lanes loaded is:

$$mg_{moment}^{MI} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12Lt_s^3}\right)^{0.1}$$

Simple modifications also are provided for skew, e.g., Skew Adjustment = $1 - c_1(\tan \theta)^{1.5}$

$$c_1 = 0.25 \left(\frac{K_g}{12Lt_s^3}\right)^{0.25} \left(\frac{S}{L}\right)^{0.5}$$

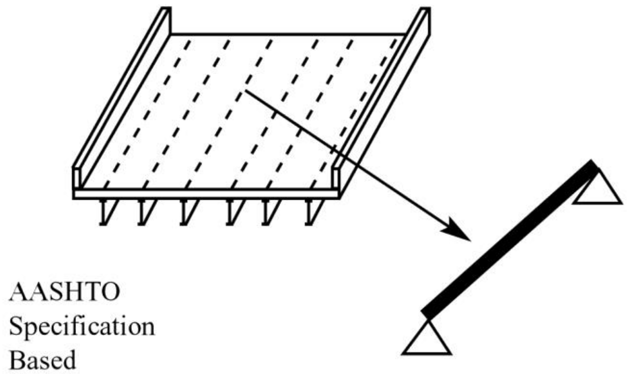


Figure 2 Reduction in Dimensionality to a Line Girder with AASHTO Distribution Factors [31].

For additional formulas and methods, see Section 4 of the AASHTO LRFD BDS [1].

2.4 Multiple Presence Factors

Within the AASHTO LRFD BDS [1], the distribution factor equations in Article 3.6.1.1.2 implicitly contain the effects of load from adjacent traffic lanes. The development of these equations included one-, two-, three-, etc. loaded lanes. In each analysis, the multiple presence factors were applied as outlined in NSBA's *Steel Bridge Design Handbook: Loads and Load Combinations* [4] (i.e., m =1.2, 1.0, 0.85, 0.65).

In addition to distribution factor equations, the AASHTO methods require consideration of the lever rule or the rigid rotation method in certain circumstances as outlined in Articles 4.6.2.2.1 and 4.6.2.2.2d, respectively, of the AASHTO LRFD BDS [1]. In these cases, the multiple presence factors must be explicitly applied during the analysis for the number of lanes considered in that analysis. For example, if the lever rule is used for one loaded lane, then a multiple presence factor of m = 1.2 must be directly applied.

In a similar manner, when a rigorous analysis is used, the multiple presence factors must be applied by the designer to determine the controlling number of loaded lanes. This is one of the complicating features of 2-D and 3-D modeling as critical load placement, multiple presence, and enveloping all of this for one and multiple lanes loaded are required.

2.5 Live Load Positioning

AASHTO LRFD BDS [1] requires that a standard design vehicle (and lane load) have a 6 ft gage, or spacing, between wheel lines. A truck is positioned within a design lane 10 feet wide which in turn is positioned within a 12 foot traffic lane to maximize loading effects. Additional requirements include the following:

- The outside wheel shall be placed no closer than 2 ft from edge of the lane or the face of the barrier except as noted below
- The outside wheel should not be placed closer than 1 ft from the face of the barrier for deck overhang design
- The minimum spacing between wheel lines of adjacent trucks is 4 ft
- The transverse location of one truck or several adjacent trucks can be anywhere within the transverse cross section that meets the above.

For practical application of the above, automated algorithms within analysis programs are typically used to place the truck, apply multiple presence factors, and envelope the results.

2.6 Centrifugal Force Effects

In addition to modeling vertical live load effects, since most analyses include a relatively refined live load model where individual wheel lines are used as the live load tracks, the transverse load and overturning effects of centrifugal force and superelevation in curved bridges should be included in the analysis. A simple method for including these effects is presented in the FHWA's *NHI Course No. 130095: Analysis and Design of Skewed and Curved Steel Bridge with LRFD – Reference Manual* [6]. Generally, the analysis should consider the presence and absence of centrifugal force effects, since it is a function of speed. This approach of evaluating both no centrifugal force effects and centrifugal force effects based on design speed will assist to envelope predicted conditions.

2.7 Modeling of Other Loads

2.7.1 Dead Loads

Self-weight of girders can be modeled manually as a distributed load in most analysis methods and is often calculated automatically by computer analysis programs. Self-weight of other members such as diaphragms, cross-frames, bracing, stiffeners, etc., are often approximated in simpler analysis methods as a percentage of girder self-weight and assumed to be applied uniformly to the girders. In more rigorous analysis methods (such as 2-D or 3-D analyses), the self-weight of many of the larger components (such as diaphragms, cross-frames, or bracing) may be calculated automatically by the computer analysis if these members are individually entered in the application. It should be noted that when the method of estimating the self-weight of other system components is performed by the use of a percentage of the main-girder load, the assumption used at the beginning of the modeling should then be compared with the final details to determine if the assumed magnitude reasonably correlates with the final details. If not, a re-analysis may be warranted, particularly for a more improved prediction of the girder deflections. If additional loads beyond the self-weight of the beams are unknown at the time of analysis, usually during preliminary design, it may be prudent to rely on plans for similar, existing bridges and compute the weight of the various attachments and secondary components of a bridge (shear studs, cross-frames, stiffeners, splices, etc.), which can then be verified during final design of the beams.

Even in the most rigorous 3-D analyses though, it is often the case that smaller members such as stiffeners are not directly modeled, and their self-weight is typically applied as a percentage of the girder self-weight, usually either by increasing the self-weight of the girders or by applying an additional line load to the girders.

The calculation of force effects in the girders and cross-frames due to dead loads is straightforward, particularly when a refined analysis is being used. However, in skewed and/or curved bridges, locked-in force effects can be induced in the structure, depending on the chosen fit condition and how the structural steel framing is assembled. Cross-frame forces and lateral bending stresses are particularly sensitive to these effects. This topic is discussed the AASHTO LRFD BDS [1] in Article 6.7.2 and its associated Commentary. For more information, see NSBA's *Skewed and Curved Steel I-Girder Bridge Fit* [7, 8].

Other non-composite loads are usually modeled as distributed line loads on the girders, e.g., the self-weight of concrete haunches. The self-weight of the deck warrants discussion and careful consideration. When designing straight bridges with little or no skew using line girder analysis methods, the self-weight of the deck may be uniformly distributed among all of the girders, assuming the girders are well-connected by cross-frames provided at a reasonable spacing. This is addressed in the AASHTO LRFD BDS [1] in Article 4.6.2.2.1, where it says, "Where bridges meet the conditions specified herein, permanent loads of and on the deck may be distributed uniformly among the beams and/or stringers." This reflects the fact that in a straight bridge with little or no skew and well-connected girders, the girders can be expected to deflect uniformly and carry loads uniformly under the effects of a distributed load such as the weight of the wet concrete deck. Regardless of the distribution for design, girder camber should consider a more uniform distribution of non-composite dead loads to obtain proper fit.

When refined analysis methods are being used, on the other hand, the self-weight of the concrete deck is usually assigned to each girder based on simple span behavior (i.e., tributary area) and then modeled as distributed line loads on the girders. In this case, the actual response of the girders to the self-weight of the wet concrete deck (i.e., the actual distribution of the self-weight of the deck) is determined by the model, based on the relative stiffnesses of the girders and cross-frames and the overall geometry (curvature, skew, etc.).

One complication that arises in assessing the effects of deck and haunch dead load is the need to consider staged deck placement, particularly in larger, longer bridges. In such cases, the concrete deck is often placed sequentially in sections which are allowed to harden prior to placement of the next deck section. In addition, the concrete strength varies for each placement since it is time-dependent, and the concrete does not achieve its full strength before subsequent sections are placed. The calculation of load effects should consider the changing stiffness and the implications of load effects introduced to various parts of the structure in their non-composite condition, with later loads being applied to a structure which is partly composite and partly non-composite.

A distinct but important topic is the need to address flange lateral bending which can occur during deck placement due to the application of deck overhang bracing reactions to the bottom flange. The "Design for Concrete Deck Overhang Loads" [9] offers a good treatment of this subject, and NSBA's *Steel Bridge Design Handbook: Design Example 1: Three-Span Continuous Straight Composite Steel I-Girder Bridge* [10] provides example calculations.

The modeling of load effects carried in a composite manner is a topic with some degree of subjectivity involved. The AASHTO LRFD BDS [1] allow for the distribution of composite loads equally to all girders in a cross section. However, many designers use this provision only on narrow structures; when using line girder analysis methods for the design of wider structures they will give some consideration to the location and nature of the load. For example, in a wide superstructure, distributing barrier loads to only a few of the exterior girders may be more reasonable than assuming uniform distribution of those loads over all of the girders in the cross-section. Several owner/agencies have specific guidelines on this topic. In most cases, these simplifying assumptions are based primarily on engineering judgment, although some designers use a simplified method such as the lever rule. For the case of bridges with more complex geometry (e.g., curved and/or severely skewed), when a refined analysis method (a 2-D or 3-D analysis) is used, the composite loads can be applied at their appropriate locations, and the analysis model will determine the distribution of the loads based on the relative stiffness of the girders and cross-frames and the overall geometry (curvature, skew, etc.).

2.7.2 Wind Loads

Wind loads should be considered in the analysis. The key task is determining the appropriate method for addressing these loads.

In many cases it may be appropriate to address the effects of wind loads separately from the primary analysis of the girders. The primary wind load effects on simpler, shorter span structures are to induce flange lateral bending stresses to the bottom flanges and to contribute to cross-frame and diaphragm loads.

A common assumption is to distribute the wind load resulting from wind pressure on the girder web equally to the top and bottom of the girder. The load distributed to the top of the girder is directly carried into the deck with no effect on the exterior girder. The load effect in the deck itself is typically ignored, as the deck is assumed to act as a wide (deep) horizontal diaphragm for transfer of the wind loads to supports. The support shear connectors and diaphragms transfer the wind load, as they do transverse seismic loads, to the bearings (if non-integral supports are used) and the substructure.

The wind load distributed to the bottom of the girder is applied to the bottom flange as a transverse distributed load along the length of the girder. The bottom flange may be modeled as a continuous beam on multiple pinned supports (because the cross-frames are relatively stiff) and the resulting flange lateral bending stresses are based upon the lateral flexural bending moment. These stresses are superposed, typically according to the one-third rule, with the girder flange primary bending stresses due to gravity loads and thus included in the girder design as appropriate. In many cases it can be shown that these added flange lateral bending stresses do not control the design due to the nature of the load factors in the AASHTO load combinations. Section 3 of the AASHTO LRFD BDS [1] includes these stresses in the load effects and outlines their application in detail for various limit states.

The reactions from the wind model at each cross-frame are calculated and applied as loads to the cross-frames. The cross-frames act to transfer this load to the top of the next adjacent girder via either truss action or shear diaphragm action depending on whether the cross-frame is a truss-type cross-frame or a plate-like member (e.g., a channel, bent plate, or deep W-section). Again, the load transferred to the top of the girder is assumed to be directly transferred into the deck as previously described. The loads induced in the cross-frames are added to any other load effects as appropriate. In the case of straight, non-skewed girders, this wind load is often the only quantified load in the intermediate cross-frame analysis. Transverse seismic load is of interest for support cross-frames, as discussed later.

In some cases, it may be appropriate to include the wind load effects directly in the main superstructure analysis. Wind load may have a more pronounced influence on the girder design in long span, deep girders, or girders with unusual framing plans or unusual substructure conditions. Experience and engineering judgment are required in determining when it may be appropriate to include wind loading directly in the main girder design.

Evaluation of the construction sequence should address wind load effects. Often, large parts of a steel girder bridge are erected and exposed to wind loading prior to deck placement and often prior to complete installation of all girders and cross-frames. Insufficiently braced girders can suffer damage or even collapse due to wind loading during construction. Additionally, some nominal deflection check under a normal wind should be conducted to verify that the entire system is not so flexible as to jeopardize worker safety or create work delays. AASHTO's *Guide Specifications for Wind Loads on Bridges During Construction* [11] provides additional information for wind loads during construction.

2.7.3 Thermal Loads

Thermal effects can also cause loads in steel girders. Whether these loads are significant or negligible depends on several factors such as the structural configuration of the bridge, the articulation of the bearings, and the nature of the thermal effects.

For example, in bridges with relatively simple framing plans and bearings that freely allow thermal expansion and contraction of the superstructure, it is reasonable to neglect the effects of routine temperature rise and fall in the design of the superstructure. But in bridges with complex framing plans, the bearings may provide some restraint to thermal movements. Similarly, bridges that feature unusual bearing conditions or bridges that use integral substructures may also present cases where thermal movements are restrained and may induce internal loads and stresses. Careful assessment of the restraints, particularly as a result of bearing type and configuration, and their effect on thermally induced loading effects is advised in such cases.

A thermal gradient is another potential cause of temperature-related loads. If, due to the nature of the thermal effect or the nature of the structure, a thermal gradient occurs across the cross section of a girder, internal stresses will develop. For example, if part of a girder is exposed to significant solar radiation, while another part of the girder is shaded, the heated part of the girder will expand relative to the shaded portion, causing internal stresses due to the variation of temperature with depth.

2.7.4 Seismic Loads

For routine bridges, seismic loads, like wind loads, are treated as equivalent static lateral loads. These lateral loads in turn are a function of the seismicity of the site and the weight of the bridge. All other things being equal, steel bridges have an inherent advantage seismically since they typically weigh less than half of a concrete superstructure alternative.

There are two distinct philosophies for addressing seismic effects on structures: i) ductile "capacity design" and ii) seismic isolation. In ductile capacity design, each element in the seismic load path must possess adequate resistance and ductility to transmit inertial loads from the superstructure mass into the ground. For typical steel bridges, most of the mass is in the deck. Thus, the transverse seismic load path over the supports consists of shear connectors, diaphragms, bearings, substructures and their interconnections. In seismic isolation design, the uncoupling of the superstructure from the substructure reduces the magnitude of the seismic forces that must be transmitted through the elements in the seismic load path, albeit at the expense of larger movements. Sources addressing these concerns specific to steel superstructures include references [12] through [16].

A complete discussion of the effects of seismic loads on steel girder bridge superstructures is beyond the scope of this document. However, it can be said that seismic effects on superstructures should be considered for bridges in regions of high seismicity, particularly if an unusual superstructure configuration is used. Noncomposite structures such as those used for freight rail or light rail transit bridges, or bridges featuring unusually large or very concentrated loads with potential for significant inertial loading, also represent cases that may warrant such consideration.

In many cases the owner/agency may have specific guidance for the design and analysis of bridges in earthquake prone regions.

3.0 BEHAVIOR CONSIDERATIONS

The behavior of steel girder bridges can be broadly divided into two categories:

- Primary Bending — Vertical shear, vertical moment, vertical deflections and end rotations, as are experienced by all bridges.

- Horizontal Curvature and Skew Effects — Torsional stresses, warping and flange lateral bending, load shifting, and warping and twisting deformations

In the following sections, many effects are discussed as curvature effects, but similar twisting and flange lateral bending effects occur in straight skewed steel girder bridges.

3.1 Primary Bending

Curved and/or skewed steel girder bridges experience the same gravity loading effects (dead load and live load) as straight girder bridges. All bridges are subject to primary shear and bending moment effects as well as vertical deflections and end rotations. These effects are familiar to bridge engineers, so an extensive discussion is not warranted; however, it is important to mention them because they are an essential component of stress and deformation for straight, curved, and/or skewed steel girder bridges.

3.2 Torsional Stress Effects

In addition to the basic vertical shear and bending effects described above, a curved girder is also subjected to torsional effects. The torsion in curved girders arises from the fact that the center of loading (center of gravity) of each span in a curved girder is offset from a chord line drawn between the supports for that span. This offset represents an eccentricity which, when multiplied by a given vertical load (dead load or live load), results in a torque on the girder (Figure 3).

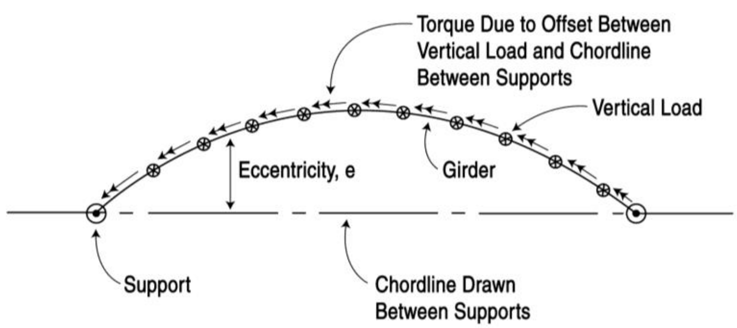
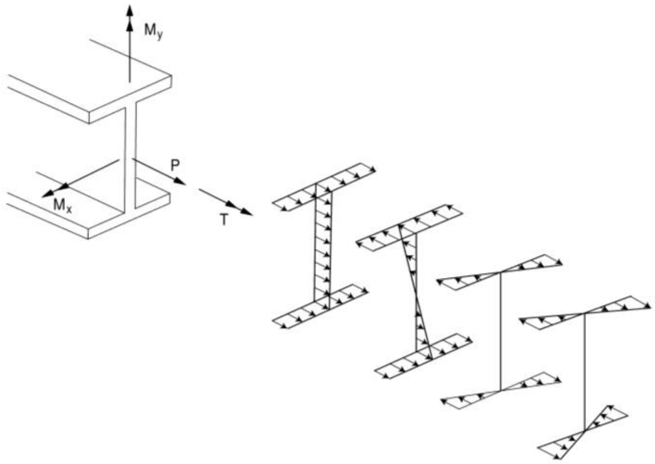


Figure 3 Plan view of the development of torque in a curved girder



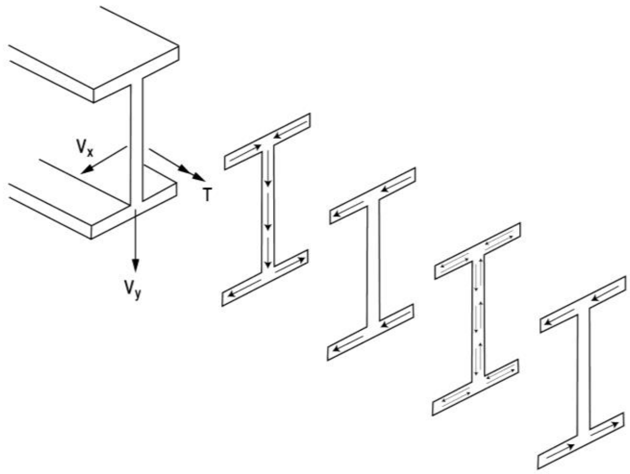
$$\text{Total Normal Stress} = \sigma = \frac{P}{A} + \frac{M_x\, y}{I_x} + \frac{M_y\, x}{I_y} + \text{Warping Normal Stress}$$

Figure 4 Normal stresses in a curved or skewed I-shaped girder.

Torsion in steel girders causes normal stresses and shear stresses. Because I-shaped girders and box-shaped girders carry these stresses in different ways, they are considered separately.

I-shaped girders have low St. Venant torsional stiffness, so warping stresses are the primary torsional load effect. Therefore, the total state of normal stress in an I-shaped girder is a combination of axial stress, primary vertical bending stress, horizontal bending stress and warping normal stress (Figure 4).

The total state of shear stress in an I-shaped girder is a combination of vertical shear stress, horizontal shear stress, a small St. Venant torsional shear stress, and warping shear stress (Figure 5).



$$\text{Total Shear Stress} = \tau = \frac{V_y Q_x}{I_x t} + \frac{V_x Q_y}{I_y t} + \text{St. Venant Torsion} + \text{Warping Torsion}$$

Figure 5 Shear stresses in a curved or skewed I-shaped girder.

The relatively low St. Venant torsional stiffness of I-shaped girders is a result of their open cross-section geometry. The St. Venant torsional shear flow around the perimeter of the cross section can only develop force couples across the thickness of any given segment. Since they do not have a significant force couple distance between these shear flows, the ability of I-shaped girders to carry torque via St. Venant torsional response is low.

In contrast, box-shaped girders are closed-cell structures in their final condition (prior to casting the deck, top lateral bracing may be used to create a quasi-closed shape). Closed cells are well-suited for carrying torsion by means of St. Venant torsional shear flow because the shear flow around the circumference of the box has relatively large force couple distances (Figure 6).

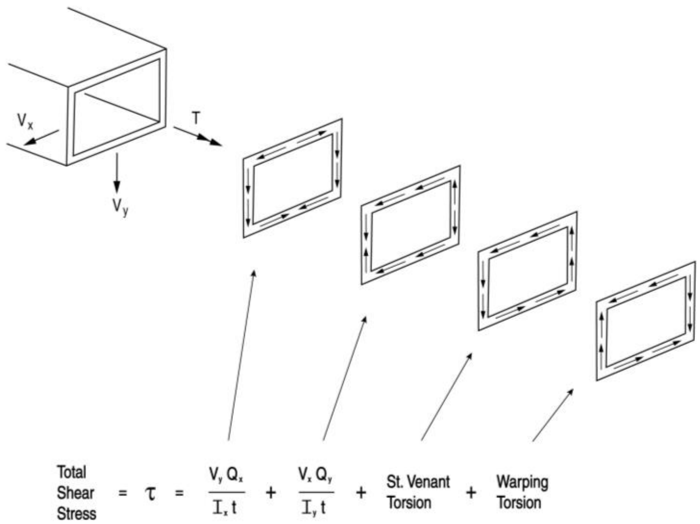


Figure 6 Shear stresses in a curved or skewed box-shaped girder.

Thus, a box-shaped girder can carry relatively large torques with relatively low shear flows. The shear flow around the circumference of the box follows a consistent direction (clockwise or counterclockwise) at any given location along the length of the girder. As a result, when combined with vertical shear in the webs, this shear flow always adds to the shear stress in one web and subtracts in the other.

As in an I-shaped girder, the total state of normal stress in a box-shaped girder is a combination of any axial stress, primary vertical bending stress, horizontal bending stress and warping normal stress (Figure 7). The total state of shear stress in a box-shaped girder is a combination of vertical shear stress, horizontal shear stress, St. Venant torsional shear stress and warping shear stress (Figure 6).

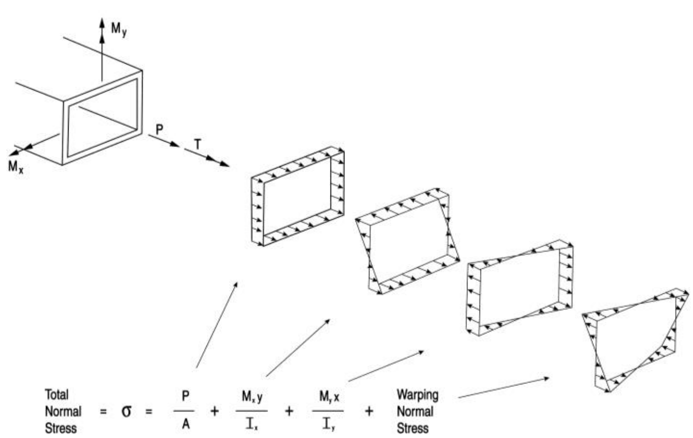


Figure 7 Normal stresses in a curved or skewed box-shaped girder.

3.3 Flange Lateral Bending

Many practical effects result from the way girders carry torsion. For example, the warping normal stresses caused by torsion as previously described for I-girders represent one source of what are called flange lateral bending stresses. Flange lateral bending stresses are an important part of the design equations for flange stresses in I-girders. Most curved I-girder analysis techniques include, as a key feature, some method of calculating flange lateral bending stresses, and most formulae for girder design (applied loads/stresses vs. load/stress capacity) include an accounting of flange lateral bending stresses.

Note that curvature is not the only source of flange lateral bending stresses. Other causes include wind loads, construction loads and differential girder displacement. Related to the latter and of interest to this discussion, though, is the effect of skew in causing flange lateral bending moments.

As mentioned previously, skewed bridges exhibit many of the same behaviors as curved girders. For example, in a bridge with straight girders, but with an overall skew and right (non-skewed) cross-frames, the cross-frames cause flange lateral bending.

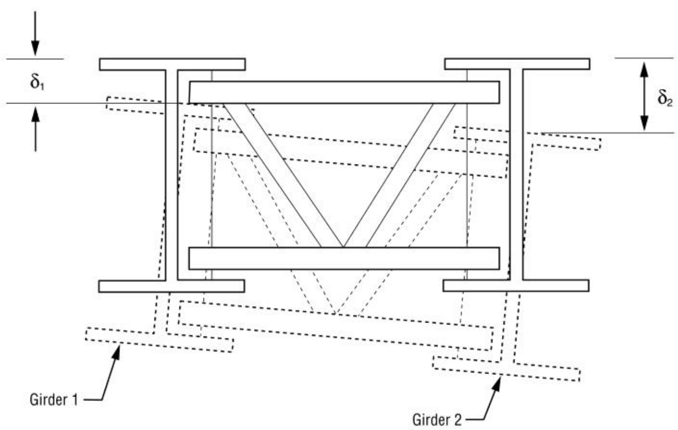


Figure 8 **System deformation of girders and non-skewed cross-frames in a skewed bridge.**

Right (non-skewed) cross-frames in skewed bridges connect adjacent girders at different positions along the length of each girder, with each girder experiencing different displacements at the point of connection. At these locations, the deflections of the girders attempt to cause forced racking displacements in the cross-frames (Figure 8), but the cross-frames, being much stiffer in the transverse direction than the girders, resist these deformations by developing internal loads and instead undergo an in-plane rotation, which causes flange lateral bending in the girders.

The cross-frame loads include horizontal components that induce flange lateral bending effects, analogous to the effects that are the basis of the *V-Load* method of curved girder analysis discussed later.

Furthermore, near the ends of the girders in skewed bridges, cross-frames begin to act as alternate load paths as their stiffness approaches or exceeds that of the girders. Even if select cross-frames are oriented along the skew, or if select cross-frames are omitted [17], the remainder of the cross-frames still undergo this type of behavior and cause the skewed girder system to exhibit characteristics similar to a curved girder system, even if the girders themselves are straight.

It may initially seem that this effect can be avoided by skewing the cross-frames so that girders are not connected at points of differential deflection, but this does not completely eliminate the introduction of cross-frame-induced flange lateral bending. Bending rotations (rotations about the horizontal transverse axis of the girder) are associated with vertical deflections of the girders caused by primary vertical bending. These primary bending rotations are well-known. Assuming uniform bending along the skewed bridge cross-section, skewed cross-frames would connect the girders at points of identical deflection and rotation.

However, as the cross-frames rotate to match the primary girder rotations, they also try to rack because they are trying to rotate about the transverse axis of the girders, which is not coincident with the centerline axis of the cross-frames (because the cross-frames are skewed). However, due to their high in-plane stiffness, the cross-frames resist this racking deformation by developing internal forces and instead experience an in-plane rotation. So as the top and bottom corners of the cross-frames move forward and backward to follow the primary girder rotation, they also move outward and inward along the plane of the cross-frame (Figure 9), inducing flange lateral bending in the girder flanges. Figure 10 shows a more detailed view of the relative displacements of the top and bottom struts of the cross-frames as they rotate with the girder. Beckman and Medlock [18] provide a detailed discussion of this behavior and other skewed steel bridge considerations.

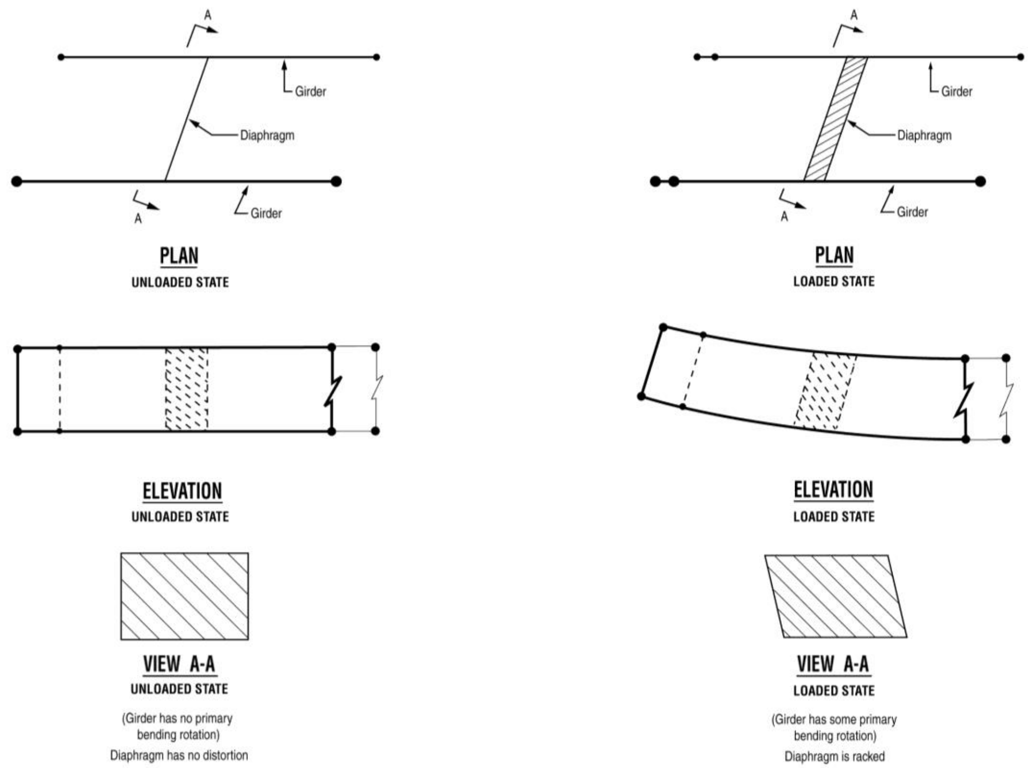


Figure 9 System deformation of girders and cross-frames skewed to match the bridge skew.

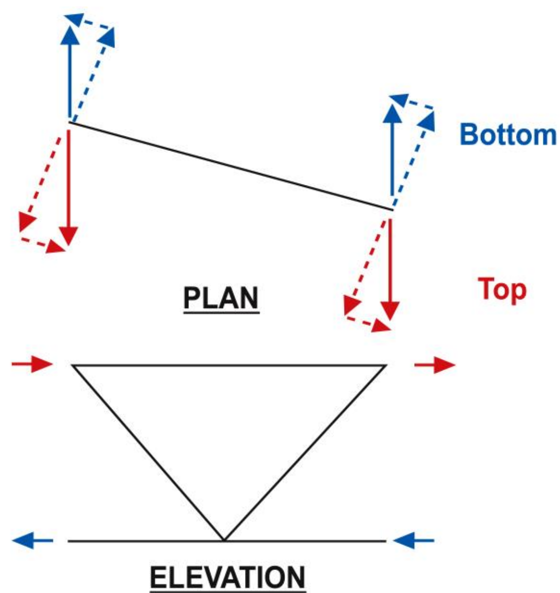


Figure 10 Detail of skewed cross-frame deflections during girder rotation.

The previous examples are just a sample of how a straight bridge with skewed supports exhibits behavior similar to that of a curved girder bridge and why it should be designed using many of the same approaches. Several references [6, 19 to 22] offer good discussions of the effects of curvature and skew in steel girder bridges.

3.4 Torsional Deformation Effects

In addition to causing significant stresses in both I-shaped and box-shaped girders, torsion also causes significant deformations. Curved girders not only deflect vertically, they also twist. They not only experience end rotations, they also warp (Figure 11). Depending on the sharpness of the curvature, the length of the spans, the framing of the bridge and the magnitude of the loads, these deformations can become significant; sometimes large enough to be a serious consideration affecting the constructor's ability to assemble adjacent girders in the field.

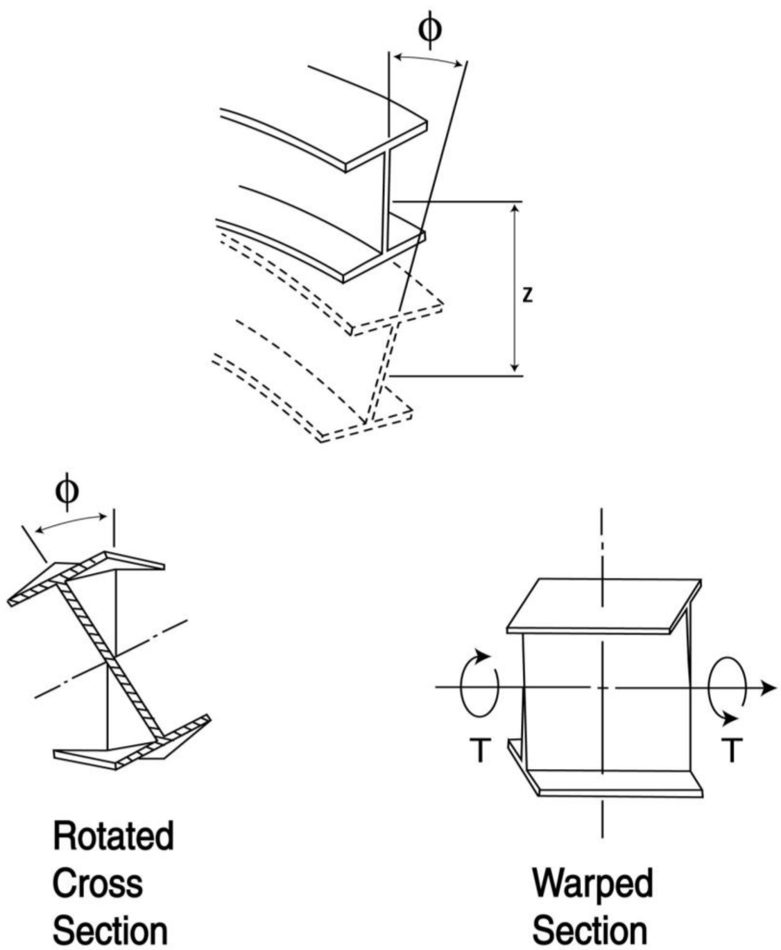


Figure 11 Vertical deflection, twisting deformation, and warping deformation in curved I-shaped girders.

Curved girder bridges behave as an integral system, where the movement of one individual girder influences that of the others. The sequence of erection, as well as the number of girders in place and connected by cross-frames at any given time during erection affects their response to loading. Contract plans should clearly indicate the assumed erection sequence [23]. If the constructor chooses to erect the girders in a different sequence, an engineer should review (and reanalyze) the proposed erection sequence in detail during shop drawing review.

The AASHTO LRFD BDS [1] explicitly require designers to assess these deformations, address them as appropriate on their plans and indicate the assumed erection sequence and intended girder positions at various stages of construction.

Again, note that these deformation issues are not limited exclusively to curved girders. Skewed bridges experience many of the same phenomena.

3.5 Load Shifting

As was mentioned previously, individual curved girders experience torsion because their center of loading (center of gravity) is offset from the chord line drawn between their supports (Figure 3). This is also true for systems of curved girders, in which global overturning causes a load shifting effect whereby the girders on the outside of the curve carry different loads than those on the inside.

This effect is comparable to how groups of piles carry vertical loads and overturning moments in pile-supported footings. Another analogy is the way bolts carry loads in an eccentrically loaded bolt group. In all cases, the model used is a rigid-body in which the applied moment is resolved into force couples that are additive to the primary loads at some points (i.e., additive to the loads in some piles, additive to the loads in some bolts, or additive to the loads in some girders) and relieving at other points.

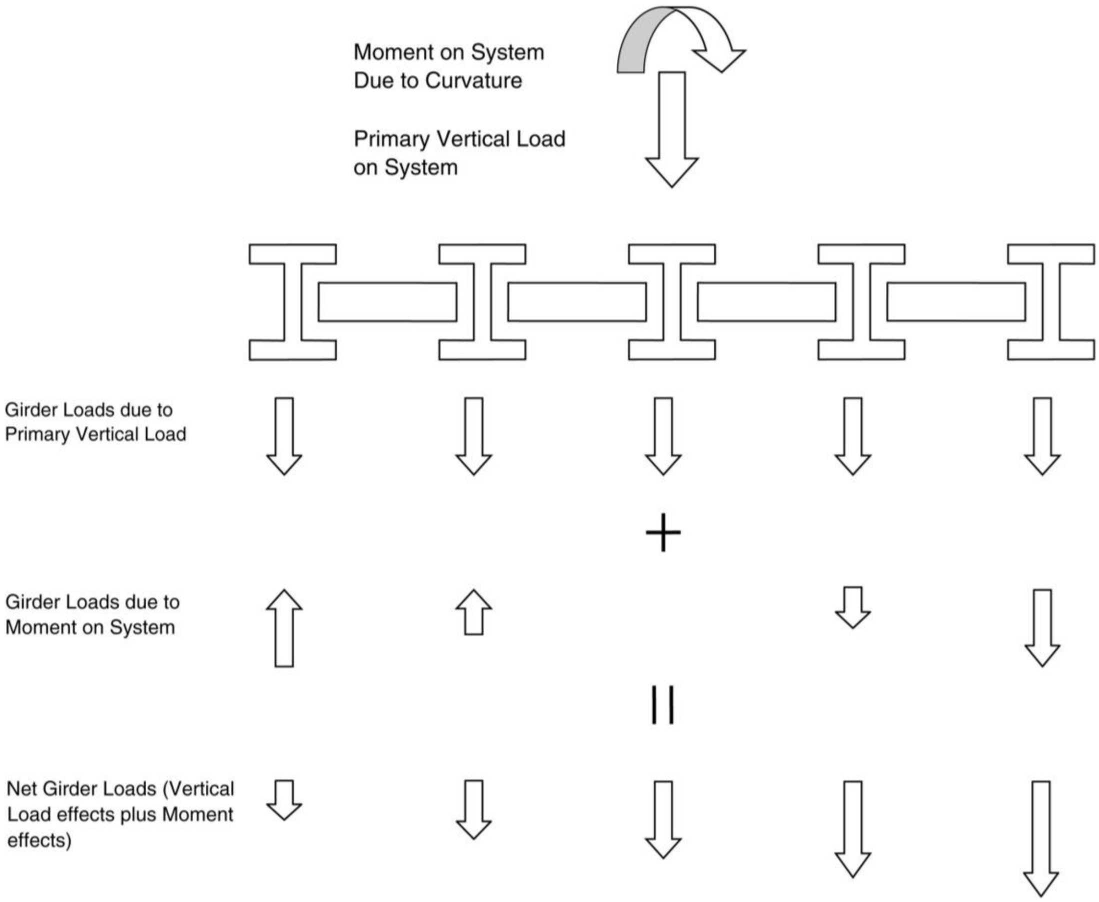


Figure 12 **Load shifting in curved girder bridge systems.**

As an example, this behavior in a simple-span curved girder bridge results in the girders on the outside of the curve carrying more load (Figure 12). This behavior characteristic generally holds for most curved girder bridges, but exceptions exist where variations in the direction of this type of behavior can occur depending on issues such as the span length balance in multiple-span continuous girder bridges [24].

Not only is this load shifting phenomenon itself significant, but the specific load path for achieving this load shifting is also important. Loads are transferred from one girder to the next through the cross-frames, which are thus primary load carrying members and must be designed, fabricated, and inspected as such.

4.0 SIMPLIFIED ANALYSIS METHODS

Approximate analysis techniques for curved and skewed steel girder bridges cover a range of methods. The most commonly used methods are described here.

Approximate analysis techniques have several advantages that make them attractive for specific applications, even in this age of computerization. Many of the approximate analysis techniques are based on equilibrium approaches and are quite transparent, giving the engineer a good feel for an approximate distribution of forces through a bridge. Most of the approximate analysis techniques are also relatively simple and quick to use, making them valuable for preliminary designs or as approximate tools for validating more complex analyses.

However, approximate analysis techniques should only be considered rough tools for bridges with anything beyond the most basic geometry and framing. The simplifications and approximations involved in applying the approximate analysis techniques to more complex bridges tend to reduce the accuracy of their results, particularly with regard to the prediction of structural deformations. Use of approximate analysis techniques should be limited to preliminary design or the design of relatively simple structures.

4.1 Girder Modeling

How to approach the modeling of individual girders depends greatly on the type of analysis method being used and whether girders are considered individually, or the entire superstructure system is modeled. In the case of most of the simplified analysis methods, however, the typical approach is to model a single girder, rather than modeling the entire multiple-girder cross section.

For such an analysis, the designer must determine appropriate cross-sectional properties. The non-composite steel girder cross-sectional properties are generally easy to quantify as the dimensions of the girder are clearly and unambiguously identified. For the composite case, the problem becomes slightly more complex, with an important parameter being the “effective width” of the composite deck. The AASHTO LRFD BDS [1] offer explicit guidelines for effective width estimates, which are typically one-half the distance to the adjacent girder on each side of the girder being evaluated for interior girders or one-half the distance to the adjacent girder plus the full overhang width for exterior girders.

Box-shaped sections deserve some special mention when discussing effective width of the composite deck because each girder has two webs and two top flanges, and thus two sets of effective deck widths. Overlaps of the calculated effective deck width between flanges in a single box section should be neglected in calculating the girder’s composite cross section properties; however, gaps in effective deck widths when wide web spacings are used may exist. Again, the AASHTO LRFD BDS [1] offer guidance, which are typically tributary widths perpendicular to the axis of the member.

A subset of the modeling of composite section properties is the appropriate treatment of the cross section in negative moment regions. In those areas, because the deck is in tension, it is often cracked, and its effectiveness as part of the cross section may be subject to some debate. In general, if the concrete deck is composite, it can be assumed effective over the entire span length for analysis purposes consistent with AASHTO LRFD BDS [1]. For noncomposite bridges and bridges that are noncomposite in negative moment regions, engineering judgement is generally required to determine the method of analysis. Designers are advised to discuss the preference of the appropriate owner-agency with regard to this issue.

Another consideration in modeling a single girder is quantifying the lateral support of the girder. Typically, the calculation of loading effects (moments, shears, deflections, and stresses) is not affected by lateral support conditions, but the calculation of the capacity of various elements, particularly compression flanges, is strongly dependent on lateral support. Load-dependent second-order effects may be estimated with codified magnifications. Most commonly, this issue breaks down to simply modeling the cross-frame spacing as this is usually the primary consideration in determining the unbraced lengths of compression flanges.

What constitutes a lateral support is also an important issue. Recent research into lateral bracing of steel compression members has shown that minimum stiffness and strength requirements must be met in order for a lateral support to adequately brace a compression flange. Bracing design is outlined in detail within this handbook in NSBA's *Steel Bridge Design Handbook: Bracing System Design* [25]. More recently, NCHRP Report 962 [26] investigated minimum strength and stiffness requirements for steel bridge cross-frames and provides a detailed discussion of the underlying mechanics, practical design examples, and proposed AASHTO specification language.

The modeling of support conditions is of great importance when modeling a single girder, just as it is when modeling the entire superstructure in more rigorous analysis methods. Consideration should be given to bearing configurations and any restraints offered by bearings, as well as by integral substructures, when used.

Keep in mind that simply using an "integral" substructure does not necessarily mean that the support offers rotational restraint to the girder. Most designers and owner-agencies neglect any restraint from integral abutments in superstructure design. Typically, the design and detailing of integral end bents/abutments focuses on providing flexibility for accommodating bridge thermal movements without inducing excessive loads into thew end bent/abutment. For this reason, most engineers have concluded that the relative stiffness of integral end bents/abutments is much less than that of the superstructure and that the girder can be assumed to behave like a simply supported structure at the end bent/abutment.

However, integral supports do offer restraint against torsional rotations associated with lateral torsional buckling. Additionally, maintenance-prone joints are often reduced, eliminated, or relocated off the structure through the use of integral substructures.

4.2 Live Load Modeling

Component dead loads on bridges are generally static (permanent), unchanging loading conditions which are usually relatively easy to deal with. Live loads, on the other hand, are much harder to quantify as they represent a myriad number of loads applied in an infinite number of positions and combinations of positions across the length and width of the bridge. How live loads are addressed can make the analysis either simple to perform and understand or make it unmanageable and overwhelming.

There are two primary ways to address live load modeling for bridge structures. First is what can be called the "brute force" method, which involves running analyses of multiple live load cases. In computer applications, this is accomplished using a live load generator — a computer routine that produces literally hundreds or thousands of live load cases, each representing a different load (truck load, lane load, combinations of multiple truck or lane loads, etc.) applied at different positions along the structure. For each live load case, the analysis model is fully calculated and shear and moment results for all key members are developed. The multiple live load case method generates a huge pool of results to develop the load effect envelopes for various members in the structure.

An alternative to the multiple live load case method is the influence line, or influence surface method. An influence surface is an influence line approach applied in two dimensions rather than just one dimension. A full explanation of the influence surface method is beyond the scope of this module, but a summary description is warranted.

In this approach, the response of a given point in the model (e.g., a point on a girder, deck, cross-frame, etc.) is calculated for a unit load vertically applied to the deck surface at numerous locations, usually in a grid. Instead of presenting these responses in terms of the results of multiple iterative analyses, however, the responses are directly presented in terms of the maximum and minimum response. The influence surface approach to modeling live load effects thus allows focus on the maximum/minimum loading responses of the structure at given locations. The amount of output from an influence surface analysis is much less, and the designer can focus on the critical loading effects rather than spending substantial time collating thousands or millions of numbers to determine envelope results.

The specific influence surface approach and the associated algorithms used by different applications can vary significantly. Some applications position for critical effects, some will move a truck or group of trucks longitudinally along the bridge (influence surface), while others will move the same transversely. The computational effort required within the application is strongly dependent upon the type of algorithm used.

The value of an influence surface approach becomes apparent for the more complicated and involved levels of analysis discussed later.

4.3 Line Girder Analysis Using Classical Beam Analysis Methods

Straight, non-skewed bridges are ideal candidates for using classical beam analysis techniques such as the moment distribution method, the moment-area theorem, etc. These types of structures have a structural behavior that is reasonably modeled in one dimension only and can often be characterized simply by beam analysis. Typically, this is referred to as *line girder analysis*.

Classical beam analysis techniques have several advantages that make them attractive for specific uses. Many of the classical beam analysis techniques are based on direct equilibrium and are quite transparent, clearly illustrating the distribution of forces through a bridge. Most classical beam analysis techniques also are simple and quick to use, making them valuable for preliminary designs or as approximate tools for validating more complex analyses.

However, line girder analysis techniques should only be considered approximate tools for bridges with characteristics beyond the most basic geometry and framing. The simplifications and approximations involved in applying the classical beam analysis techniques to more complex bridges tend to reduce the accuracy of their results, particularly with regard to the prediction of structural deformations. Use of line girder analysis techniques should be limited to the design of relatively simple structures.

In addition, before undertaking an analysis using the classical beam analysis techniques by hand, designers are advised to consider the complications which may arise from the need to model moving live loads, as described in detail above. Similarly, designers should consider the complexity associated with performing the AASHTO specified capacity calculations at all potentially critical sections along the length of the girder. Performing these code checks by hand for strength, service, fatigue, deflection and other issues can be simple for shorter structures but may become a very tedious task for longer bridges.

Prudence is required in applying simple line girder analysis methods to bridges with complicated framing plans. While these simpler methods may be more transparent and easier to implement, the number of simplifying assumptions and approximations needed to adapt a line girder analysis to a complex framing plan may affect the accuracy and understanding of important behavior.

A full discussion of the classical beam analysis methods is not warranted in the Steel Bridge Design Handbook as these methods are documented in numerous textbooks.

4.4 Computerized Approaches Based on Classical Beam Analysis Methods

One way to address the complexity of live load modeling in a line girder analysis is to computerize the analysis calculations. There are numerous ways to incorporate computer technology into a line girder analysis. Usually, the extent of the design process covered in the computer application determines the effort required to create and use that tool. An application may be as simple as a spreadsheet that tabulates moments and shears for the beam, or it may be as complicated as a custom program which covers the entire design process, from generation of geometric parameters and cross section properties, quantification of framing considerations, calculations of shears, moments, stresses, and deflections, and comparisons to specification-derived capacities and other code provisions.

Spreadsheets can offer flexibility and control in line girder analyses. The limitations to the complexity and extent of the spreadsheet are based primarily on the time and money available and the skills of the designer in programming the spreadsheet. However, care should be exercised in using spreadsheets. As with any computer application, a spreadsheet can become a "black box" approach that is difficult to document, check, and interpret. As a result, quality control and validation processes should be carefully followed.

In addition, the multiple paths and decision points in the current AASHTO steel girder design provisions may prove to be difficult to program efficiently in a spreadsheet. Spreadsheets are efficient for calculating a large number of values using simple formulas but are much less efficient when used for calculating values that may result from complex formulas or processes often dependent on the value of one or more other variables with several conditional execution paths.

A more efficient approach to addressing the complexities in steel girder design may be found in applications that are developed using a formal programming language where spreadsheet constraints are removed. However, both significant computer science and engineering experience are required to address the complexities of the current steel girder design provisions. Applications for line girder analysis are most often commercially licensed or otherwise developed by specialized consultants. Several steel line girder design applications are available, both from commercial software companies and from various bridge owner-agencies (either for free or at a nominal cost).

In all cases, any computerized approach, whether a spreadsheet or a program, whether commercially purchased or "home-grown," should always be validated and spot-checked by independent calculations performed either by hand or by an independent computer tool. Most computer applications are complex tools and even minor programming errors can drastically affect the results of engineering computations. Details about validation of applications can be found in references [27, 28].

4.5 Approximate Modeling of Curvature Effects

A line girder analysis can also serve as the basis for an approximate curved girder analysis. First, the curved girder is modeled as a straight girder line, using developed span lengths (i.e., using span lengths measured along the curve arc length of that girder). Next, the effects calculated by the line girder analysis are increased by factors that account for the effects of curvature (i.e., for the "load shifting" that occurs in curved girder bridges where the girders on the outside of the curve carry more load than those on the inside of the bridge [6, 17].
Note that when the central angle is small enough, the AASHTO LRFD BDS [1] allow for the effects of curvature to be neglected in the calculation of gravity load effects in curved girder bridges. Whether the global effects of curvature are included or neglected, designers must always account for flange lateral bending effects in curved girders, regardless of how slight the curvature. When using approximate methods to address curvature, this is most easily done using flange lateral bending equations.

The most common expression of this equation is:

$$M_{Lat} = \frac{Md^2}{10Rh}$$

where:

M_{Lat} = Flange Lateral Bending Moment
M = Girder Primary (Vertical) Bending Moment
d = *Cross-frame* spacing
R = Radius of Curvature
h = Depth of the Girder

4.6 V-Load Method (for curved I-shaped girders)

The V-Load method is a technique for analysis of curved steel I-girder bridges originally developed in the 1960s by United States Steel. The V-Load method is based on free-body diagrams and static equilibrium equations, is readily learned and understood, and is still popular as a teaching tool since it clearly illustrates gravity load effects. A complete description of the V-Load method can be found in several references [6, 29].

The V-Load method gets its name from the shears in the cross-frames, the “V-Loads” in the analysis. The key free-body diagram in the V-Load method is of a cross-frame between two adjacent girders. The shear transferred across the cross-frame represents the load being shifted from the girder closer to the inside of the curve to the girder closer to the outside of the curve. The forces are balanced by a horizontal force couple directly associated with the flange lateral bending effects found in curved I-girders. While computer applications are still available to automate the V-Load method, it is primarily discussed herein from a historical perspective. The method has many limitations, including challenges addressing skew and composite systems, and has generally been superseded by more refined analyses.

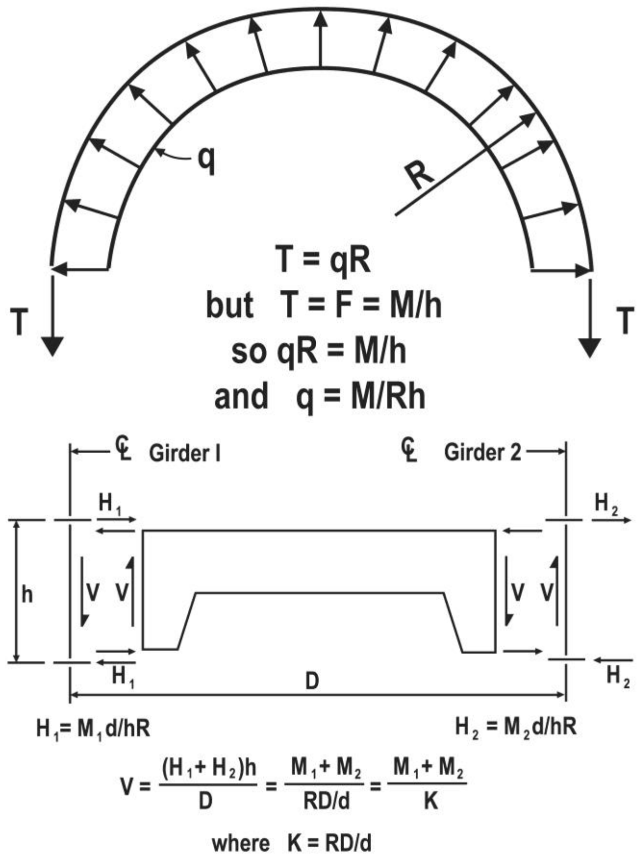


Figure 13 Basic free-body diagrams of the V-Load method [6] (shown in a positive moment region)

The V-Load method offers the advantage of a simple, transparent, method based only on equilibrium for assessing force effects in curved girder bridges. While theoretically applicable to complex framing plans (variable girder spacing, flares, bifurcated girders, etc.), using it on such structures is not recommended.

4.7 M/R Method (for curved box-shaped girders)

As mentioned above, the V-Load method is applicable to I-girders. The corresponding method for tub and box girders is the M/R method developed in 1970 by Tung and Fountain [30]. Like the V-Load method, it is an equilibrium-based technique.

The M/R method is most useful for analysis of single tub or box girders, making it applicable for erection analysis of single girders, complete analysis of a narrow bridge with only one tub girder in its cross section, or a single girder as part of a phased-construction plan. Because the M/R method also calculates tub girder rotations, theoretically, it is possible to use it to solve for loads in multiple adjacent girders. For practical purposes, however, the calculations for multiple girder systems are too cumbersome unless computer applications are used.

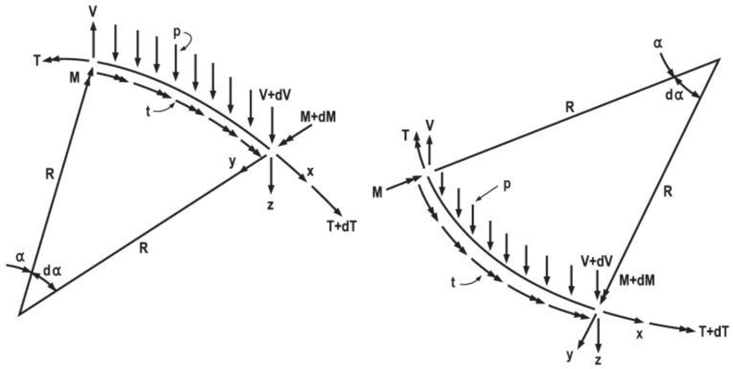


Figure 14 Basic free-body diagrams of the M/R method [30].

4.8 Computerized Approaches to the V-Load and M/R Methods

As mentioned above, commercially sold applications are available which automate the V-Load method.

In addition, it is feasible for designers to create their own “home-grown” applications to automate either the V-Load method or the M/R method. As with the development of any steel girder design software, the two most challenging aspects to creating an application are programming routines to generate the live load results and programming routines to perform structural capacity code checks. Typically, commercial V-Load-based applications are readily available for simpler curved I-girder bridges, as are 2-D and 3-D analysis tools for more complicated curved I-girder bridges. Meanwhile, the practical limitations of the M/R method and the inherent additional complexity of box-shaped girders suggest that 2-D and 3-D analysis tools may be more appropriate for most curved box-shaped girder designs.

What may be more appropriate for individual designers who are so inclined might be the development of simplified V-Load or M/R applications for use in preliminary design or for performing independent validation of more complicated analyses. Such efforts may be able to reasonably approximate live load results and might skip performing a full set of structural capacity code checks.

5.0 RIGOROUS ANALYSIS METHODS

The most commonly used rigorous analysis methods can be broadly divided into two categories: 2-D and 3-D analysis methods.

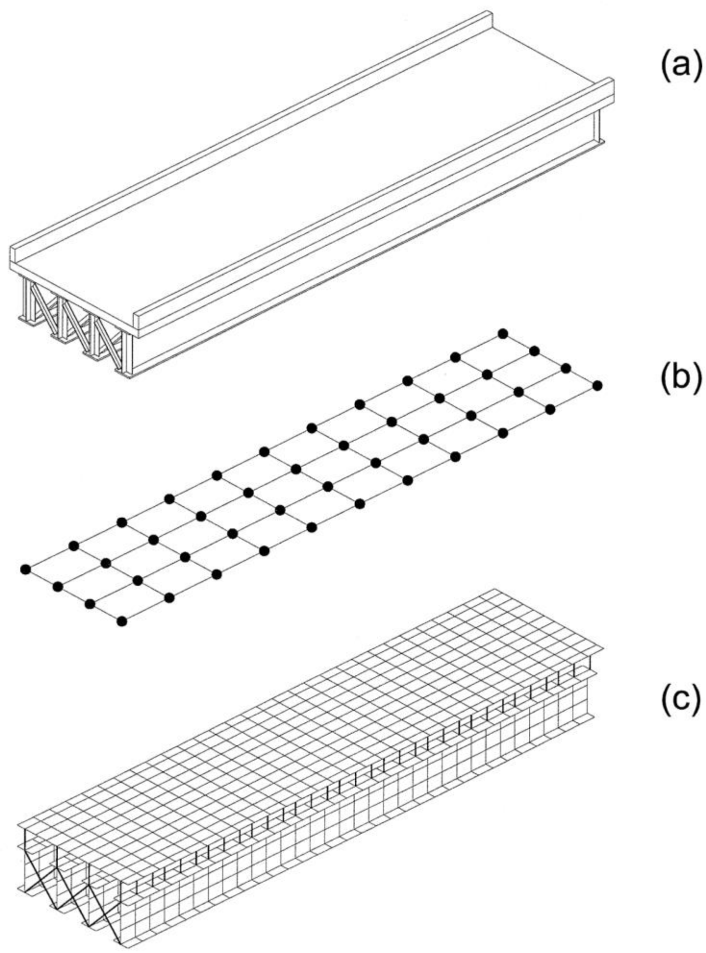


Figure 15 Rigorous analysis method models. (a) Actual bridge, (b) 2-D analysis model, (c) 3-D analysis model.

5.1 2-D "Grid" Analysis Methods

The “entry level” finite element model used by the building structural engineer is the plane frame. Plane frames are routinely used to model the behavior of regular structures for the distribution of gravity and lateral loads.

The “entry level” finite element model used by the bridge structural engineer is the plane grid for superstructures and plane frame for substructures. This approach is called a “grillage” or “grid” analysis. Space frames are routinely used as well. Figure 16 illustrates a typical two-node plane-frame element where each node has three degrees of freedom (two translations and a rotation).

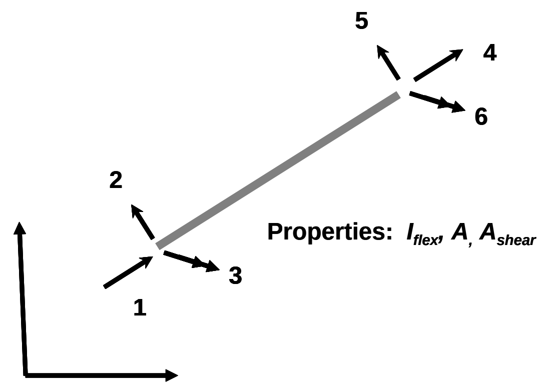


Figure 16 Plane Frame Element.

Results from grillage models can be used in several ways:

- Direct computation of live load envelopes and subsequent combination with dead loads for all load combinations. Downstream computation may include specification checking, etc.

- Indirect computation of a distribution factor for the subsequent use in a line girder program that then does the downstream computations and specification checking.

- Direct computation of live and/or dead loads for use in hand-calculation-based or other downstream processes.

Figure 17 illustrates the plane-grid element. This element typically has three degrees of freedom (one translation and two rotations). Some finite element programs may offer an additional degree of freedom to address warping of the element. The loads in a plane grid (a.k.a., grillage) are applied out of plane and the bending moment and torques are applied in plane. Variations exist. The plane grid is often used to model the deck, girders, and other transverse elements such as cross-frames, diaphragms, and also miscellaneous components such as barriers, rails, etc.

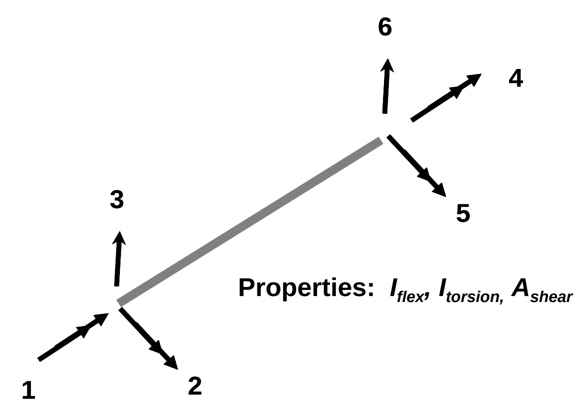


Figure 17 Plane Grid Element

The longitudinal elements typically represent the girders, while transverse elements represent the deck, transverse diaphragms and/or cross-frames. The stiffness of such is briefly outlined below.

5.1.1 Stiffness Modeling for Grid Analysis

FHWA's *Manual for Refined Analysis in Bridge Design and Evaluation* [3] offers an introductory reference on modeling with grillages. In addition, suggestions regarding the creation of meshes are paraphrased below [31]:

- Consider how the bridge behaves and place beam elements along lines of strength/stiffness, for example, parallel to girders, along edge beams and barriers.

- The total number of elements can vary widely. It can be one element in the longitudinal direction if the bridge is narrow and behaves similarly to a beam, or it can be numerous elements for the girders and other elements for the deck for wider bridges where the system is dominated by the behavior of the deck. Elements need not be spaced closer than two to three times the slab thickness.

The spacing of the transverse elements should be sufficiently small to distribute the effect of concentrated wheel loads and reactions. In the vicinity of such loads, the spacing can be decreased for improved results.

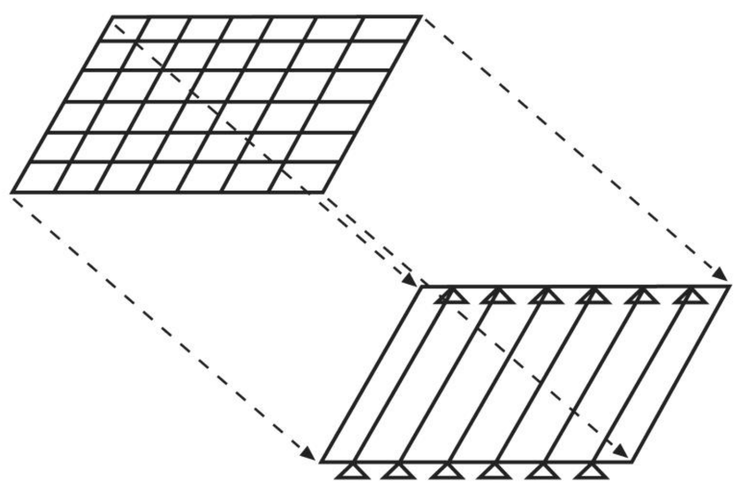


Figure 18 A slab-on-girder bridge with a possible grillage mesh.

For modeling the deck, the transverse element cross-sectional properties are usually based on the gross or uncracked section and are calculated on a per unit length basis. These properties are multiplied by the center-to-center spacing of the elements to obtain the element properties, called the tributary length. Two properties are required for the grillage model: flexural moment of inertia and the torsional constant. The moment of inertia is the familiar second moment of area, which is equal to

$$i_{deck} = \frac{bt^3}{12}$$

The deck transverse member torsional constant for a grillage element is

$$j_{deck} = \frac{bt^3}{6} = 2i_{deck}$$

The moment of inertia I_{girder} for a beam element is determined in the usual way, while its eccentricity e_g (for a composite beam) is accounted for by

$$I = I_{girder} + e_g^2 A_{girder} + I_{deck}$$

For noncomposite systems, e_g is zero, the beam is assumed to be at the middle surface of the deck, and the beam's axial stiffness does not contribute.

For open sections that are comprised of thin rectangular shapes (such as a wide flange or plate girder) in curved and/or skewed steel I-girder bridges with geometries that exceed specified "connectively index" and "skew index" values, Article 4.6.3.3.2 of the AASHTO LRFD BDS [1] requires that the warping rigidity of the I-girders be considered in 2-D analysis methods such as grid or PEB analysis. NCHRP Report 725 [32] presents an approximate method of considering the girder's warping rigidity. As will be discussed later, the warping rigidity is inherently addressed when using 3D modeling methods since the lateral stiffness of the flanges is addressed by either modeling the flanges using beam elements or using plate or shell elements.

Note that some grid analysis applications may not feature elements which model warping directly. In such cases, the effects of warping must be addressed outside the application.

For closed sections, such as box girders, see references on advanced mechanics of materials for procedures to compute the torsional constant. For such sections, the torsional stiffness is significant and should be carefully calculated.

Cross-frames, e.g, X- and K-frames, are typically modeled in 2-D analysis methods using a line (beam) element equivalent stiffness. Article 4.6.3.3.4 of the AASHTO LRFD BDS [1] requires that both cross-frame flexure and shear deformations be considered when determining the equivalent beam element stiffness, and the associated Commentary to this Article discusses available methods for achieving this. The same article also required that the influence of end connection eccentricities be considered in the calculation of the equivalent axial stiffness of single angle and flange-connected T-section cross-frame members.

5.1.2 Live Load Modeling for Grid Analysis

Some applications apply live load directly to the finite element model and create a new load case for each load placement. For more information, see the previous discussion regarding live load placement and number of possible load cases. Relatively accurate live load distribution to the girders can be achieved with a fairly coarse mesh; however, the direct application of concentrated wheel loads to the grillage is facilitated by a finer mesh where a node is more often close to the location of load application.

Alternatively, applications generate influence surfaces and direct load placement is on the surface. In this case, the application typically applies unit vertical loads to each node and computes the load effect throughout the system. In a subsequent computation the influence surface is used to compute the load effect. An example of an influence surface is illustrated in Figure 19.

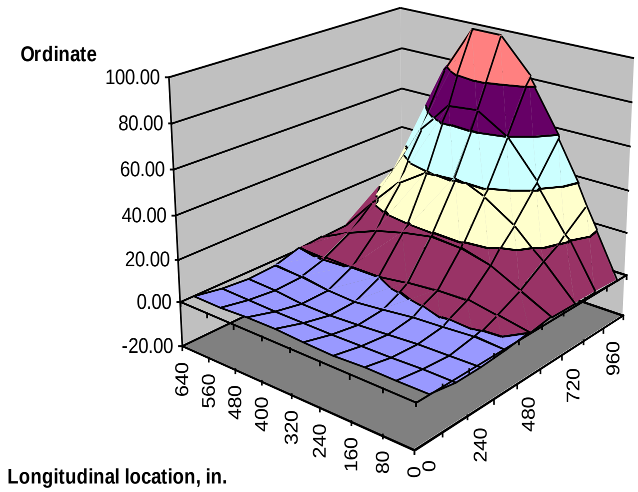


Figure 19 Example of an influence surface.

Note that most live loading routines place the load and interpolate the results based upon the surrounding nodes. For example, an area coordinate interpolation is illustrated in Figure 20. Area coordinates are described in finite analysis textbooks.

Either method is appropriate; each has advantages. With present computer speeds, computational effort for linear problems such as this is not of significant concern; rather the engineer's effort to model the bridge and interpret the results is.

5.1.3 Advantages and Disadvantages of Grid Analysis

Grid analysis has the advantage of being relatively simple and fast to perform, typically requiring less effort than a 3-D analysis of the same structure. Grid analysis often provides adequate refinement/accuracy for a wide range of structures.

Many engineers find grid analyses easier to understand than 3-D analyses as they are already comfortable with the plane frame analysis for pier, culverts, and other transportation-related structures. Furthermore, the flow of forces through the model is readily observed with shear, bending, and torsional moment diagrams that the software produces. Equilibrium is easily checked for the entire structure, at any cross section for the entire bridge, and for any element or joint. This is often not the case with 3-D finite element programs where shell or brick elements are used.

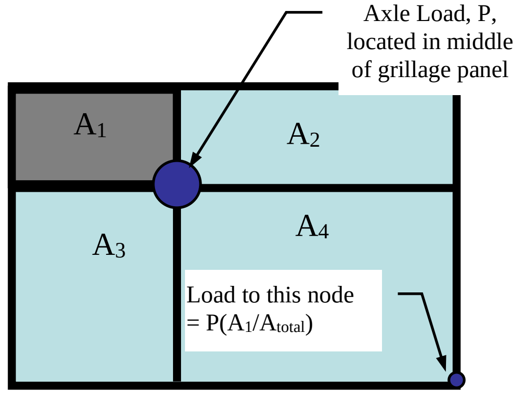


Figure 20 Illustration of the distribution of a wheel load to various nodes in a grid analysis using area coordinate interpolation.

Grillage modeling is well-suited for the determination of the load effects in a global context, such as determining loads carried by individual girders. However, grid analysis is less suited for determining actions in cross-frames, stresses or capacities in the deck, and local behavior where the deck and girder may share a load path into a support (joint, bearings).

Other disadvantages result from the fact that many simplifying assumptions are required, the most important of which is discretization of the deck (a continuum) into grid elements (frame elements) and condensing a cross-frame with up to four physical elements into one frame element. Some see the grillage method as dated because these types of assumptions are not required when the deck continuum can be modeled with shell or brick elements in increasingly accessible 3-D analysis.

More importantly, depending on the complexity of the structure under consideration, the simplifying assumptions and approximations involved in a grid analysis may result in an analysis which is not sufficiently refined or rigorous to adequately quantify girder deflections and rotations and load effects in individual elements such as cross-frame, diaphragms, bracing, or decks.

5.1.4 Limitations of Grid Analysis

Theoretically, virtually any steel girder bridge structure is a candidate for grid analysis. NCHRP Report 725 [32] and AASHTO/NSBA's *Guidelines for Steel Girder Bridge Analysis* [2] provide a quantitative "scorecard" approach to evaluating appropriate analysis methods (1-D vs. 2-D grid vs. 2-D PEB vs. 3-D) for various bridge geometries. In general terms, due to the disadvantages listed above, grid analysis generally is not recommended for the structures listed below:

- Bridges with more severe curvature and/or skew where load effects in bracing members are key design results, and where the accurate calculation of deflections, rotations and deformations becomes more significant. One definition of "significant" would be when deflections and rotations are large enough to warrant particular consideration in detailing and erection of girders, cross-frames, and bracing. In such cases, there is no "conservative" calculation of these values – overestimation or underestimation are equally unacceptable.

- Bridges with long span girders where relative stiffness effects become more important and where the accurate calculation of deflections, rotations and deformations becomes more significant (as described above).

- Bridges with deep girders, variable depth girders, or complex framing such as variable girder spacing, where the simplifications introduced in reducing the structural properties down to the single line element level (both for girders and for cross-frames) may lead to inaccurate results.

- Bridges with complex framing such as bifurcated girders, particularly when the girders bifurcate away from the end of a unit, where direct analysis of the load effects in all members may produce more accurate results than could be achieved by hand analysis of the framing at the location of bifurcation.

- Bridges with unusual or particularly complicated loading (e.g., perhaps the need to model a thermal gradient over the height of the girders) where the nature of the loading would require significant simplifying assumptions for application to a grid analysis.

5.1.5 Improvements to 2D-Grid Analysis for I-Girders

NCHRP Report 725 [32] offers a number of recommendations for improvements to steel girder bridge 2D-grid analysis methods. The improvements provide substantial benefits related to the accuracy of results, with little additional computational effort. These recommendations include:

- ***Improved I-girder Torsion Model for 2D-Grid Analysis***: Conventional use of just the St. Venant term (GJ/L) in characterizing the torsional stiffness of I-girders results in a significant underestimation of the actual girder torsional stiffness, due to the neglect of the girder warping stiffness (EC_w), which is substantial compared to the contribution of the St. Venant torsional stiffness (GJ) in I-girders. The work presented in NCHRP Report 725 [32] includes the development of an equivalent torsion constant J_{eq} which significantly improves the accuracy of 2D-Grid analyses for I-girders. This reference is implemented in AASHTO LRFD BDS [1] Article 4.6.3.3.2 and C4.6.3.3.2.

- ***Improved Equivalent Beam Cross-Frame Model***: Most 2D-Grid analysis software packages use approximations to model the stiffness of truss-type cross-frames using a beam-type element. These methods typically capture either only an approximate flexural stiffness or approximate shear stiffness of the entire cross-frame. Due to these stiffness approximations, an inaccurate representation of the stiffness of the system will result, and incorrect prediction of cross-frame forces, girder loads and deflections, etc. NCHRP Report 725 [32] demonstrates the potential improvements that could be achieved by using a shear-deformable (Timoshenko) beam element to represent the cross-frame, and describes the development of an "exact" equivalent beam element via the use of a user-defined element may be available in commercial frame analysis software program. This reference is implemented in AASHTO LRFD BDS [1] Article 4.6.3.3.4 and C4.6.3.3.4.

For a complete discussion of these recommendations, the designer is encouraged to review NCHRP Report 725 [32] , FHWA's *Manual for Refined Analysis in Bridge Design and Evaluation* [3], or as provided in the AASHTO/NSBA's *Guidelines for Steel Girder Bridge Analysis* [2].

5.2 3-D Analysis Methods

Typically, the most analytically complicated level of analysis is 3-D analysis. Like a grid analysis, a 3-D analysis is a finite element modeling technique. But instead of limiting the model to a 2-D grid of nodes and line elements, a 3-D analysis models the superstructure in detail (flanges, webs, deck, bracing members, etc.) in three dimensions.

Commercial applications are available to perform part or all of a 3-D analysis. Such applications are available which build a 3-D model of a steel girder bridge and run a live load analysis using either influence surface or live load generator methods. Commercial software modeling services exist which have the capability to build, run and post-process 3-D FEMs, including features that perform AASHTO LRFD BDS [1] checks on the girders and features that summarize the deflections, cross-frames forces and bearing reactions.

In addition, a 3-D analysis can be performed using any general-purpose finite element analysis program, including a number of commercially available applications. A significant amount of effort might be involved in building and post-processing the analysis model and performing AASHTO code checks on the girders, cross-frames and other elements — particularly considering 3-D shell models provide girder results in terms of flange and web stresses or forces, while many AASHTO design equations are written in terms of overall girder moments and shears – thus requiring post processing to sum elemental force effects on a line girder cross section.

One of the most applicable uses of shell elements is to model the deck, where the girders are modeled with shell and/or frame elements located eccentrically with respect to this deck. Shell elements can adequately model the load distribution to the girder and shear lag (effective flange) effects with finer meshes (e.g., [33]). However, modeling the deck stresses or flexural bending within a deck is difficult. Significant arching action, cracking, etc. cannot be properly modeled with most shell elements in readily available applications. For a detailed analysis of deck stresses, brick elements are likely required. Fortunately, such analyses are usually confined to research endeavors.

Three-dimensional modeling requires that quite a bit of consideration be given to numerous significant modeling issues. Several of these are described below.

5.2.1 Stiffness Modeling for 3-D Analysis

Typically, girder flanges are modeled using beam or plate/shell elements, webs are modeled using plate elements, and cross-frames and bracing are modeled using truss or plate elements (as appropriate for the given cross-frame or bracing configuration). Additionally, the deck typically is modeled using eight-node solid (brick) elements or four-node shell elements. Herein, the term shell elements shall be used to describe all elements which have both in-plane membrane and out-of-plane plate bending behavior.

The modeling of girder flanges using beam elements vs. using shell elements has been the subject of some debate among designers. For box-shaped girders, the bottom flange should be modeled using shell elements in order to correctly quantify the lateral and torsional stiffness of the cross section. For box-shaped girder top flanges and for I shaped girder top and bottom flanges though, there is a choice to be made. Using plate elements to model these flanges allows for direct calculation of in-plane stresses in detail. However, the value of this much detail in stress calculation is limited when using component-based design specification that require internal actions of shear, bending moment, etc. Beam elements can adequately model most, or all, of the required stiffness parameters of the flanges, and calculation of stresses from the flange force results is relatively simple.

Recently, more modern computer applications are permitting the use of shell elements complemented by a query at the section to compute the internal actions in terms of girder shears and moments. These applications can allow for simultaneous refinement of the modeling approach and practicality of addressing specification design equations.

Correct modeling of the deck is critical in 3-D analyses. Non-composite deck dead loads (primarily the weight of the wet concrete deck and haunch) are typically modeled either automatically or manually as distributed loads on the girders with load distribution based on simple span behavior in the deck. For the composite condition, the deck is typically modeled using 8-node brick elements, or sometimes using 4-node plate elements which have sufficient stiffness modeling provisions to capture all of the structural function of the deck, including in-plane axial stiffness and out-of-plane bending and shear stiffness as well as in-plane shear stiffness. Correct modeling of the axial stiffness – including accurate modeling of the offset of the deck from the top flange of the girder – is necessary to properly capture the vertical section properties of the composite girder cross-section. Correct modeling of the out-of-plane bending and shear stiffness is necessary to properly model the deck's critical function in transverse load distribution among the girders.

Consideration of connection details, particularly the end connections of cross-frame and bracing members is warranted. Automatically assuming pinned connections may not be appropriate; particularly if the members have large bending stiffness and if welded or slip-critical bolted connections with significant moment capacity are used. Likewise, cross-frame and bracing members typically comprise angle and tee shapes connected on one flange, imparting a local eccentricity into the bracing member. In lieu of a detailed investigation, the effective axial stiffness may be reduced to 65% of the nominal axial stiffness [1, 2, 3] to account for the end eccentricity of the bracing member. More recently, NCHRP Report 962 [26] addresses this is more detail, and the guidance will likely appear in a future AASHTO LRFD BDS.

The alignment of the neutral axes of the various cross-frame and bracing members is also important – significant eccentricities between these members and the girder flanges should be modeled as they can directly affect the calculated stiffness of, and loads in, the cross-frames or bracing.

Special consideration should be given to the modeling of bearings in 3-D analysis. The design should carefully consider not only the vertical stiffness of the bearings, but also the horizontal and rotational stiffness values of the bearings and their orientation. As previously mentioned, the modeling of bearing stiffness (in all directions) can have a significant influence on the behavior and load effects within the girders. consideration should be given to modeling substructures as part of the superstructure 3-D analysis. As previously mentioned, substructure stiffness can have significant effects on the behavior of the superstructure, particularly for seismic loading.

5.2.2 Load Modeling for 3-D Analysis

Much of the previous discussion on modeling of live loads, dead loads, and other loads is directly applicable to a discussion of 3-D analysis methods. There are no special considerations unique to 3-D analysis for the decisions as to whether to include consideration of staged deck placements, wind loads, thermal expansion/contraction, thermal gradients, or seismic loading in the superstructure analysis. It should be noted that 3-D analysis models lend themselves to more direct modeling of many of these effects. Simpler techniques require the designer to either use approximations and/or simplifications when applying these loads to the analysis or require the designer to manually add these loading effects to the analysis outside of the main analysis model.

5.2.3 Advantages and Disadvantages of 3-D Analysis

Because all of the components of the bridge are directly modeled, a 3-D analysis has the advantage of being rigorous and internally consistent among all portions of the structure. A 3-D analysis directly models the stiffness characteristics, and direct analysis results are available for all elements of the structure. Complex structural configurations are modeled in detail, rather than approximating the overall stiffness parameters with estimated single values. For example, to model a steel tub girder in a 3-D analysis, the bottom flange is modeled with separate elements, as are the two webs, the two top flanges, the internal cross-frames and the top flange lateral bracing. In contrast, in a grid analysis the entire tub girder (flanges, webs, internal cross-frames and top flange lateral bracing) is modeled as a single line element, with the stiffness of all associated complex framing approximated using simplified calculations or empirical estimates. This greater detail and rigor in a 3-D model theoretically leads to more accurate analysis results.

However, 3-D analysis involves much greater modeling effort than grid analysis. Additionally, the resulting model is significantly more complex, and as a result, an increased possibility exists that errors may inadvertently be introduced. Furthermore, the greater detail and greater volume of direct results for each and every element can be a two-edged sword. While there is value in having direct results for all elements of the structure, the sheer volume of the results can become overwhelming in terms of the required post-processing effort (since, e.g., results of interest, such as moments and shears, may not be directly available otherwise). Many designers find 3-D analysis results much less intuitive and harder to visualize and understand and validate. As a result, risk of inadvertent mistakes or misinterpreted analysis results is greater. In summary, an important question is associated with 3-D analysis: Is the greater implied accuracy and detail worth the effort?

5.2.4 Limitations of 3-D Analysis

There are virtually no limitations to the type or complexity of structures that can be modeled in a 3-D analysis. The limitations come down to time and money available to perform the analysis, which brings up a key concern. While theoretically more "accurate", a 3-D analysis is only as accurate as the assumptions made during the building and running of the analysis model. 3-D models are complex and, as mentioned above, it is easier to introduce errors in more complex analyses. While a 3-D analysis does offer an increase in "refinement" over simpler analysis methods, designers should not confuse "refinement" with "accuracy". An unfounded sense of confidence can be too easily gained – designers are encouraged to refer to a particularly apt article by Bakht and Jaeger [34].

5.3 Beyond Linear Elastic Behavior and Analysis

Up to this point, this module has focused solely on first order, linear elastic behavior and analysis. However, it is routinely accepted in modern bridge design to allow some level of inelastic behavior under certain circumstances. In fact, current AASHTO LRFD BDS [1] allow for redistribution of negative moment in steel girder bridges, which implicitly permits inelastic behavior.

Designers are cautioned to take advantage of inelastic behavior only with significant thought and consideration given to the associated need for appropriate structural analysis. A non-linear analysis should not be undertaken lightly as accounting for non-linear, second order effects greatly complicates what may already be a fairly complex analysis. Most commercial steel girder modeling software packages do not have provisions for direct assessment of inelastic behavior, and accounting for inelastic behavior using manual methods or "home-grown" software may prove to be a daunting task.

In many cases, it may be sufficient for designers to perform only a simple inelastic analysis outside of the main analysis to address cases where stresses exceed the yield stress of the material to some minor level in localized areas. Addressing significant inelastic behavior is beyond the scope of the Steel Bridge Design Handbook and should only be undertaken by experienced designers with access to appropriate analysis methods and analytical tools.

6.0 CLOSING REMARKS

The role of structural analysis in the design process is important, yet a broad perspective is necessary. Designers should take some comfort in the lower bound theorem for structural analysis [35]:

> *A load computed on the basis of an equilibrium moment distribution in which the moments are nowhere greater than the plastic moment is less than or equal to the true plastic limit load.*

In simpler terms, a structure designed to behave elastically, based on an analysis which satisfies all equilibrium requirements, will have some degree of inelastic redundancy.

Although stated in terms of bending moment, the lower bound theorem is valid for any type of action or stress. The essential requirements of the theorem are [31]:

> *Calculated internal actions and applied forces should be in equilibrium everywhere, and*
>
> *Materials and section/member behavior must be ductile; that is, the material must be able to yield without fracture or instability.*

In all analyses, the goal is to predict the actual response with sufficient accuracy while incurring only modest expense. Nonetheless, uncertainty of residual stresses, construction stresses, creep of concrete, deck shrinkage, etc. are often not considered in the structural modeling even though they are all present in the actual structure. The structural engineer routinely, and subtly, uses the lower-bound theorem with significant advantage because many load distribution mechanisms are not accounted for in analyses.

However, the lower-bound theorem does not address constructibility, deformations, serviceability and critical aspects of stability – all important reasons to attempt to model the response of the bridge as well as practically possible.

The keys to successful design include understanding the behavior of the structure, understanding the various analysis techniques available, and developing an awareness of the advantages and limitations of each approach. With this knowledge in hand, designers can better choose and execute the appropriate analysis method for each individual structure.

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